#### **Problem 1: Basics of Neural Networks**

- Learning Objective: In this problem, you are asked to implement a basic multilayer fully connected neural network from scratch, including forward and backward passes of certain essential layers, to perform an image classification task on the CIFAR100 dataset. You need to implement essential functions in different indicated python files under directory lib.
- **Provided Code:** We provide the skeletons of classes you need to complete. Forward checking and gradient checkings are provided for verifying your implementation as well.
- **TODOs:** You are asked to implement the forward passes and backward passes for standard layers and loss functions, various widely-used optimizers, and part of the training procedure. And finally we want you to train a network from scratch on your own. Also, there are inline questions you need to answer. See **README.md** to set up your environment.

```
In [ ]: from lib.mlp.fully_conn import *
        from lib.mlp.layer_utils import *
        from lib.datasets import *
        from lib.mlp.train import *
        from lib.grad_check import *
        from lib.optim import *
        import numpy as np
        import matplotlib.pyplot as plt
        %matplotlib inline
        plt.rcParams['figure.figsize'] = (10.0, 8.0) # set default size of plots
        plt.rcParams['image.interpolation'] = 'nearest'
        plt.rcParams['image.cmap'] = 'gray'
        # for auto-reloading external modules
        # see http://stackoverflow.com/questions/1907993/autoreload-of-modules-in
        %load ext autoreload
        %autoreload 2
```

The autoreload extension is already loaded. To reload it, use: %reload\_ext autoreload

# Loading the data (CIFAR-100 with 20 superclasses)

In this homework, we will be classifying images from the CIFAR-100 dataset into the 20 superclasses. More information about the CIFAR-100 dataset and the 20 superclasses can be found here.

Download the CIFAR-100 data files here, and save the \_\_mat files to the data/cifar100 directory.

Load the dataset.

```
In []: data = CIFAR100_data('data/cifar100/')
    for k, v in data.items():
        if type(v) == np.ndarray:
            print ("Name: {} Shape: {}, {}".format(k, v.shape, type(v)))
        else:
            print("{}: {}".format(k, v))
        label_names = data['label_names']
        mean_image = data['mean_image'][0]
        std_image = data['std_image'][0]
```

```
Name: data_train Shape: (40000, 32, 32, 3), <class 'numpy.ndarray'>
Name: labels_train Shape: (40000,), <class 'numpy.ndarray'>
Name: data_val Shape: (10000, 32, 32, 3), <class 'numpy.ndarray'>
Name: labels_val Shape: (10000,), <class 'numpy.ndarray'>
Name: data_test Shape: (10000, 32, 32, 3), <class 'numpy.ndarray'>
Name: labels_test Shape: (10000,), <class 'numpy.ndarray'>
label_names: ['aquatic_mammals', 'fish', 'flowers', 'food_containers', 'fruit_and_vegetables', 'household_electrical_devices', 'household_furni ture', 'insects', 'large_carnivores', 'large_man-made_outdoor_things', 'large_natural_outdoor_scenes', 'large_omnivores_and_herbivores', 'mediu m_mammals', 'non-insect_invertebrates', 'people', 'reptiles', 'small_mam mals', 'trees', 'vehicles_1', 'vehicles_2']
Name: mean_image Shape: (1, 1, 1, 3), <class 'numpy.ndarray'>
Name: std_image Shape: (1, 1, 1, 3), <class 'numpy.ndarray'>
```

#### Implement Standard Layers

You will now implement all the following standard layers commonly seen in a fully connected neural network (aka multi-layer perceptron, MLP). Please refer to the file <code>lib/mlp/layer\_utils.py</code>. Take a look at each class skeleton, and we will walk you through the network layer by layer. We provide results of some examples we precomputed for you for checking the forward pass, and also the gradient checking for the backward pass.

### FC Forward [2pt]

In the class skeleton flatten and fc in lib/mlp/layer\_utils.py , please complete the forward pass in function forward . The input to the fc layer may not be of dimension (batch size, features size), it could be an image or any higher dimensional data. We want to convert the input to have a shape of (batch size, features size). Make sure that you handle this dimensionality issue.

```
In []: %reload_ext autoreload

# Test the fc forward function
input_bz = 3 # batch size
input_dim = (7, 6, 4)
output_dim = 4

input_size = input_bz * np.prod(input_dim)
weight_size = output_dim * np.prod(input_dim)
```

Difference: 4.02601593296122e-09

### FC Backward [2pt]

Please complete the function backward as the backward pass of the flatten and fc layers. Follow the instructions in the comments to store gradients into the predefined dictionaries in the attributes of the class. Parameters of the layer are also stored in the predefined dictionary.

```
In [ ]: %reload ext autoreload
        # Test the fc backward function
        inp = np.random.randn(15, 2, 2, 3)
        w = np.random.randn(12, 15)
        b = np.random.randn(15)
        dout = np.random.randn(15, 15)
        flatten layer = flatten(name="flatten test")
        x = flatten layer.forward(inp)
        single_fc = fc(np.prod(x.shape[1:]), 15, init_scale=5e-2, name="fc_test")
        single_fc.params[single_fc.w_name] = w
        single_fc.params[single_fc.b_name] = b
        dx num = eval numerical gradient array(lambda x: single fc.forward(x), x,
        dw_num = eval_numerical_gradient_array(lambda w: single_fc.forward(x), w,
        db_num = eval_numerical_gradient_array(lambda b: single_fc.forward(x), b,
        out = single_fc.forward(x)
        dx = single fc.backward(dout)
        dw = single fc.grads[single fc.w name]
        db = single_fc.grads[single_fc.b_name]
        dinp = flatten_layer.backward(dx)
        # The error should be around 1e-9
        print("dx Error: ", rel_error(dx_num, dx))
        # The errors should be around 1e-10
```

```
print("dw Error: ", rel_error(dw_num, dw))
print("db Error: ", rel_error(db_num, db))
# The shapes should be same
print("dinp Shape: ", dinp.shape, inp.shape)

dx Error: 2.5392988113439708e-09
dw Error: 4.225718303852159e-09
db Error: 3.426033680946141e-11
dinp Shape: (15, 2, 2, 3) (15, 2, 2, 3)
```

### GeLU Forward [2pt]

In the class skeleton gelu in lib/mlp/layer\_utils.py , please complete the forward pass.

GeLU is a smooth version of ReLU and it's used in pre-training LLMs such as GPT-3 and BERT.

$$\mathrm{GeLU}(x) = x\Phi(x) pprox 0.5x(1+\mathrm{tanh}(\sqrt{2/\pi}(x+0.044715x^3)))$$

Where  $\Phi(x)$  is the CDF for standard Gaussian random variables. You should use the approximate version to compute forward and backward pass.

Difference: 1.8037541876132445e-08

### GeLU Backward [2pt]

Please complete the backward pass of the class gelu.

```
In []: %reload_ext autoreload

# Test the relu backward function
x = np.random.randn(15, 15)
dout = np.random.randn(*x.shape)
gelu_b = gelu(name="gelu_b")

dx_num = eval_numerical_gradient_array(lambda x: gelu_b.forward(x), x, do

out = gelu_b.forward(x)
dx = gelu_b.backward(dout)
```

```
# The error should not be larger than 1e-4, since we are using an approxi
print ("dx Error: ", rel_error(dx_num, dx))
```

dx Error: 7.964181048913853e-10

### **Dropout Forward [2pt]**

In the class dropout in lib/mlp/layer\_utils.py , please complete the forward pass.

Remember that the dropout is **only applied during training phase**, you should pay attention to this while implementing the function.

Important Note1: The probability argument input to the function is the "keep probability": probability that each activation is kept.

Important Note2: If the keep\_prob is set to 1, make it as no dropout.

```
In [ ]: %reload_ext autoreload
        x = np.random.randn(100, 100) + 5.0
        print ("----
        for p in [0, 0.25, 0.50, 0.75, 1]:
            dropout_f = dropout(keep_prob=p)
            out = dropout_f.forward(x, True)
            out test = dropout f.forward(x, False)
            # Mean of output should be similar to mean of input
            # Means of output during training time and testing time should be sim
            print ("Dropout Keep Prob = ", p)
            print ("Mean of input: ", x.mean())
            print ("Mean of output during training time: ", out.mean())
            print ("Mean of output during testing time: ", out_test.mean())
            print ("Fraction of output set to zero during training time: ", (out
            print ("Fraction of output set to zero during testing time: ", (out t
            print ("--
```

```
Dropout Keep Prob = 0
Mean of input: 4.997419378745024
Mean of output during training time: 4.997419378745024
Mean of output during testing time: 4.997419378745024
Fraction of output set to zero during training time: 0.0
Fraction of output set to zero during testing time: 0.0
Dropout Keep Prob = 0.25
Mean of input: 4.997419378745024
Mean of output during training time: 4.952777134045768
Mean of output during testing time: 4.997419378745024
Fraction of output set to zero during training time: 0.7519
Fraction of output set to zero during testing time: 0.0
Dropout Keep Prob = 0.5
Mean of input: 4.997419378745024
Mean of output during training time: 5.078228393829265
Mean of output during testing time: 4.997419378745024
Fraction of output set to zero during training time: 0.4925
Fraction of output set to zero during testing time: 0.0
Dropout Keep Prob = 0.75
Mean of input: 4.997419378745024
Mean of output during training time: 4.964557353998986
Mean of output during testing time: 4.997419378745024
Fraction of output set to zero during training time: 0.2545
Fraction of output set to zero during testing time: 0.0
Dropout Keep Prob = 1
Mean of input: 4.997419378745024
Mean of output during training time: 4.997419378745024
Mean of output during testing time: 4.997419378745024
Fraction of output set to zero during training time: 0.0
Fraction of output set to zero during testing time: 0.0
```

