Untitled

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0.1 PG Diploma - Machine Learning and Artifical Intelligence

1 Assignment - Linear Regression - Solution

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Problem Statement A Chinese automobile company Geely Auto aspires to enter the US market by setting up their manufacturing unit there and producing cars locally to give competition to their US and European counterparts.They have contracted an automobile consulting company to understand the factors on which the pricing of cars depends. Specifically, they want to understand the factors affecting the pricing of cars in the American market, since those may be very different from the Chinese market.

The company wants to know: * Which variables are significant in predicting the price of a car. * How well those variables describe the price of a car

Based on various market surveys, the consulting firm has gathered a large dataset of different types of cars across the American market.

Business Goal We are required to model the price of cars with the available independent variables. It will be used by the management to understand how exactly the prices vary with the independent variables. They can accordingly manipulate the design of the cars, the business strategy etc. to meet certain price levels.

```
In [1]: #importing the necessary libraries

        import numpy as np
        import pandas as pd
        import matplotlib.pyplot as plt
        import seaborn as sns
        import pandas_profiling

        import warnings
        warnings.filterwarnings('ignore')

        from sklearn.model_selection import train_test_split
        from sklearn.preprocessing import MinMaxScaler
        import statsmodels.api as sm
        from statsmodels.stats.outliers_influence import variance_inflation_factor
```

```
from sklearn.feature_selection import RFE
from sklearn.linear_model import LinearRegression
```

In [2]:
```
#Reading the data as a Pandas dataframe
car = pd.read_csv('CarPrice_Assignment.csv')
car.head()
```

Out[2]:
```
   car_ID  symboling                   CarName fueltype aspiration doornumber  \
0       1          3        alfa-romero giulia      gas        std        two
1       2          3       alfa-romero stelvio      gas        std        two
2       3          1  alfa-romero Quadrifoglio      gas        std        two
3       4          2               audi 100 ls      gas        std       four
4       5          2                audi 100ls      gas        std       four

       carbody drivewheel enginelocation  wheelbase  ...  enginesize  \
0  convertible        rwd          front       88.6  ...         130
1  convertible        rwd          front       88.6  ...         130
2    hatchback        rwd          front       94.5  ...         152
3        sedan        fwd          front       99.8  ...         109
4        sedan        4wd          front       99.4  ...         136

   fuelsystem  boreratio  stroke compressionratio horsepower  peakrpm citympg  \
0        mpfi       3.47    2.68              9.0        111     5000      21
1        mpfi       3.47    2.68              9.0        111     5000      21
2        mpfi       2.68    3.47              9.0        154     5000      19
3        mpfi       3.19    3.40             10.0        102     5500      24
4        mpfi       3.19    3.40              8.0        115     5500      18

   highwaympg    price
0          27  13495.0
1          27  16500.0
2          26  16500.0
3          30  13950.0
4          22  17450.0

[5 rows x 26 columns]
```

In [3]:
```
#gathering an overview of the database using the 'pandas_profiling' library.
car.profile_report(style={'full_width':True})
```

```
<IPython.core.display.HTML object>
```

Out[3]:

In [4]:
```
car.shape
```

Out[4]:
```
(205, 26)
```

In [5]:
```
car.info()
```

```
<class 'pandas.core.frame.DataFrame'>
RangeIndex: 205 entries, 0 to 204
Data columns (total 26 columns):
car_ID              205 non-null int64
symboling           205 non-null int64
CarName             205 non-null object
fueltype            205 non-null object
aspiration          205 non-null object
doornumber          205 non-null object
carbody             205 non-null object
drivewheel          205 non-null object
enginelocation      205 non-null object
wheelbase           205 non-null float64
carlength           205 non-null float64
carwidth            205 non-null float64
carheight           205 non-null float64
curbweight          205 non-null int64
enginetype          205 non-null object
cylindernumber      205 non-null object
enginesize          205 non-null int64
fuelsystem          205 non-null object
boreratio           205 non-null float64
stroke              205 non-null float64
compressionratio    205 non-null float64
horsepower          205 non-null int64
peakrpm             205 non-null int64
citympg             205 non-null int64
highwaympg          205 non-null int64
price               205 non-null float64
dtypes: float64(8), int64(8), object(10)
memory usage: 41.7+ KB
```

