α–β–(γ) Filter

Prateek Garg

October 28th, 2021

α–β Filter:

Objective: Track a constant velocity aircraft

- State of the system is described by two attributes: range and velocity. More generally we are observing a quantity with a rate of change expected to be constant.
- We “sample” the measurements after a fixed interval.
- We initialise the values of attributes to the expected values.
- On the actual measurements of range we get different range from what is expected.
- There can be two reasons for these deviations:
    - Imprecisions in measuring instruments
    - The velocity of aircraft changes.
- To account for these deviations, we use Alpha and Beta to estimate range and velocity iteratively.

α–β Filter Implementation:

```
clear;
clc;
alpha=0.2;
beta=0.1;
t_samp=5; %track-to-track interval

%initialise values
x_init=30000;
v_init=40;

z=load('ab.txt'); %measurements
n=size(z,2);
```

```
t=linspace(t_samp,t_samp*n,n); %time values

x=zeros(1,n); %range estimates of aircraft
v=zeros(1,n); %velocity estimates of aircraft

x_est=zeros(1,n); %range estimates of aircraft
v_est=zeros(1,n); %velocity estimates of aircraft

%Estimate initial values
x_temp=x_init+v_init*t_samp;
v_temp=v_init;
for i = 1:n
    %Save temporary predicted estimates
    x_est(i)=x_temp;
    v_est(i)=v_temp;


    %State Extrapolation Equations:
        %x(i)=x_temp;
        %v(i)=v_temp;

    %Present estimate after measurement
    x(i)=x_temp+alpha*(z(i)-x_temp);
    v(i)=v_temp+beta*(z(i)-x_temp)/t_samp;

    x_temp=x(i)+v(i)*t_samp;
    v_temp=v(i);

end

format shortG;
%format LONG E ;
[t./5;z;x;v;x_est;v_est] %#ok<NOPTS>
figure
plot(t,x,t,z,t,x_init+v_init*t,t,x_est, 'linewidth',1.5);
legend;
figure
plot(t,v,t,v_est, 'linewidth',1.5);
legend;
```

Observations:

1. Alpha:
   The present estimate after measurement is written as:
   Pres. Est.= Past Est.+alpha*(Measured value−Past Est.)
   = (1−alpha)*Past Est.+alpha*(Measured value)

   This shows us that to calculate the present estimate, we use a weighted average between past estimate and measured value. For a good estimate alpha <1
   If the measuring instrument is very precise 0.5< alpha<1 (Since measured value will be close to true value) and else 0<alpha<0.5
2. Beta:
   Beta handles disparities in velocity estimates:
   Pres. Est.= Past Est. +Beta*(Measured value of Range−Past Est. fo range)/Δt

   Again if the precision is high, Beta should be kept high to value the influence of measurements more than the past prediction. If precision is low the beta should be kept low.

α−β−γ Filter:

Objective: Tracking accelerating aircraft

- State of the system is described by three attributes: range, velocity and acceleration. More generally we are observing a quantity with a rate of change expected to be varying.

α−β−γ Filter Implementation:

```
clear;
clc;

alpha=0.5;
beta=0.4;
gamma=0.1;
t_samp=5; %track-to-track interval

%initialise values
x_init=30000;
v_init=40;
a_init=0;
```

```
z=load('abr.txt'); %measurements
n=size(z,2);
t=linspace(t_samp,t_samp*n,n); %time values

x=zeros(1,n); %range estimates of aircraft
v=zeros(1,n); %velocity estimates of aircraft
a=zeros(1,n); %acceleration estimates of aircraft


x_est=zeros(1,n); %range estimates of aircraft
v_est=zeros(1,n); %velocity estimates of aircraft
a_est=zeros(1,n); %acceleration estimates of aircraft

%Estimate initial values
x_temp=x_init+v_init*t_samp+a_init*t_samp*t_samp/2;
v_temp=v_init+a_init*t_samp;
a_temp=a_init;

for i = 1:n
    %Save temporary predicted estimates
    x_est(i)=x_temp;
    v_est(i)=v_temp;
    a_est(i)=a_temp;

    %State Extrapolation Equations:
        %x(i)=x_temp;
        %v(i)=v_temp;
        %a(i)=a_temp;

    %Present estimate after measurement
    x(i)=x_temp+alpha*(z(i)-x_temp);
    v(i)=v_temp+beta*(z(i)-x_temp)/t_samp;
    a(i)=a_temp+2*gamma*(z(i)-x_temp)/t_samp/t_samp;
    x_temp=x(i)+v(i)*t_samp+a(i)*t_samp*t_samp/2;
    v_temp=v(i)+a(i)*t_samp;
    a_temp=a(i)
end
```

```
format shortG;
%format LONG E ;
[t./5;z;x;v;a;x_est;v_est;a_est] %#ok<NOPTS>
figure
plot(t,x,t,z,t,x_est,'linewidth',1.5);
figure
plot(t,v,t,v_est);
figure
plot(t,a,t,a_est);
```

Observations:

1. On using an Alpha-Beta Filter, we see the estimate deviate from true values considerably even when alpha is set close to 1(Measurements are very close to true value). This anomaly is explained by the fact that we aren't including acceleration term in state extrapolation equations.
2. In the given data, acceleration is actually changing at a time step. This change isn't caused by imprecion in instruments. So using a larger value than the one given in example yields better results.