### **Dropout Backward [2pt]**

Please complete the backward pass. Again remember that the dropout is only applied during training phase, handle this in the backward pass as well.

```
In []: %reload_ext autoreload

x = np.random.randn(5, 5) + 5
dout = np.random.randn(*x.shape)

keep_prob = 0.75
dropout_b = dropout(keep_prob, seed=100)
out = dropout_b.forward(x, True, seed=1)
dx = dropout_b.backward(dout)
dx_num = eval_numerical_gradient_array(lambda xx: dropout_b.forward(xx, T

# The error should not be larger than 1e-10
print ('dx relative error: ', rel_error(dx, dx_num))
```

dx relative error: 3.0031126762278686e-11

### Testing cascaded layers: FC + GeLU [2pt]

Please find the TestFCGeLU function in lib/mlp/fully\_conn.py .

You only need to complete a few lines of code in the TODO block.

Please design an Flatten  $\rightarrow$  FC  $\rightarrow$  GeLU network where the parameters of them match the given x, w, and b.

Please insert the corresponding names you defined for each layer to param\_name\_w, and param\_name\_b respectively. Here you only modify the param\_name part, the \_w , and \_b are automatically assigned during network setup

```
In [ ]: %reload_ext autoreload
      x = np.random.randn(3, 5, 3) # the input features
      w = np.random.randn(15, 5) # the weight of fc layer
      b = np.random.randn(5) # the bias of fc layer
      dout = np.random.randn(3, 5) # the gradients to the output, notice the sh
      tiny_net = TestFCGeLU()
      # TODO: param name should be replaced accordingly #
      tiny_net.net.assign("fc1_w", w)
      tiny_net.net.assign("fc1_b", b)
      END OF YOUR CODE
      out = tiny_net.forward(x)
      dx = tiny_net.backward(dout)
      # TODO: param name should be replaced accordingly #
      dw = tiny_net.net.get_grads("fc1_w")
      db = tiny_net.net.get_grads("fc1_b")
      END OF YOUR CODE
      dx_num = eval_numerical_gradient_array(lambda x: tiny_net.forward(x), x,
      dw_num = eval_numerical_gradient_array(lambda w: tiny_net.forward(x), w,
      db_num = eval_numerical_gradient_array(lambda b: tiny_net.forward(x), b,
      # The errors should not be larger than 1e-7
      print ("dx error: ", rel_error(dx_num, dx))
print ("dw error: ", rel_error(dw_num, dw))
      print ("db error: ", rel_error(db_num, db))
      dx error: 9.046975743330595e-09
      dw error: 5.087598846422757e-08
```

### SoftMax Function and Loss Layer [2pt]

db error: 1.6245664279883916e-09

In the <code>lib/mlp/layer\_utils.py</code>, please first complete the function <code>softmax</code>, which will be used in the function <code>cross\_entropy</code>. Then, implement <code>corss\_entropy</code> using <code>softmax</code>. Please refer to the lecture slides of the mathematical expressions of the cross entropy loss function, and complete its forward pass and backward pass. You should also take care of <code>size\_average</code> on whether or not to divide by the batch size.

```
In []: %reload_ext autoreload
    num_classes, num_inputs = 6, 100
    x = 0.001 * np.random.randn(num_inputs, num_classes)
    y = np.random.randint(num_classes, size=num_inputs)

    test_loss = cross_entropy()

    dx_num = eval_numerical_gradient(lambda x: test_loss.forward(x, y), x, veloss = test_loss.forward(x, y)
    dx = test_loss.backward()

# Test softmax_loss function. Loss should be around 1.792
# and dx error should be at the scale of 1e-8 (or smaller)
    print ("Cross Entropy Loss: ", loss)
    print ("dx error: ", rel_error(dx_num, dx))
# print(dx_num[:10])
# print(dx[:10])
```

Cross Entropy Loss: 1.7917287629103629 dx error: 7.914942159348867e-09

### Test a Small Fully Connected Network [2pt]

Please find the SmallFullyConnectedNetwork function in lib/mlp/fully\_conn.py .

Again you only need to complete few lines of code in the TODO block.

Please design an FC --> GeLU --> FC network where the shapes of parameters match the given shapes.

Please insert the corresponding names you defined for each layer to param\_name\_w, and param\_name\_b respectively.

Here you only modify the param\_name part, the \_w , and \_b are automatically assigned during network setup.

```
In []: %reload_ext autoreload

seed = 1234
np.random.seed(seed=seed)

model = SmallFullyConnectedNetwork()
loss_func = cross_entropy()

N, D, = 4, 4 # N: batch size, D: input dimension
H, C = 30, 7 # H: hidden dimension, C: output dimension
std = 0.02
x = np.random.randn(N, D)
```

```
v = np.random.randint(C, size=N)
print ("Testing initialization ... ")
# TODO: param name should be replaced accordingly #
w1_std = abs(model.net.get_params("fc1_w").std() - std)
b1 = model.net.get_params("fc1_b").std()
w2 std = abs(model.net.get params("fc2 w").std() - std)
b2 = model.net.get params("fc2 b").std()
END OF YOUR CODE
assert w1_std < std / 10, "First layer weights do not seem right"</pre>
assert np.all(b1 == 0), "First layer biases do not seem right"
assert w2_std < std / 10, "Second layer weights do not seem right"</pre>
assert np.all(b2 == 0), "Second layer biases do not seem right"
print ("Passed!")
print ("Testing test-time forward pass ... ")
w1 = np.linspace(-0.7, 0.3, num=D*H).reshape(D, H)
w2 = np.linspace(-0.2, 0.2, num=H*C).reshape(H, C)
b1 = np.linspace(-0.6, 0.2, num=H)
b2 = np.linspace(-0.9, 0.1, num=C)
# TODO: param name should be replaced accordingly #
model.net.assign("fc1_w", w1)
model.net.assign("fc1_b", b1)
model.net.assign("fc2 w", w2)
model.net.assign("fc2_b", b2)
END OF YOUR CODE
feats = np.linspace(-5.5, 4.5, num=N*D).reshape(D, N).T
scores = model.forward(feats)
correct_scores = np.asarray([[-2.33881897, -1.92174121, -1.50466344, -1.0])
                        [-1.57214916, -1.1857013, -0.79925345, -0.4]
                        [-0.80178618, -0.44604469, -0.0903032, 0.2]
                        [-0.00331319, 0.32124836, 0.64580991, 0.9]
scores_diff = np.sum(np.abs(scores - correct_scores))
assert scores_diff < 1e-6, "Your implementation might be wrong!"</pre>
print ("Passed!")
print ("Testing the loss ...",)
y = np.asarray([0, 5, 1, 4])
loss = loss_func.forward(scores, y)
dLoss = loss_func.backward()
correct_loss = 2.4248995879903195
assert abs(loss - correct_loss) < 1e-10, "Your implementation might be wr</pre>
print ("Passed!")
print ("Testing the gradients (error should be no larger than 1e-6) ...")
din = model.backward(dLoss)
for layer in model.net.layers:
   if not layer.params:
```

```
continue
    for name in sorted(layer.grads):
        f = lambda _: loss_func.forward(model.forward(feats), y)
        grad_num = eval_numerical_gradient(f, layer params[name], verbose
        print ('%s relative error: %.2e' % (name, rel_error(grad_num, lay
Testing initialization ...
Passed!
Testing test-time forward pass ...
Passed!
Testing the loss ...
Passed!
Testing the gradients (error should be no larger than 1e-6) ...
fc1_b relative error: 4.59e-09
fc1 w relative error: 6.79e-09
fc2_b relative error: 4.01e-10
fc2_w relative error: 2.50e-08
```

## Test a Fully Connected Network regularized with Dropout [2pt]

Please find the DropoutNet function in fully\_conn.py under lib/mlp directory.

For this part you don't need to design a new network, just simply run the following test code.