In [6]: `car.describe()`

Out[6]:

	car_ID	symboling	wheelbase	carlength	carwidth	carheight	\
count	205.000000	205.000000	205.000000	205.000000	205.000000	205.000000	
mean	103.000000	0.834146	98.756585	174.049268	65.907805	53.724878	
std	59.322565	1.245307	6.021776	12.337289	2.145204	2.443522	
min	1.000000	-2.000000	86.600000	141.100000	60.300000	47.800000	
25%	52.000000	0.000000	94.500000	166.300000	64.100000	52.000000	
50%	103.000000	1.000000	97.000000	173.200000	65.500000	54.100000	
75%	154.000000	2.000000	102.400000	183.100000	66.900000	55.500000	
max	205.000000	3.000000	120.900000	208.100000	72.300000	59.800000	

	curbweight	enginesize	boreratio	stroke	compressionratio	\
count	205.000000	205.000000	205.000000	205.000000	205.000000	
mean	2555.565854	126.907317	3.329756	3.255415	10.142537	

```
std      520.680204   41.642693    0.270844    0.313597           3.972040
min     1488.000000   61.000000    2.540000    2.070000           7.000000
25%     2145.000000   97.000000    3.150000    3.110000           8.600000
50%     2414.000000  120.000000    3.310000    3.290000           9.000000
75%     2935.000000  141.000000    3.580000    3.410000           9.400000
max     4066.000000  326.000000    3.940000    4.170000          23.000000

       horsepower      peakrpm     citympg  highwaympg         price
count  205.000000   205.000000  205.000000  205.000000    205.000000
mean   104.117073  5125.121951   25.219512   30.751220  13276.710571
std     39.544167   476.985643    6.542142    6.886443   7988.852332
min     48.000000  4150.000000   13.000000   16.000000   5118.000000
25%     70.000000  4800.000000   19.000000   25.000000   7788.000000
50%     95.000000  5200.000000   24.000000   30.000000  10295.000000
75%    116.000000  5500.000000   30.000000   34.000000  16503.000000
max    288.000000  6600.000000   49.000000   54.000000  45400.000000
```

In [7]:
```
# Finding the percentage of missing values for all the columns separately
round(car.isnull().sum()/len(car.index), 2)*100
```

Out[7]:
```
car_ID              0.0
symboling           0.0
CarName             0.0
fueltype            0.0
aspiration          0.0
doornumber          0.0
carbody             0.0
drivewheel          0.0
enginelocation      0.0
wheelbase           0.0
carlength           0.0
carwidth            0.0
carheight           0.0
curbweight          0.0
enginetype          0.0
cylindernumber      0.0
enginesize          0.0
fuelsystem          0.0
boreratio           0.0
stroke              0.0
compressionratio    0.0
horsepower          0.0
peakrpm             0.0
citympg             0.0
highwaympg          0.0
price               0.0
dtype: float64
```

In [8]:
```
# Finding the duplicates(if any) and dropping them
```

```
car=car.drop_duplicates()
```

```
In [9]: #The attribute car_ID isn't needed for the price modeling. So,we drop it.
        car.drop('car_ID',axis=1,inplace=True)
```

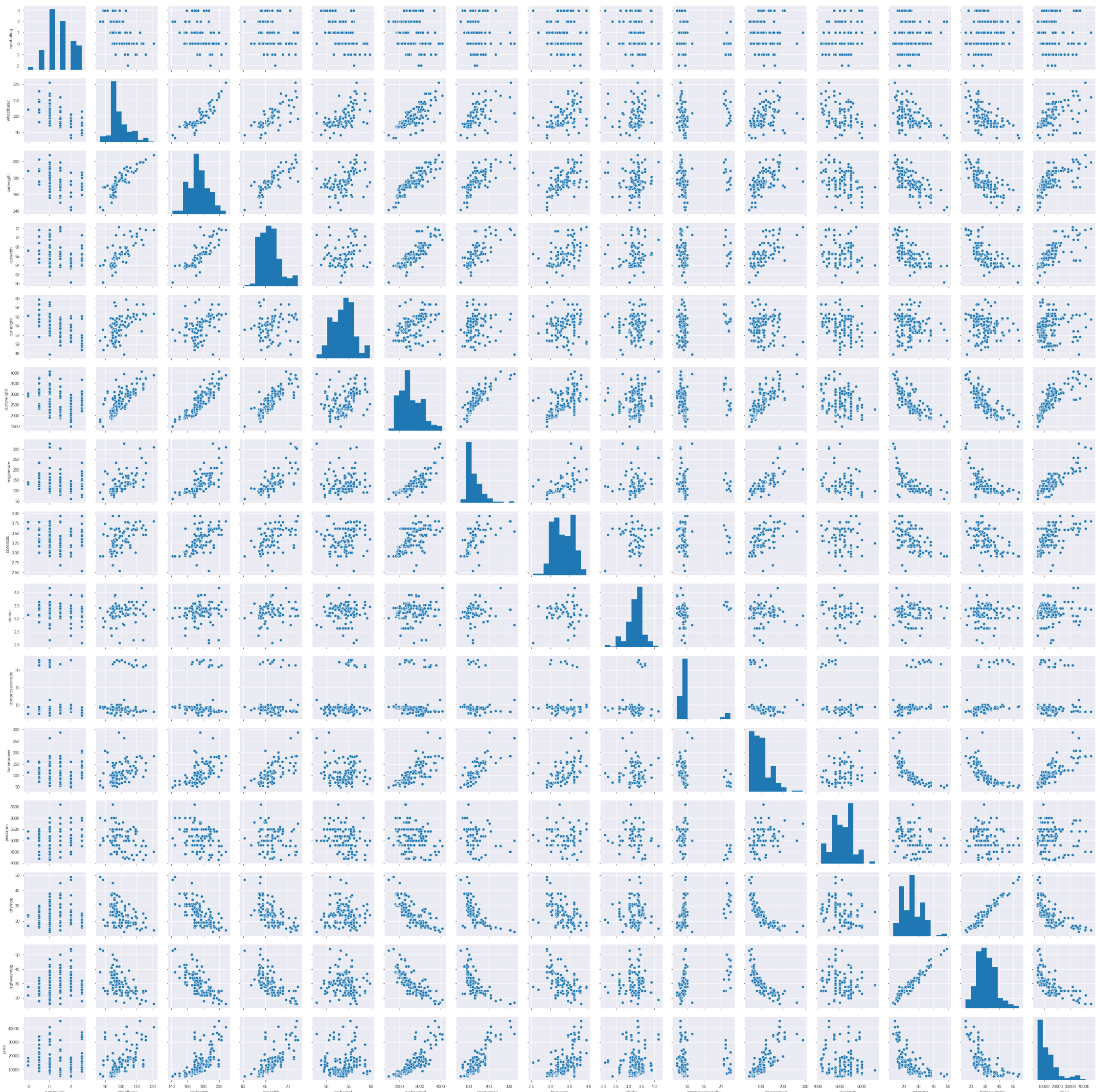
```
In [10]: #Function to fit Linear Regression using the statsmodels python package
         def fit_LR(X_train):
             X_train = sm.add_constant(X_train)
             lm = sm.OLS(y_train,X_train).fit()
             print(lm.summary())
             return lm
```

```
In [11]: #Function to Calculate the VIFs for the newly created model
         def VIF_get(X_train):
             vif = pd.DataFrame()
             X = X_train
             vif['Features'] = X.columns
             vif['VIF'] = [variance_inflation_factor(X.values, i) for i in range(X.shape[1])]
             vif['VIF'] = round(vif['VIF'], 2)
             vif = vif.sort_values(by = "VIF", ascending = False)
             return(vif)
```

1.1 Data Analysis :)

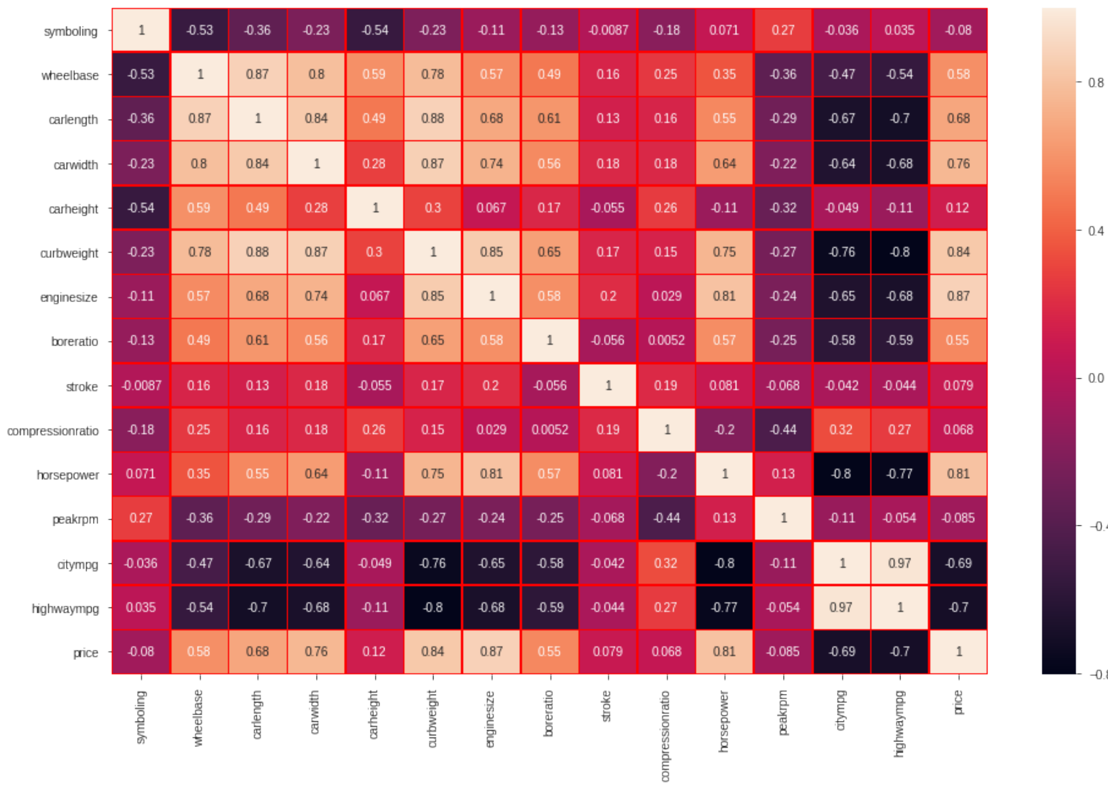
```
In [12]: sns.pairplot(car)
```

```
Out[12]: <seaborn.axisgrid.PairGrid at 0x7fb77312aeb8>
```



```
In [13]: plt.figure(figsize = (16, 10))
         sns.heatmap(car.corr(), annot = True,linewidths=0.5,linecolor='r')
```

```
Out[13]: <matplotlib.axes._subplots.AxesSubplot at 0x7fb74f366940>
```