If something goes wrong, you might want to double check your dropout implementation.

```
In [ ]: %reload ext autoreload
        seed = 1234
        np.random.seed(seed=seed)
        N, D, C = 3, 15, 10
        X = np.random.randn(N, D)
        y = np.random.randint(C, size=(N,))
        for keep prob in [0, 0.25, 0.5]:
            np.random.seed(seed=seed)
            print ("Dropout p =", keep_prob)
            model = DropoutNet(keep_prob=keep_prob, seed=seed)
            loss_func = cross_entropy()
            output = model.forward(X, True, seed=seed)
            loss = loss func.forward(output, y)
            dLoss = loss func.backward()
            dX = model.backward(dLoss)
            grads = model.net.grads
            print ("Error of gradients should be around or less than 1e-3")
            for name in sorted(grads):
                if name not in model.net.params.keys():
                    continue
                f = lambda _: loss_func.forward(model.forward(X, True, seed=seed)
                grad_num = eval_numerical_gradient(f, model.net.params[name], ver
                print ("{} relative error: {}".format(name, rel_error(grad_num, g
            print ()
```

Dropout p = 0

```
Error of gradients should be around or less than 1e-3
fc1_b relative error: 9.824168401316639e-08
fc1_w relative error: 4.706355839616906e-06
fc2 b relative error: 1.1334028569620202e-08
fc2_w relative error: 3.167223148481584e-05
fc3_b relative error: 2.05181811870711e-10
fc3_w relative error: 2.0977137288159317e-06
Dropout p = 0.25
Error of gradients should be around or less than 1e-3
fc1 b relative error: 1.894959190154108e-07
fc1_w relative error: 3.428714290278309e-06
fc2_b relative error: 1.6435765943706225e-07
fc2_w relative error: 4.5207268168886446e-05
fc3 b relative error: 2.1474160887299336e-10
fc3 w relative error: 7.9382903586546e-07
Dropout p = 0.5
Error of gradients should be around or less than 1e-3
fc1_b relative error: 3.83711938852427e-07
fc1 w relative error: 4.6044287598007907e-07
fc2 b relative error: 1.7902142485966373e-08
fc2 w relative error: 9.01845983573985e-06
fc3_b relative error: 3.285178756580047e-10
fc3 w relative error: 1.1129125846844918e-05
```

### Training a Network

In this section, we defined a TinyNet class for you to fill in the TODO block in lib/mlp/fully\_conn.py .

- Here please design a two layer fully connected network with Leaky ReLU activation (Flatten --> FC --> GeLU --> FC).
- You can adjust the number of hidden neurons, batch\_size, epochs, and learning rate decay parameters.
- Please read the lib/train.py carefully and complete the TODO blocks in the train\_net function first. Codes in "Test a Small Fully Connected Network" can be helpful.
- Implement SGD in lib/optim.py , you will be asked to complete weight decay and Adam in the later sections.

```
In []: # Arrange the data
data_dict = {
    "data_train": (data["data_train"], data["labels_train"]),
    "data_val": (data["data_val"], data["labels_val"]),
    "data_test": (data["data_test"], data["labels_test"])
}

In []: print("Data shape:", data["data_train"].shape)
    print("Flattened data input size:", np.prod(data["data_train"].shape[1:])
    print("Number of data classes:", max(data['labels_train']) + 1)
```

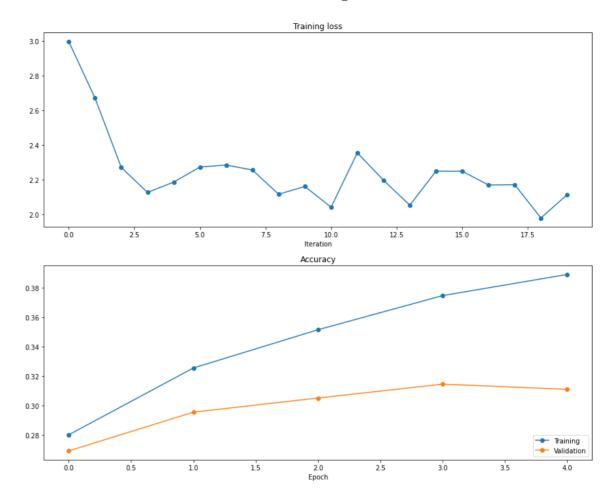
Data shape: (40000, 32, 32, 3) Flattened data input size: 3072 Number of data classes: 20

## Now train the network to achieve at least 30% validation accuracy [5pt]

You may only adjust the hyperparameters inside the TODO block

```
In [ ]: %autoreload
In [ ]: %reload_ext autoreload
      seed = 123
      np.random.seed(seed=seed)
      model = TinyNet()
      loss_f = cross_entropy()
      optimizer = SGD(model.net, 0.1)
       results = None
      # TODO: Use the train net function you completed to train a network
      batch size = 100
      epochs = 5
       lr decay = 0.99
      lr decay every = 100
      ***
                               END OF YOUR CODE
      results = train_net(data_dict, model, loss_f, optimizer, batch_size, epoc
                       lr_decay, lr_decay_every, show_every=10000, verbose=T
      opt_params, loss_hist, train_acc_hist, val_acc_hist = results
                   | 6/400 [00:00<00:34, 11.27it/s]
       (Iteration 1 / 2000) Average loss: 2.995578984299478
             400/400 [00:08<00:00, 49.76it/s]
       (Epoch 1 / 5) Training Accuracy: 0.28035, Validation Accuracy: 0.2694
      100% | 400/400 [00:05<00:00, 77.48it/s]
       (Epoch 2 / 5) Training Accuracy: 0.325675, Validation Accuracy: 0.2957
                  400/400 [00:06<00:00, 59.34it/s]
       (Epoch 3 / 5) Training Accuracy: 0.35165, Validation Accuracy: 0.3053
                   | 5/400 [00:00<00:20, 19.26it/s]/Users/pckraftwrek/Librar
      y/Mobile Documents/com~apple~CloudDocs/USC/DL/Assignment-1/csci566-assig
      nment1/lib/mlp/layer_utils.py:247: RuntimeWarning: overflow encountered
        sech_{term} = 1 / np.cosh(np.sqrt(2 / np.pi) * (x + 0.044715 * x**3))
      100% | 400/400 [00:06<00:00, 62.48it/s]
       (Epoch 4 / 5) Training Accuracy: 0.3748, Validation Accuracy: 0.3147
      100% | 400/400 [00:05<00:00, 70.60it/s]
       (Epoch 5 / 5) Training Accuracy: 0.389175, Validation Accuracy: 0.3112
In [ ]: # Take a look at what names of params were stored
```

```
print (opt_params.keys())
        dict_keys(['fc1_w', 'fc1_b', 'fc2_w', 'fc2_b'])
In [ ]:
       # Demo: How to load the parameters to a newly defined network
        model = TinyNet()
        model.net.load(opt_params)
        val_acc = compute_acc(model, data["data_val"], data["labels_val"])
        print ("Validation Accuracy: {}%".format(val_acc*100))
        test_acc = compute_acc(model, data["data_test"], data["labels_test"])
        print ("Testing Accuracy: {}%".format(test_acc*100))
        Loading Params: fc1_w Shape: (3072, 128)
        Loading Params: fc1_b Shape: (128,)
        Loading Params: fc2_w Shape: (128, 20)
        Loading Params: fc2 b Shape: (20,)
        Validation Accuracy: 31.11999999999997%
        Testing Accuracy: 30.9%
In [ ]: # Plot the learning curves
        plt.subplot(2, 1, 1)
        plt.title('Training loss')
        loss_hist_ = loss_hist[1::100] # sparse the curve a bit
        plt.plot(loss_hist_, '-o')
        plt.xlabel('Iteration')
        plt.subplot(2, 1, 2)
        plt.title('Accuracy')
        plt.plot(train_acc_hist, '-o', label='Training')
        plt.plot(val_acc_hist, '-o', label='Validation')
        plt.xlabel('Epoch')
        plt.legend(loc='lower right')
        plt.gcf().set size inches(15, 12)
        plt.show()
```



### Different Optimizers and Regularization Techniques

There are several more advanced optimizers than vanilla SGD, and there are many regularization tricks. You'll implement them in this section. Please complete the TODOs in the lib/optim.py.

### SGD + Weight Decay [2pt]

The update rule of SGD plus weigh decay is as shown below:

$$\theta_{t+1} = \theta_t - \eta \nabla_{\theta} J(\theta_t) - \lambda \theta_t$$

Update the SGD() function in lib/optim.py, and also incorporate weight decay options.

```
In []: %reload_ext autoreload

# Test the implementation of SGD with Momentum
seed = 1234
np.random.seed(seed=seed)

N, D = 4, 5
test_sgd = sequential(fc(N, D, name="sgd_fc"))

w = np.linspace(-0.4, 0.6, num=N*D).reshape(N, D)
dw = np.linspace(-0.6, 0.4, num=N*D).reshape(N, D)
```

The following errors should be around or less than 1e-6 updated\_w error: 8.677112905190533e-08