Now, we can have an overall idea that which features are related to price. We observe that some features have some kind of positive relationship with *'price'*. These are given as follows:

- wheelbase
- carlength
- carwidth
- curbweight
- enginesize
- boreratio
- horsepower

However, there are some attributes/variables that show a negative relationship with *price*: * citympg * highwaympg

We can alsoe observe some multicollinearity visible between the predictor variables:

- *carlength* with *wheelbase*, *carwidth*, *curbweight*
- *curbweight* with *enginesize*, *carlength*, *carwidth*, *wheelbase*
- *enginesize* with *horsepower*, *crubweight* and dimensions of car
- *highwaympg* and *citympg* are highly correlated (~ 0.97).

Let us observe the categorical variables present in our data.

Symboling is a categorical variable which has been considered as a numeric *int64* type variable. Let us convert the *symboling* variable.

Here Symboling is the assigned insurance risk rating - A value of +3 indicates that the auto is risky, -3 that it is probably pretty safe.

Let us define the category clearly by classifying it on the basis of the risk:

- -ve symboling as safe
- 0, 1 as moderate
- 2,3 as risky

```
In [14]: car['symboling'] = car['symboling'].map({-2: 'safe',-1: 'safe',0: 'moderate',1: 'modera
```

CarName comprises of 2 parts: * the first word is the name of 'car company'. * the second is the 'car model'.

For example, chevrolet impala has 'chevrolet' as the car company name and 'impala' as the car model name. We need to only consider the compnay name as the independent variable for making the model.

```
In [15]: car['car_company'] = car['CarName'].apply(lambda x: x.split( )[0])
         car.drop('CarName',axis=1,inplace=True)

In [16]: car['car_company'].unique()

Out[16]: array(['alfa-romero', 'audi', 'bmw', 'chevrolet', 'dodge', 'honda',
                'isuzu', 'jaguar', 'maxda', 'mazda', 'buick', 'mercury',
                'mitsubishi', 'Nissan', 'nissan', 'peugeot', 'plymouth', 'porsche',
                'porcshce', 'renault', 'saab', 'subaru', 'toyota', 'toyouta',
                'vokswagen', 'volkswagen', 'vw', 'volvo'], dtype=object)
```

We notice that certain company names have been misspelled here. Let us correct them in our dataset.