### Comparing SGD and SGD with Weight Decay [2pt]

Run the following code block to train a multi-layer fully connected network with both SGD and SGD plus Weight Decay. You are expected to see Weight Decay have better validation accuracy than vinilla SGD.

```
In [ ]: seed = 1234
        # Arrange a small data
        num train = 20000
        small_data_dict = {
            "data_train": (data["data_train"][:num_train], data["labels_train"][:
            "data_val": (data["data_val"], data["labels_val"]),
            "data_test": (data["data_test"], data["labels_test"])
        }
        reset_seed(seed=seed)
        model_sgd
                    = FullyConnectedNetwork()
                       = cross entropy()
        loss f sqd
        optimizer_sgd = SGD(model_sgd.net, 0.01)
        print ("Training with Vanilla SGD...")
        results_sgd = train_net(small_data_dict, model_sgd, loss_f_sgd, optimizer
                                max_epochs=50, show_every=10000, verbose=True)
        reset seed(seed=seed)
                     = FullyConnectedNetwork()
        model_sgdw
        loss f sqdw
                     = cross entropy()
        optimizer_sgdw = SGD(model_sgdw.net, 0.01, 1e-4)
        print ("\nTraining with SGD plus Weight Decay...")
        results_sgdw = train_net(small_data_dict, model_sgdw, loss_f_sgdw, optimi
                                 max_epochs=50, show_every=10000, verbose=True)
        opt_params_sgd, loss_hist_sgd, train_acc_hist_sgd, val_acc_hist_sgd =
        opt_params_sgdw, loss_hist_sgdw, train_acc_hist_sgdw, val_acc_hist_sgdw =
```