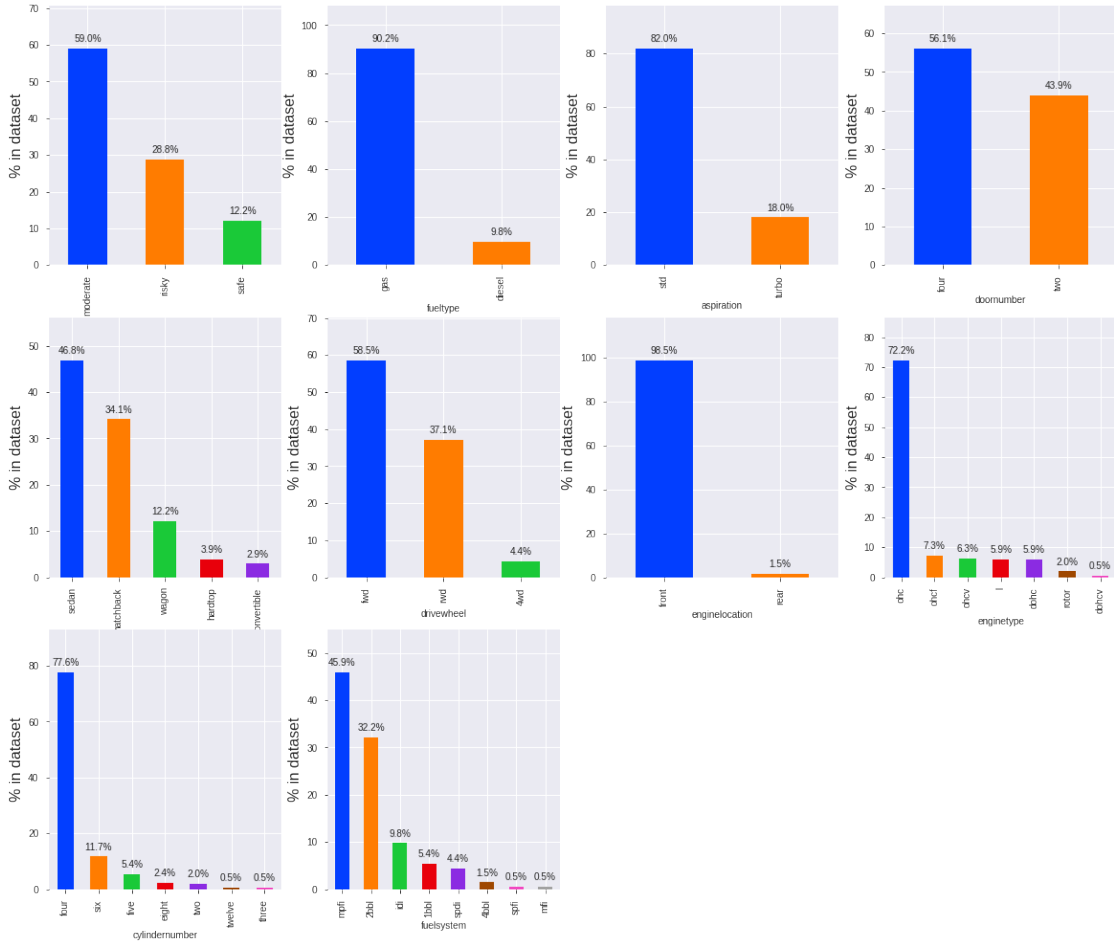
```
In [17]: car['car_company'].replace('maxda','mazda',inplace=True)
         car['car_company'].replace('Nissan','nissan',inplace=True)
         car['car_company'].replace('porcshce','porsche',inplace=True)
         car['car_company'].replace('toyouta','toyota',inplace=True)
         car['car_company'].replace(['vokswagen','vw'],'volkswagen',inplace=True)

In [18]: cat_vars = list(car.columns[car.dtypes == 'object'])
         cat_vars

Out[18]: ['symboling',
          'fueltype',
          'aspiration',
          'doornumber',
          'carbody',
          'drivewheel',
          'enginelocation',
          'enginetype',
          'cylindernumber',
          'fuelsystem',
          'car_company']
```

In [19]:
```
#Taken from https://stackoverflow.com/a/48372659 and edited
def showLabels(ax, d=None):
    plt.margins(0.2, 0.2)
    rects = ax.patches
    i = 0
    locs, labels = plt.xticks()
    counts = {}
    if not d is None:
        for key, value in d.items():
            counts[str(key)] = value

    # For each bar: Place a label
    for rect in rects:
        # Get X and Y placement of label from rect.
        y_value = rect.get_height()
        x_value = rect.get_x() + rect.get_width() / 2

        # Number of points between bar and label. Change to your liking.
        space = 5
        # Vertical alignment for positive values
        va = 'bottom'

        # If value of bar is negative: Place label below bar
        if y_value < 0:
            # Invert space to place label below
            space *= -1
            # Vertically align label at top
            va = 'top'

        # Use Y value as label and format number with one decimal place
        if d is None:
            label = "{:.1f}%".format(y_value)
        else:
            try:
                label = "{:.1f}%".format(y_value) + "\nof " + str(counts[str(labels[i].
            except:
                label = "{:.1f}%".format(y_value)

        i = i+1

        # Create annotation
        plt.annotate(
            label,                      # Use `label` as label
            (x_value, y_value),         # Place label at end of the bar
            xytext=(0, space),          # Vertically shift label by `space`
            textcoords="offset points", # Interpret `xytext` as offset in points
            ha='center',                # Horizontally center label
            va=va)                      # Vertically align label differently for
```

```
                                            # positive and negative values.
```

In [20]:
```
def plot(df,var_list, sortbyindex=False):
    #Plot the percentage of car's in the US market with respect to different car featur
    #plt.figure(figsize=(20, 17))
    for var in var_list:
        plt.subplot(3,4,var_list.index(var)+1)
        values = (df[var].value_counts(normalize=True)*100)
        if sortbyindex:
                values = values.sort_index()
        ax = values.plot.bar(color=sns.color_palette('bright', 16))
        ax.set_ylabel('% in dataset', fontsize=16)
        ax.set_xlabel(var, fontsize=10)
        showLabels(ax)


def plot_cat(var_list):
    #Function to plot a list of categorical variables together
    plt.figure(figsize=(20, 15))
    for var in var_list:
        plt.subplot(3,4,var_list.index(var)+1)
        sns.boxplot(x = var, y = 'price', data = car)
    plt.show()
```

In [21]:
```
#plotting market percentage of car's with respect to features
plt.figure(figsize=(20, 17))
plot(car,cat_vars[:-1])
```

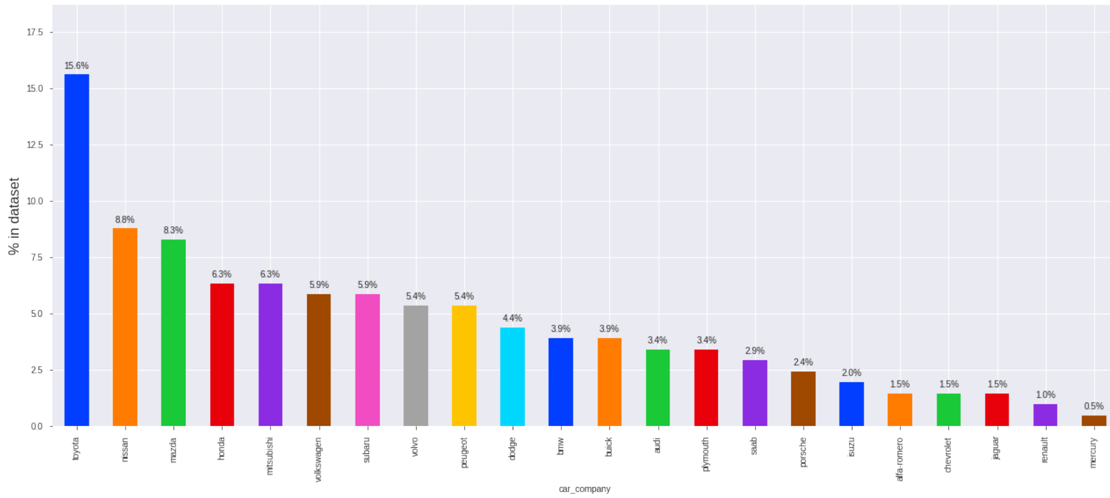


We can see that some of the car features are popular in the US Automobile Market. Geely Automotives can consider these analysis results to satisfy the market needs and make the required changes to their manufacturing line.

- symboling: moderate (0,1)
- Carbody: Sedan
- fueltype: gas
- aspiration: standard
- doornumbers: four
- drivewheel: forward
- engine location: front
- engine type: ohc
- cylinderNumber: four
- fuelSystem: mpfi