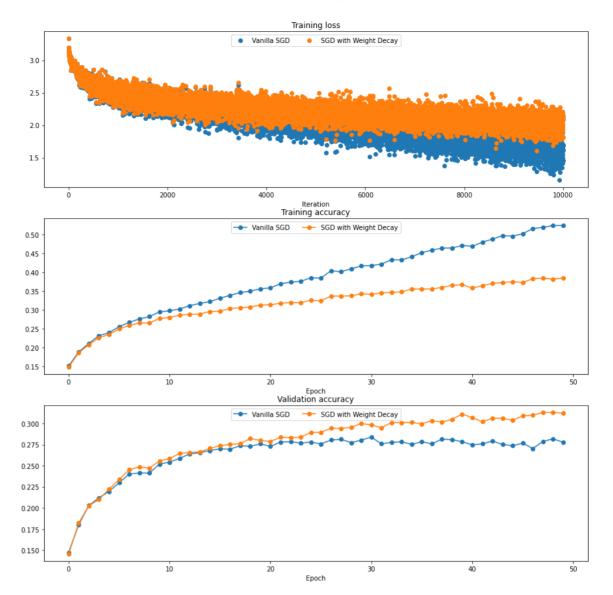
```
plt.subplot(3, 1, 1)
plt.title('Training loss')
plt.xlabel('Iteration')
plt.subplot(3, 1, 2)
plt.title('Training accuracy')
plt.xlabel('Epoch')
plt.subplot(3, 1, 3)
plt.title('Validation accuracy')
plt.xlabel('Epoch')
plt.subplot(3, 1, 1)
plt.plot(loss_hist_sgd, 'o', label="Vanilla SGD")
plt.subplot(3, 1, 2)
plt.plot(train_acc_hist_sgd, '-o', label="Vanilla SGD")
plt.subplot(3, 1, 3)
plt.plot(val_acc_hist_sgd, '-o', label="Vanilla SGD")
plt.subplot(3, 1, 1)
plt.plot(loss_hist_sgdw, 'o', label="SGD with Weight Decay")
plt.subplot(3, 1, 2)
plt.plot(train_acc_hist_sgdw, '-o', label="SGD with Weight Decay")
plt.subplot(3, 1, 3)
plt.plot(val_acc_hist_sgdw, '-o', label="SGD with Weight Decay")
for i in [1, 2, 3]:
 plt.subplot(3, 1, i)
 plt.legend(loc='upper center', ncol=4)
plt.gcf().set_size_inches(15, 15)
plt.show()
Training with Vanilla SGD...
             | 7/200 [00:00<00:13, 13.98it/s]
(Iteration 1 / 10000) Average loss: 3.3332154539088985
100%| 200/200 [00:07<00:00, 27.54it/s]
(Epoch 1 / 50) Training Accuracy: 0.15095, Validation Accuracy: 0.1474
100% | 200/200 [00:03<00:00, 53.25it/s]
(Epoch 2 / 50) Training Accuracy: 0.18815, Validation Accuracy: 0.1805
100% | 200/200 [00:03<00:00, 61.04it/s]
(Epoch 3 / 50) Training Accuracy: 0.2107, Validation Accuracy: 0.2029
       200/200 [00:14<00:00, 13.93it/s]
(Epoch 4 / 50) Training Accuracy: 0.2314, Validation Accuracy: 0.212
100% | 200/200 [00:03<00:00, 53.62it/s]
(Epoch 5 / 50) Training Accuracy: 0.23915, Validation Accuracy: 0.2197
       200/200 [00:03<00:00, 60.89it/s]
(Epoch 6 / 50) Training Accuracy: 0.2552, Validation Accuracy: 0.2298
             200/200 [00:03<00:00, 57.62it/s]
(Epoch 7 / 50) Training Accuracy: 0.26645, Validation Accuracy: 0.2403
100% | 200/200 [00:03<00:00, 62.14it/s]
(Epoch 8 / 50) Training Accuracy: 0.27555, Validation Accuracy: 0.2414
100% | 200/200 [00:03<00:00, 57.63it/s]
(Epoch 9 / 50) Training Accuracy: 0.28185, Validation Accuracy: 0.2413
100% | 200/200 [00:03<00:00, 66.18it/s]
(Epoch 10 / 50) Training Accuracy: 0.2944, Validation Accuracy: 0.252
       200/200 [00:07<00:00, 27.69it/s]
(Epoch 11 / 50) Training Accuracy: 0.29735, Validation Accuracy: 0.2543
```

```
100% | 200/200 [00:06<00:00, 32.12it/s]
(Epoch 12 / 50) Training Accuracy: 0.3021, Validation Accuracy: 0.2587
100%| 200/200 [00:04<00:00, 46.54it/s]
(Epoch 13 / 50) Training Accuracy: 0.31105, Validation Accuracy: 0.2641
         200/200 [00:04<00:00, 47.10it/s]
(Epoch 14 / 50) Training Accuracy: 0.3168, Validation Accuracy: 0.2653
       200/200 [00:04<00:00, 42.93it/s]
(Epoch 15 / 50) Training Accuracy: 0.3217, Validation Accuracy: 0.2681
       200/200 [00:03<00:00, 51.77it/s]
(Epoch 16 / 50) Training Accuracy: 0.3307, Validation Accuracy: 0.2699
        200/200 [00:03<00:00, 51.53it/s]
(Epoch 17 / 50) Training Accuracy: 0.33835, Validation Accuracy: 0.2696
100% | 200/200 [00:03<00:00, 55.31it/s]
(Epoch 18 / 50) Training Accuracy: 0.34565, Validation Accuracy: 0.2737
100% | 200/200 [00:03<00:00, 50.72it/s]
(Epoch 19 / 50) Training Accuracy: 0.3495, Validation Accuracy: 0.2729
            200/200 [00:03<00:00, 53.03it/s]
(Epoch 20 / 50) Training Accuracy: 0.35565, Validation Accuracy: 0.2758
       200/200 [00:05<00:00, 37.54it/s]
(Epoch 21 / 50) Training Accuracy: 0.35825, Validation Accuracy: 0.2729
100% | 200/200 [00:05<00:00, 38.20it/s]
(Epoch 22 / 50) Training Accuracy: 0.36895, Validation Accuracy: 0.278
100% | 200/200 [00:03<00:00, 52.26it/s]
(Epoch 23 / 50) Training Accuracy: 0.3734, Validation Accuracy: 0.2783
         200/200 [00:03<00:00, 52.19it/s]
(Epoch 24 / 50) Training Accuracy: 0.3756, Validation Accuracy: 0.2768
100%| 200/200 [00:03<00:00, 52.27it/s]
(Epoch 25 / 50) Training Accuracy: 0.38495, Validation Accuracy: 0.278
100%| 200/200 [00:05<00:00, 37.02it/s]
(Epoch 26 / 50) Training Accuracy: 0.38415, Validation Accuracy: 0.2757
         200/200 [00:04<00:00, 45.97it/s]
(Epoch 27 / 50) Training Accuracy: 0.40365, Validation Accuracy: 0.2804
100% | 200/200 [00:04<00:00, 48.25it/s]
(Epoch 28 / 50) Training Accuracy: 0.40105, Validation Accuracy: 0.2812
     200/200 [00:03<00:00, 50.68it/s]
(Epoch 29 / 50) Training Accuracy: 0.40885, Validation Accuracy: 0.2773
100% | 200/200 [00:03<00:00, 51.42it/s]
(Epoch 30 / 50) Training Accuracy: 0.4163, Validation Accuracy: 0.2803
100% | 200/200 [00:03<00:00, 52.95it/s]
(Epoch 31 / 50) Training Accuracy: 0.41745, Validation Accuracy: 0.2838
100% | 200/200 [00:03<00:00, 52.21it/s]
(Epoch 32 / 50) Training Accuracy: 0.42125, Validation Accuracy: 0.2758
            200/200 [00:03<00:00, 52.88it/s]
(Epoch 33 / 50) Training Accuracy: 0.433, Validation Accuracy: 0.2777
            200/200 [00:03<00:00, 54.17it/s]
(Epoch 34 / 50) Training Accuracy: 0.4322, Validation Accuracy: 0.2782
100% | 200/200 [00:05<00:00, 35.08it/s]
(Epoch 35 / 50) Training Accuracy: 0.44095, Validation Accuracy: 0.2753
100% | 200/200 [00:04<00:00, 49.69it/s]
(Epoch 36 / 50) Training Accuracy: 0.4517, Validation Accuracy: 0.2783
100% | 200/200 [00:03<00:00, 51.83it/s]
(Epoch 37 / 50) Training Accuracy: 0.4583, Validation Accuracy: 0.2759
```

```
100% | 200/200 [00:03<00:00, 56.39it/s]
(Epoch 38 / 50) Training Accuracy: 0.4637, Validation Accuracy: 0.2815
100%| 200/200 [00:03<00:00, 55.54it/s]
(Epoch 39 / 50) Training Accuracy: 0.4642, Validation Accuracy: 0.2808
         200/200 [00:03<00:00, 55.72it/s]
(Epoch 40 / 50) Training Accuracy: 0.47055, Validation Accuracy: 0.2784
       200/200 [00:03<00:00, 55.67it/s]
(Epoch 41 / 50) Training Accuracy: 0.4684, Validation Accuracy: 0.2747
       200/200 [00:04<00:00, 40.94it/s]
(Epoch 42 / 50) Training Accuracy: 0.4795, Validation Accuracy: 0.2758
        200/200 [00:09<00:00, 20.17it/s]
(Epoch 43 / 50) Training Accuracy: 0.48745, Validation Accuracy: 0.2793
100% | 200/200 [00:04<00:00, 49.83it/s]
(Epoch 44 / 50) Training Accuracy: 0.49715, Validation Accuracy: 0.2751
100% | 200/200 [00:03<00:00, 56.15it/s]
(Epoch 45 / 50) Training Accuracy: 0.49545, Validation Accuracy: 0.2736
            200/200 [00:03<00:00, 56.24it/s]
(Epoch 46 / 50) Training Accuracy: 0.50175, Validation Accuracy: 0.2767
       200/200 [00:03<00:00, 55.62it/s]
(Epoch 47 / 50) Training Accuracy: 0.51565, Validation Accuracy: 0.2704
100% | 200/200 [00:04<00:00, 45.22it/s]
(Epoch 48 / 50) Training Accuracy: 0.51875, Validation Accuracy: 0.2786
100% | 200/200 [00:04<00:00, 42.92it/s]
(Epoch 49 / 50) Training Accuracy: 0.5235, Validation Accuracy: 0.2818
       200/200 [00:05<00:00, 34.56it/s]
(Epoch 50 / 50) Training Accuracy: 0.52375, Validation Accuracy: 0.2778
Training with SGD plus Weight Decay...
              | 3/200 [00:00<00:27, 7.09it/s]
(Iteration 1 / 10000) Average loss: 3.3332154539088985
        200/200 [00:07<00:00, 26.04it/s]
(Epoch 1 / 50) Training Accuracy: 0.148, Validation Accuracy: 0.1458
       200/200 [00:05<00:00, 34.01it/s]
(Epoch 2 / 50) Training Accuracy: 0.186, Validation Accuracy: 0.1822
100% | 200/200 [00:06<00:00, 30.72it/s]
(Epoch 3 / 50) Training Accuracy: 0.2073, Validation Accuracy: 0.2027
100% | 200/200 [00:03<00:00, 60.68it/s]
(Epoch 4 / 50) Training Accuracy: 0.22575, Validation Accuracy: 0.2101
100%| 200/200 [00:03<00:00, 57.59it/s]
(Epoch 5 / 50) Training Accuracy: 0.2345, Validation Accuracy: 0.2223
       200/200 [00:03<00:00, 54.66it/s]
(Epoch 6 / 50) Training Accuracy: 0.24915, Validation Accuracy: 0.2338
100% | 200/200 [00:03<00:00, 55.90it/s]
(Epoch 7 / 50) Training Accuracy: 0.2584, Validation Accuracy: 0.2451
        200/200 [00:03<00:00, 56.81it/s]
(Epoch 8 / 50) Training Accuracy: 0.2651, Validation Accuracy: 0.2488
100% | 200/200 [00:03<00:00, 57.37it/s]
(Epoch 9 / 50) Training Accuracy: 0.2648, Validation Accuracy: 0.2471
     200/200 [00:03<00:00, 60.49it/s]
(Epoch 10 / 50) Training Accuracy: 0.27685, Validation Accuracy: 0.2558
            200/200 [00:03<00:00, 60.53it/s]
(Epoch 11 / 50) Training Accuracy: 0.2792, Validation Accuracy: 0.2583
```

```
100% | 200/200 [00:03<00:00, 62.95it/s]
(Epoch 12 / 50) Training Accuracy: 0.28575, Validation Accuracy: 0.2646
100%| 200/200 [00:03<00:00, 58.26it/s]
(Epoch 13 / 50) Training Accuracy: 0.2879, Validation Accuracy: 0.2657
         200/200 [00:09<00:00, 21.61it/s]
(Epoch 14 / 50) Training Accuracy: 0.28865, Validation Accuracy: 0.2664
       200/200 [00:03<00:00, 57.46it/s]
(Epoch 15 / 50) Training Accuracy: 0.29545, Validation Accuracy: 0.2705
       200/200 [00:03<00:00, 62.39it/s]
(Epoch 16 / 50) Training Accuracy: 0.2964, Validation Accuracy: 0.2737
         200/200 [00:03<00:00, 58.99it/s]
(Epoch 17 / 50) Training Accuracy: 0.30345, Validation Accuracy: 0.2752
100%| 200/200 [00:03<00:00, 63.84it/s]
(Epoch 18 / 50) Training Accuracy: 0.30555, Validation Accuracy: 0.276
100% | 200/200 [00:03<00:00, 57.68it/s]
(Epoch 19 / 50) Training Accuracy: 0.30715, Validation Accuracy: 0.2821
            200/200 [00:03<00:00, 60.94it/s]
(Epoch 20 / 50) Training Accuracy: 0.31265, Validation Accuracy: 0.2799
       200/200 [00:04<00:00, 43.54it/s]
(Epoch 21 / 50) Training Accuracy: 0.31315, Validation Accuracy: 0.2787
100% | 200/200 [00:04<00:00, 49.01it/s]
(Epoch 22 / 50) Training Accuracy: 0.31755, Validation Accuracy: 0.2836
100% | 200/200 [00:04<00:00, 48.13it/s]
(Epoch 23 / 50) Training Accuracy: 0.3192, Validation Accuracy: 0.2833
         200/200 [00:03<00:00, 62.60it/s]
(Epoch 24 / 50) Training Accuracy: 0.31905, Validation Accuracy: 0.2837
100% | 200/200 [00:04<00:00, 41.96it/s]
(Epoch 25 / 50) Training Accuracy: 0.32525, Validation Accuracy: 0.2894
100%| 200/200 [00:03<00:00, 62.17it/s]
(Epoch 26 / 50) Training Accuracy: 0.3238, Validation Accuracy: 0.2895
         200/200 [00:03<00:00, 52.65it/s]
(Epoch 27 / 50) Training Accuracy: 0.33645, Validation Accuracy: 0.2944
100% | 200/200 [00:03<00:00, 63.99it/s]
(Epoch 28 / 50) Training Accuracy: 0.33645, Validation Accuracy: 0.2941
     200/200 [00:06<00:00, 32.67it/s]
(Epoch 29 / 50) Training Accuracy: 0.33695, Validation Accuracy: 0.2953
100% | 200/200 [00:03<00:00, 63.39it/s]
(Epoch 30 / 50) Training Accuracy: 0.3425, Validation Accuracy: 0.3
100% | 200/200 [00:03<00:00, 62.79it/s]
(Epoch 31 / 50) Training Accuracy: 0.3406, Validation Accuracy: 0.2982
100% | 200/200 [00:03<00:00, 60.04it/s]
(Epoch 32 / 50) Training Accuracy: 0.34505, Validation Accuracy: 0.2949
            200/200 [00:07<00:00, 26.45it/s]
(Epoch 33 / 50) Training Accuracy: 0.34595, Validation Accuracy: 0.3011
            200/200 [00:04<00:00, 43.08it/s]
(Epoch 34 / 50) Training Accuracy: 0.34755, Validation Accuracy: 0.301
100% | 200/200 [00:05<00:00, 39.91it/s]
(Epoch 35 / 50) Training Accuracy: 0.3548, Validation Accuracy: 0.3012
100% | 200/200 [00:03<00:00, 50.97it/s]
(Epoch 36 / 50) Training Accuracy: 0.3552, Validation Accuracy: 0.2995
100% | 200/200 [00:03<00:00, 52.79it/s]
(Epoch 37 / 50) Training Accuracy: 0.35525, Validation Accuracy: 0.3034
```

```
100% | 200/200 [00:04<00:00, 49.61it/s]
(Epoch 38 / 50) Training Accuracy: 0.3593, Validation Accuracy: 0.3017
100%| 200/200 [00:03<00:00, 61.42it/s]
(Epoch 39 / 50) Training Accuracy: 0.3648, Validation Accuracy: 0.3048
100%| 200/200 [00:04<00:00, 48.45it/s]
(Epoch 40 / 50) Training Accuracy: 0.36665, Validation Accuracy: 0.311
       200/200 [00:06<00:00, 31.40it/s]
(Epoch 41 / 50) Training Accuracy: 0.35765, Validation Accuracy: 0.3068
       200/200 [00:04<00:00, 41.78it/s]
(Epoch 42 / 50) Training Accuracy: 0.36375, Validation Accuracy: 0.302
100% | 200/200 [00:04<00:00, 48.51it/s]
(Epoch 43 / 50) Training Accuracy: 0.3702, Validation Accuracy: 0.3062
100%| 200/200 [00:04<00:00, 43.45it/s]
(Epoch 44 / 50) Training Accuracy: 0.37215, Validation Accuracy: 0.306
100% | 200/200 [00:04<00:00, 47.97it/s]
(Epoch 45 / 50) Training Accuracy: 0.37475, Validation Accuracy: 0.3037
            200/200 [00:03<00:00, 55.78it/s]
(Epoch 46 / 50) Training Accuracy: 0.37205, Validation Accuracy: 0.3089
       200/200 [00:04<00:00, 42.74it/s]
(Epoch 47 / 50) Training Accuracy: 0.3827, Validation Accuracy: 0.3097
100% | 200/200 [00:03<00:00, 51.30it/s]
(Epoch 48 / 50) Training Accuracy: 0.38395, Validation Accuracy: 0.313
100% | 200/200 [00:09<00:00, 20.79it/s]
(Epoch 49 / 50) Training Accuracy: 0.38155, Validation Accuracy: 0.3131
     200/200 [00:09<00:00, 20.80it/s]
(Epoch 50 / 50) Training Accuracy: 0.38415, Validation Accuracy: 0.3121
```