Let us see that which company most popular in the US Automobile Market

In [22]:
```
plt.figure(figsize=(100, 30))
plot(car,[cat_vars[-1]])
```

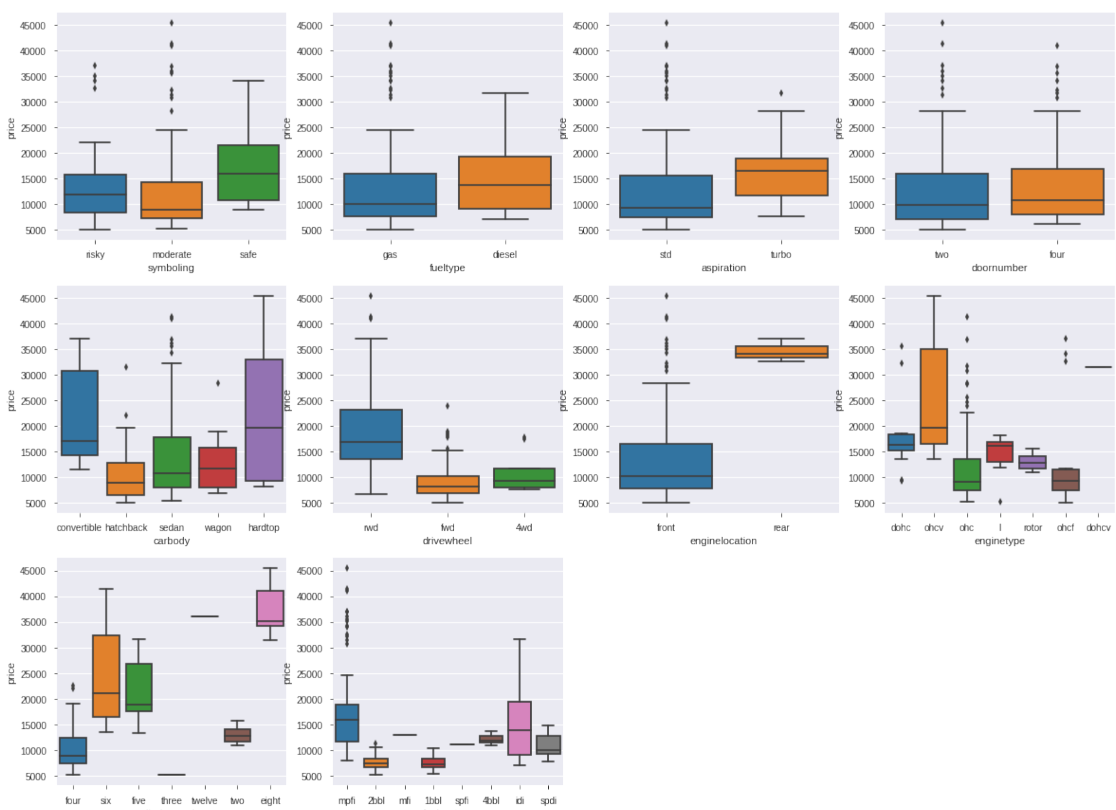


We can clearly see that Toyota clearly dominates with 15.6%.

This is in turn followed by: * Nissan with 8.8% * Mazda with 8.2% * Mitsubishi and Honda are equally popular with 6.3% market percentage.

Let's see how these categorical variables relate individually with price.

```
In [23]: # ploting all the attributes excluding car_company against price
         plot_cat(cat_vars[:-1])
```



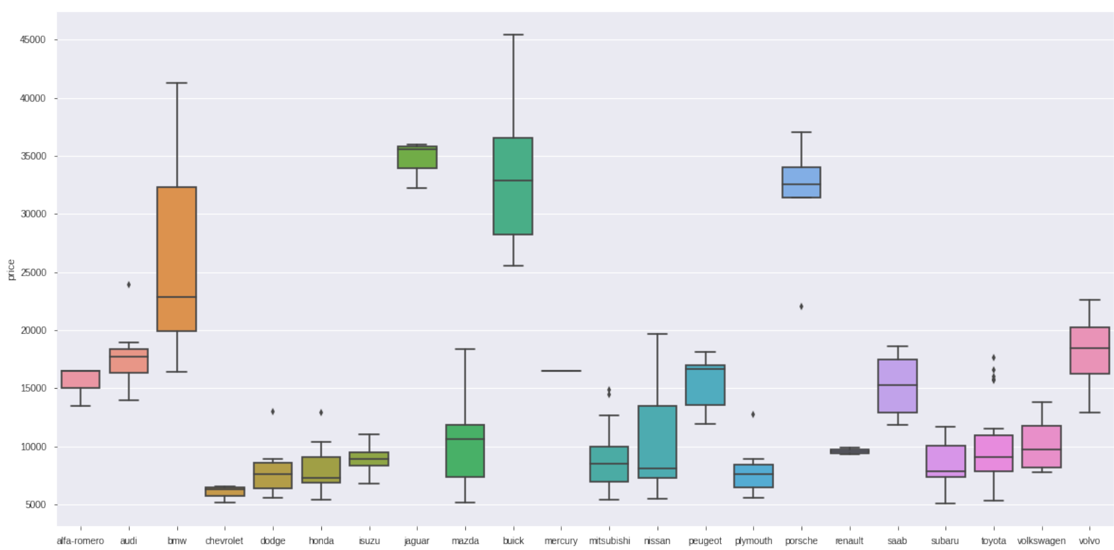
INSIGHTS:

- The fuel type tends to have an effect on the price of the cars.
- enginelocation and aspiration have a significant affect on the pricing of the car.
- The price of real wheel drive is quite higher that other drivewheel options.
- cylindernumber and engine type also seem to regulate the price of cars.
- We also observe that hardtop and convertables cars are priced quite higher than the other body types.