## SGD with L1 Regularization [2pts]

With L1 Regularization, your regularized loss becomes  $ilde{J}_{\ell_1}( heta)$  and it's defined as

$${ ilde J}_{\ell_1}( heta) = J( heta) + \lambda \| heta\|_{\ell_1}$$

where

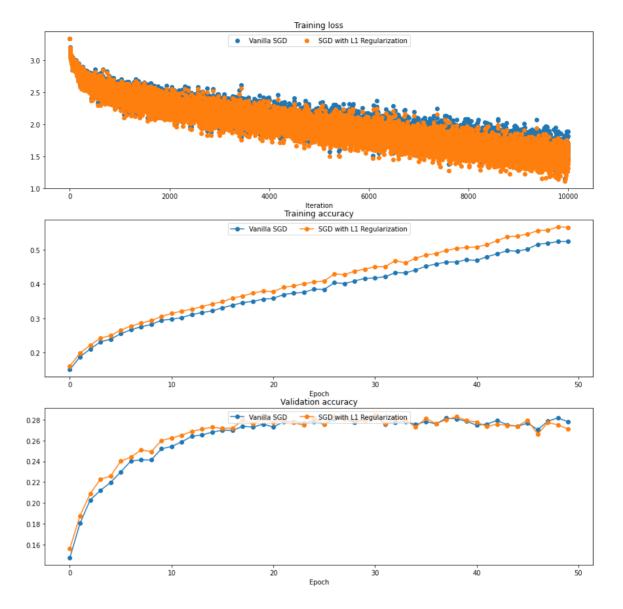
$$\| heta\|_{\ell_1} = \sum_{l=1}^n \sum_{k=1}^{n_l} | heta_{l,k}|$$

Please implmemt TODO block of <a href="mailto:apply\_l1\_regularization">apply\_l1\_regularization</a> in lib/layer\_utils. Such regularization funcationality is called after gradient gathering in the <a href="mailto:backward">backward</a> process.

```
results_sgd_l1 = train_net(small_data_dict, model_sgd_l1, loss_f_sgd_l1,
                        max_epochs=50, show_every=10000, verbose=True, r
opt_params_sgd_l1, loss_hist_sgd_l1, train_acc_hist_sgd_l1, val_acc_hist_
plt.subplot(3, 1, 1)
plt.title('Training loss')
plt.xlabel('Iteration')
plt.subplot(3, 1, 2)
plt.title('Training accuracy')
plt.xlabel('Epoch')
plt.subplot(3, 1, 3)
plt.title('Validation accuracy')
plt.xlabel('Epoch')
plt.subplot(3, 1, 1)
plt.plot(loss_hist_sgd, 'o', label="Vanilla SGD")
plt.subplot(3, 1, 2)
plt.plot(train_acc_hist_sgd, '-o', label="Vanilla SGD")
plt.subplot(3, 1, 3)
plt.plot(val_acc_hist_sgd, '-o', label="Vanilla SGD")
plt.subplot(3, 1, 1)
plt.plot(loss_hist_sgd_l1, 'o', label="SGD with L1 Regularization")
plt.subplot(3, 1, 2)
plt.plot(train_acc_hist_sgd_l1, '-o', label="SGD with L1 Regularization")
plt.subplot(3, 1, 3)
plt.plot(val_acc_hist_sgd_l1, '-o', label="SGD with L1 Regularization")
for i in [1, 2, 3]:
 plt.subplot(3, 1, i)
 plt.legend(loc='upper center', ncol=4)
plt.gcf().set_size_inches(15, 15)
plt.show()
Training with SGD plus L1 Regularization...
              | 5/200 [00:00<00:12, 15.49it/s]
(Iteration 1 / 10000) Average loss: 3.3332154539088985
             200/200 [00:05<00:00, 38.03it/s]
(Epoch 1 / 50) Training Accuracy: 0.16155, Validation Accuracy: 0.1563
100% | 200/200 [00:06<00:00, 30.83it/s]
(Epoch 2 / 50) Training Accuracy: 0.19865, Validation Accuracy: 0.1878
100% | 200/200 [00:05<00:00, 37.35it/s]
(Epoch 3 / 50) Training Accuracy: 0.2218, Validation Accuracy: 0.2089
100% | 200/200 [00:04<00:00, 44.23it/s]
(Epoch 4 / 50) Training Accuracy: 0.24215, Validation Accuracy: 0.2228
100% | 200/200 [00:07<00:00, 26.98it/s]
(Epoch 5 / 50) Training Accuracy: 0.2499, Validation Accuracy: 0.2258
100%| 200/200 [00:04<00:00, 44.14it/s]
(Epoch 6 / 50) Training Accuracy: 0.26565, Validation Accuracy: 0.2402
        200/200 [00:04<00:00, 41.88it/s]
(Epoch 7 / 50) Training Accuracy: 0.27745, Validation Accuracy: 0.244
100% | 200/200 [00:04<00:00, 40.80it/s]
(Epoch 8 / 50) Training Accuracy: 0.28575, Validation Accuracy: 0.251
100%| 200/200 [00:05<00:00, 34.32it/s]
```

```
(Epoch 9 / 50) Training Accuracy: 0.2934, Validation Accuracy: 0.2493
100% | 200/200 [00:09<00:00, 20.15it/s]
(Epoch 10 / 50) Training Accuracy: 0.30535, Validation Accuracy: 0.26
100% | 200/200 [00:06<00:00, 29.03it/s]
(Epoch 11 / 50) Training Accuracy: 0.3142, Validation Accuracy: 0.2625
        200/200 [00:07<00:00, 27.88it/s]
(Epoch 12 / 50) Training Accuracy: 0.32095, Validation Accuracy: 0.265
            200/200 [00:09<00:00, 21.20it/s]
(Epoch 13 / 50) Training Accuracy: 0.32675, Validation Accuracy: 0.2689
100% | 200/200 [00:11<00:00, 17.27it/s]
(Epoch 14 / 50) Training Accuracy: 0.33405, Validation Accuracy: 0.271
100% | 200/200 [00:08<00:00, 23.29it/s]
(Epoch 15 / 50) Training Accuracy: 0.34185, Validation Accuracy: 0.2728
         200/200 [00:20<00:00, 9.84it/s]
(Epoch 16 / 50) Training Accuracy: 0.34845, Validation Accuracy: 0.2715
            200/200 [00:15<00:00, 13.27it/s]
(Epoch 17 / 50) Training Accuracy: 0.359, Validation Accuracy: 0.272
100% | 200/200 [00:09<00:00, 20.43it/s]
(Epoch 18 / 50) Training Accuracy: 0.36515, Validation Accuracy: 0.2798
100% | 200/200 [00:09<00:00, 21.72it/s]
(Epoch 19 / 50) Training Accuracy: 0.3734, Validation Accuracy: 0.2762
100% | 200/200 [00:06<00:00, 31.57it/s]
(Epoch 20 / 50) Training Accuracy: 0.379, Validation Accuracy: 0.2809
        200/200 [00:12<00:00, 16.44it/s]
(Epoch 21 / 50) Training Accuracy: 0.3779, Validation Accuracy: 0.2773
       200/200 [00:08<00:00, 24.76it/s]
(Epoch 22 / 50) Training Accuracy: 0.39025, Validation Accuracy: 0.28
        200/200 [00:06<00:00, 29.11it/s]
(Epoch 23 / 50) Training Accuracy: 0.39425, Validation Accuracy: 0.2771
100% | 200/200 [00:07<00:00, 26.68it/s]
(Epoch 24 / 50) Training Accuracy: 0.40065, Validation Accuracy: 0.2747
100% | 200/200 [00:06<00:00, 28.58it/s]
(Epoch 25 / 50) Training Accuracy: 0.40585, Validation Accuracy: 0.2815
            200/200 [00:06<00:00, 29.96it/s]
(Epoch 26 / 50) Training Accuracy: 0.4086, Validation Accuracy: 0.2758
100% | 200/200 [00:06<00:00, 28.70it/s]
(Epoch 27 / 50) Training Accuracy: 0.42925, Validation Accuracy: 0.284
100% | 200/200 [00:13<00:00, 15.21it/s]
(Epoch 28 / 50) Training Accuracy: 0.4273, Validation Accuracy: 0.2805
100% | 200/200 [00:06<00:00, 29.05it/s]
(Epoch 29 / 50) Training Accuracy: 0.4375, Validation Accuracy: 0.2796
          200/200 [00:06<00:00, 29.97it/s]
(Epoch 30 / 50) Training Accuracy: 0.4434, Validation Accuracy: 0.2793
100% | 200/200 [00:07<00:00, 26.95it/s]
(Epoch 31 / 50) Training Accuracy: 0.4499, Validation Accuracy: 0.2844
100% | 200/200 [00:07<00:00, 25.72it/s]
(Epoch 32 / 50) Training Accuracy: 0.4508, Validation Accuracy: 0.2761
         200/200 [00:09<00:00, 21.10it/s]
(Epoch 33 / 50) Training Accuracy: 0.468, Validation Accuracy: 0.2817
100% | 200/200 [00:10<00:00, 18.76it/s]
(Epoch 34 / 50) Training Accuracy: 0.46125, Validation Accuracy: 0.2797
100% | 200/200 [00:08<00:00, 23.17it/s]
```

```
(Epoch 35 / 50) Training Accuracy: 0.47505, Validation Accuracy: 0.2729
     200/200 [00:14<00:00, 13.89it/s]
(Epoch 36 / 50) Training Accuracy: 0.48405, Validation Accuracy: 0.2814
100% | 200/200 [00:08<00:00, 23.80it/s]
(Epoch 37 / 50) Training Accuracy: 0.48875, Validation Accuracy: 0.2761
       200/200 [00:08<00:00, 22.28it/s]
(Epoch 38 / 50) Training Accuracy: 0.49765, Validation Accuracy: 0.2801
        200/200 [00:07<00:00, 26.09it/s]
(Epoch 39 / 50) Training Accuracy: 0.50345, Validation Accuracy: 0.2828
       200/200 [00:07<00:00, 25.63it/s]
(Epoch 40 / 50) Training Accuracy: 0.5064, Validation Accuracy: 0.2793
100% | 200/200 [00:07<00:00, 25.62it/s]
(Epoch 41 / 50) Training Accuracy: 0.50775, Validation Accuracy: 0.2777
       200/200 [00:06<00:00, 29.42it/s]
(Epoch 42 / 50) Training Accuracy: 0.51445, Validation Accuracy: 0.2734
            200/200 [00:06<00:00, 30.86it/s]
(Epoch 43 / 50) Training Accuracy: 0.52595, Validation Accuracy: 0.2758
100% | 200/200 [00:06<00:00, 29.24it/s]
(Epoch 44 / 50) Training Accuracy: 0.53775, Validation Accuracy: 0.2745
100% | 200/200 [00:06<00:00. 29.38it/s]
(Epoch 45 / 50) Training Accuracy: 0.53965, Validation Accuracy: 0.2738
100% | 200/200 [00:06<00:00, 28.71it/s]
(Epoch 46 / 50) Training Accuracy: 0.54555, Validation Accuracy: 0.2795
        200/200 [00:08<00:00, 22.64it/s]
(Epoch 47 / 50) Training Accuracy: 0.55535, Validation Accuracy: 0.2661
       200/200 [00:09<00:00, 20.57it/s]
(Epoch 48 / 50) Training Accuracy: 0.557, Validation Accuracy: 0.2773
        200/200 [00:09<00:00, 20.95it/s]
(Epoch 49 / 50) Training Accuracy: 0.56605, Validation Accuracy: 0.275
100% | 200/200 [00:05<00:00, 38.20it/s]
(Epoch 50 / 50) Training Accuracy: 0.5652, Validation Accuracy: 0.271
```



## SGD with L2 Regularization [2pts]

With L2 Regularization, your regularized loss becomes  $ilde{J}_{\ell_2}( heta)$  and it's defined as

$$ilde{J}_{\ell_2}( heta) = J( heta) + \lambda \| heta\|_{\ell_2}$$

where

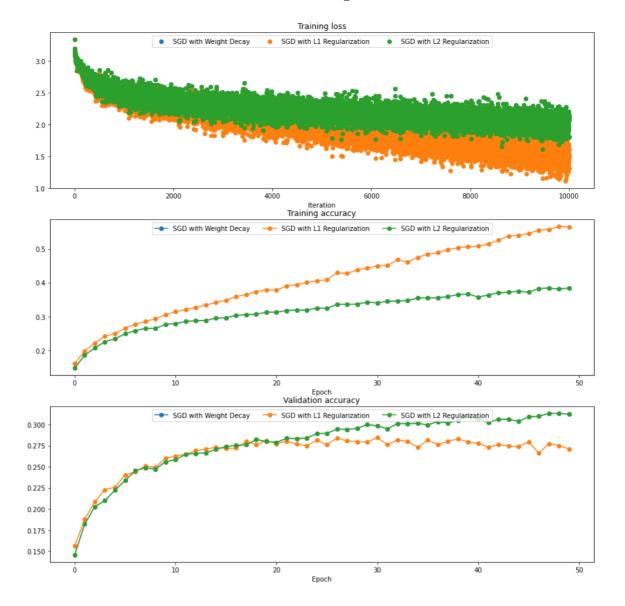
$$\| heta\|_{\ell_2} = \sum_{l=1}^n \sum_{k=1}^{n_l} heta_{l,k}^2$$

Similarly, implmemt TODO block of <code>apply\_l2\_regularization</code> in <code>lib/layer\_utils</code> . For SGD, you're also asked to find the  $\lambda$  for L2 Regularization such that it achives the EXACTLY SAME effect as weight decay in the previous cells. As a reminder, learning rate is the same as previously, and the weight decay paramter was 1e-4.

```
#### Find lambda for L2 regularization so that
                                                            ####
#### it achieves EXACTLY THE SAME learning curve as weight decay ####
12 \quad lambda = 0.005
print ("\nTraining with SGD plus L2 Regularization...")
results_sgd_l2 = train_net(small_data_dict, model_sgd_l2, loss_f_sgd_l2,
                         max_epochs=50, show_every=10000, verbose=False
opt_params_sgd_l2, loss_hist_sgd_l2, train_acc_hist_sgd_l2, val_acc_hist_
plt.subplot(3, 1, 1)
plt.title('Training loss')
plt.xlabel('Iteration')
plt.subplot(3, 1, 2)
plt.title('Training accuracy')
plt.xlabel('Epoch')
plt.subplot(3, 1, 3)
plt.title('Validation accuracy')
plt.xlabel('Epoch')
plt.subplot(3, 1, 1)
plt.plot(loss_hist_sgdw, 'o', label="SGD with Weight Decay")
plt.subplot(3, 1, 2)
plt.plot(train acc hist sqdw, '-o', label="SGD with Weight Decay")
plt.subplot(3, 1, 3)
plt.plot(val_acc_hist_sgdw, '-o', label="SGD with Weight Decay")
plt.subplot(3, 1, 1)
plt.plot(loss hist sqd l1, 'o', label="SGD with L1 Regularization")
plt.subplot(3, 1, 2)
plt.plot(train_acc_hist_sgd_l1, '-o', label="SGD with L1 Regularization")
plt.subplot(3, 1, 3)
plt.plot(val_acc_hist_sgd_l1, '-o', label="SGD with L1 Regularization")
plt.subplot(3, 1, 1)
plt.plot(loss_hist_sgd_l2, 'o', label="SGD with L2 Regularization")
plt.subplot(3, 1, 2)
plt.plot(train_acc_hist_sgd_l2, '-o', label="SGD with L2 Regularization")
plt.subplot(3, 1, 3)
plt.plot(val_acc_hist_sgd_l2, '-o', label="SGD with L2 Regularization")
for i in [1, 2, 3]:
 plt.subplot(3, 1, i)
 plt.legend(loc='upper center', ncol=4)
plt.gcf().set size inches(15, 15)
plt.show()
```

Training with SGD plus L2 Regularization...

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100%	200/200	[00:04<00:00,	48.55it/s]
100%	200/200	[00:03<00:00,	58.78it/s]
100%	200/200	[00:03<00:00,	60.95it/s]
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### Adam [2pt]

The update rule of Adam is as shown below:

$$t=t+1 \ g_t: ext{gradients at update step } t \ m_t=eta_1 m_{t-1}+(1-eta_1)g_t \ v_t=eta_2 v_{t-1}+(1-eta_2)g_t^2 \ \hat{m_t}=m_t/(1-eta_1^t) \ \hat{v_t}=v_t/(1-eta_2^t) \ heta_{t+1}= heta_t-rac{\eta \ \hat{m_t}}{\sqrt{\hat{v_t}}+\epsilon}$$

Complete the Adam() function in lib/optim.py Important Notes:

- 1. t must be updated before everything else
- 2.  $\beta_1^t$  is  $\beta_1$  exponentiated to the t'th power
- 3. You should also enable weight decay in Adam, similar to what you did in SGD

```
In [ ]: %reload ext autoreload
        seed = 1234
        np.random.seed(seed=seed)
        # Test Adam implementation; you should see errors around 1e-7 or less
        N, D = 4, 5
        test adam = sequential(fc(N, D, name="adam fc"))
        w = np.linspace(-0.4, 0.6, num=N*D).reshape(N, D)
        dw = np.linspace(-0.6, 0.4, num=N*D).reshape(N, D)
        m = np.linspace(0.6, 0.9, num=N*D).reshape(N, D)
        v = np.linspace(0.7, 0.5, num=N*D).reshape(N, D)
        test_adam.layers[0].params = {"adam_fc_w": w}
        test_adam.layers[0].grads = {"adam_fc_w": dw}
        opt_adam = Adam(test_adam, 1e-2, 0.9, 0.999, t=5)
        opt_adam.mt = {"adam_fc_w": m}
        opt_adam.vt = {"adam_fc_w": v}
        opt_adam.step()
        updated_w = test_adam.layers[0].params["adam_fc_w"]
        mt = opt adam.mt["adam fc w"]
        vt = opt_adam.vt["adam_fc_w"]
        expected_updated_w = np.asarray([
          [-0.40094747, -0.34836187, -0.29577703, -0.24319299, -0.19060977],
          [-0.1380274, -0.08544591, -0.03286534, 0.01971428,
                                                                0.0722929],
          [ 0.1248705,  0.17744702,  0.23002243,  0.28259667,
                                                               0.335169691
          [ 0.38774145, 0.44031188, 0.49288093, 0.54544852,
                                                               0.59801459]])
        expected_v = np.asarray([
                       0.68908382, 0.67851319, 0.66794809,
                                                               0.65738853,],
          [ 0.69966,
          [ 0.64683452, 0.63628604, 0.6257431,
                                                                0.60467385,],
                                                  0.61520571,
          [ 0.59414753, 0.58362676, 0.57311152, 0.56260183,
                                                                0.55209767,],
          [ 0.54159906, 0.53110598, 0.52061845, 0.51013645,
                                                                0.49966, ]])
        expected m = np.asarray([
                     0.49947368, 0.51894737, 0.53842105,
          [ 0.48,
                                                               0.55789474],
          [ 0.57736842, 0.59684211, 0.61631579, 0.63578947,
                                                                0.65526316].
          [ 0.67473684, 0.69421053, 0.71368421, 0.73315789,
                                                                0.75263158],
          [ 0.77210526, 0.79157895, 0.81105263, 0.83052632,
                                                                         11)
                                                                0.85
        print ('The following errors should be around or less than 1e-7')
        print ('updated_w error: ', rel_error(expected_updated_w, updated_w))
        print ('mt error: ', rel_error(expected_m, mt))
        print ('vt error: ', rel_error(expected_v, vt))
        The following errors should be around or less than 1e-7
```

```
The following errors should be around or less than 1e-7 updated_w error: 1.1395691798535431e-07 mt error: 4.214963193114416e-09 vt error: 4.208314038113071e-09
```

# Comparing the Weight Decay v.s. L2 Regularization in Adam [5pt]

Run the following code block to compare the plotted results between effects of weight decay and L2 regularization on Adam. Are they still the same? (we can make

them the same as in SGD, can we also do it in Adam?)

Answer. Although similar, the two methods are not identical. While in SGD we can view both methods as a multiplication operation on the model parameter by a fixed value, this is not possible in Adam due to the influence of L2 regularization in both the denominator and numerator. Therefore, we cannot simply convert the L2 regularization to a multiplication operation as in weight decay.

```
In [ ]: seed = 1234
        reset_seed(seed)
        model_adam_wd
                          = FullyConnectedNetwork()
        loss_f_adam_wd = cross_entropy()
        optimizer_adam_wd = Adam(model_adam_wd.net, lr=1e-4, weight_decay=1e-6)
        print ("Training with AdamW...")
        results_adam_wd = train_net(small_data_dict, model_adam_wd, loss_f_adam_w
                                max_epochs=50, show_every=10000, verbose=False)
        reset seed(seed)
                          = FullyConnectedNetwork()
        model_adam_l2
        loss_f_adam_l2 = cross_entropy()
        optimizer_adam_l2 = Adam(model_adam_l2.net, lr=1e-4)
        reg lambda 12 = 1e-4
        print ("\nTraining with Adam + L2...")
        results_adam_l2 = train_net(small_data_dict, model_adam_l2, loss_f_adam_l
                                 max_epochs=50, show_every=10000, verbose=False,
        opt params adam wd, loss hist adam wd, train acc hist adam wd, val acc hi
        opt_params_adam_l2, loss_hist_adam_l2, train_acc_hist_adam_l2, val_acc_hi
        plt.subplot(3, 1, 1)
        plt.title('Training loss')
        plt.xlabel('Iteration')
        plt.subplot(3, 1, 2)
        plt.title('Training accuracy')
        plt.xlabel('Epoch')
        plt.subplot(3, 1, 3)
        plt.title('Validation accuracy')
        plt.xlabel('Epoch')
        plt.subplot(3, 1, 1)
        plt.plot(loss_hist_sgd, 'o', label="Vanilla SGD")
        plt.subplot(3, 1, 2)
        plt.plot(train_acc_hist_sgd, '-o', label="Vanilla SGD")
        plt.subplot(3, 1, 3)
        plt.plot(val_acc_hist_sgd, '-o', label="Vanilla SGD")
        plt.subplot(3, 1, 1)
        plt.plot(loss_hist_sgdw, 'o', label="SGD with Weight Decay")
        plt.subplot(3, 1, 2)
        plt.plot(train_acc_hist_sgdw, '-o', label="SGD with Weight Decay")
        plt.subplot(3, 1, 3)
        plt.plot(val_acc_hist_sgdw, '-o', label="SGD with Weight Decay")
```

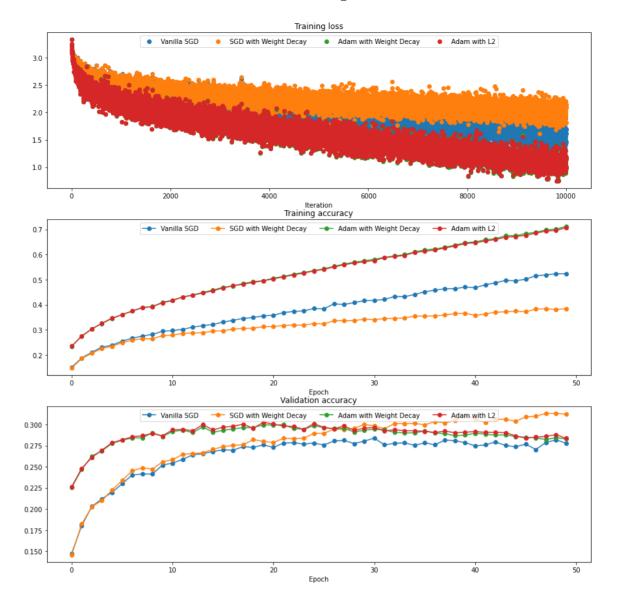
```
plt.subplot(3, 1, 1)
plt.plot(loss_hist_adam_wd, 'o', label="Adam with Weight Decay")
plt.subplot(3, 1, 2)
plt.plot(train_acc_hist_adam_wd, '-o', label="Adam with Weight Decay")
plt.subplot(3, 1, 3)
plt.plot(val_acc_hist_adam_wd, '-o', label="Adam with Weight Decay")
plt.subplot(3, 1, 1)
plt.plot(loss_hist_adam_l2, 'o', label="Adam with L2")
plt.subplot(3, 1, 2)
plt.plot(train_acc_hist_adam_l2, '-o', label="Adam with L2")
plt.subplot(3, 1, 3)
plt.plot(val_acc_hist_adam_l2, '-o', label="Adam with L2")
for i in [1, 2, 3]:
  plt.subplot(3, 1, i)
  plt.legend(loc='upper center', ncol=4)
plt.gcf().set_size_inches(15, 15)
plt.show()
```

Training with AdamW...

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Training with Adam + L2...

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### **Submission**

Please prepare a PDF document <code>problem\_1\_solution.pdf</code> in the root directory of this repository with all plots and inline answers of your solution. Concretely, the document should contain the following items in strict order:

- 1. Training loss / accuracy curves for the simple neural network training with > 30% validation accuracy
- 2. Plots for comparing vanilla SGD to SGD + Weight Decay, SGD + L1 and SGD + L2
- 3. "Comparing different Regularizations with Adam" plots

Note that you still need to submit the jupyter notebook with all generated solutions. We will randomly pick submissions and check that the plots in the PDF and in the notebook are equivalent.