Let us see how the prices vary with numerous automobile companies in the US market.

```
In [30]: plt.figure(figsize=(20,10))
         sns.boxplot(x = cat_vars[-1], y = 'price', data = car)

         print('Average US car price: ',car['price'].mean())
         print(car.groupby('car_company').price.mean().sort_values(ascending=False).head())
```

```
Average US car price:  13276.710570731706
car_company
jaguar     34600.000000
buick      33647.000000
porsche    31400.500000
bmw        26118.750000
volvo      18063.181818
Name: price, dtype: float64
```

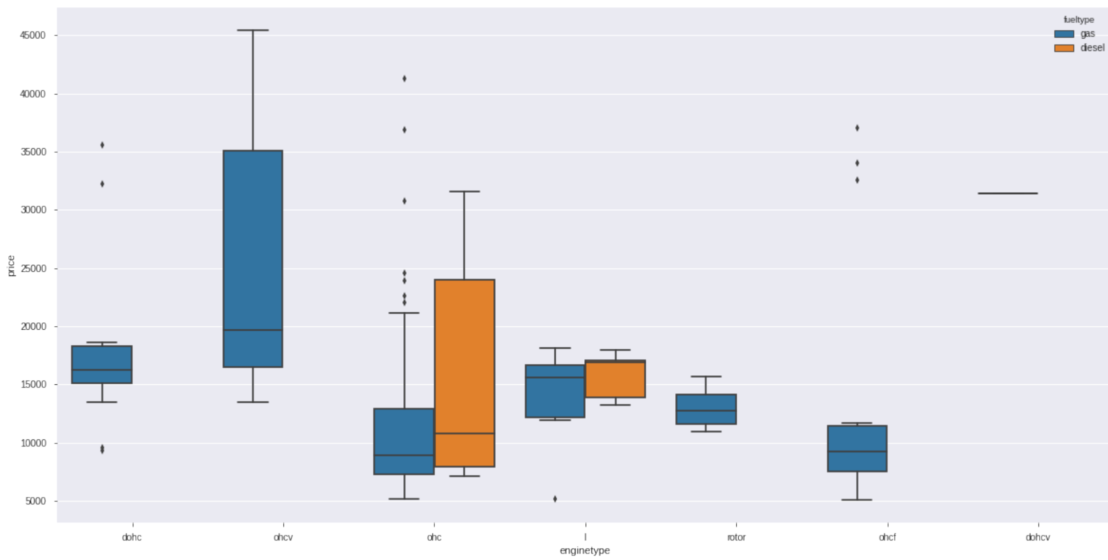


INSIGHTS:

- There are some outliers for the companies which demonstrates that they do manufacture some expensive cars that are priced above their usual market pricing range.
- Cars manufacturers like Jaguar, Buick, Porsche, BMW and Volo are on the high end side and are priced well above the other US cars which have an average price of approx. $13,000.
- Also, company name shows some affect on the price determination.

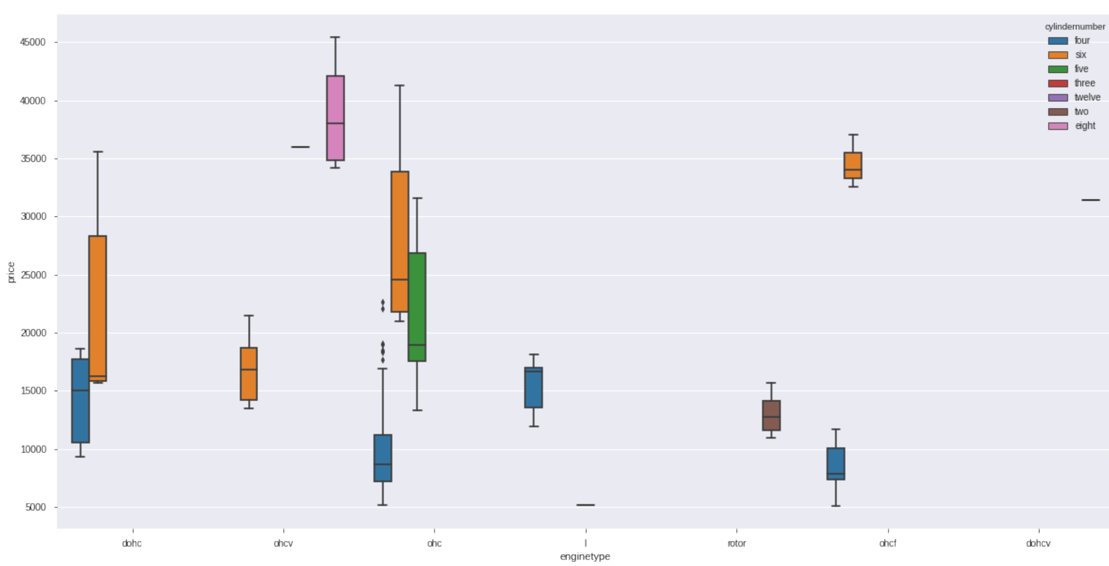
```
In [25]: plt.figure(figsize=(20,10))
         sns.boxplot(x='enginetype', y='price', hue = 'fueltype', data=car)
```

```
Out[25]: <matplotlib.axes._subplots.AxesSubplot at 0x7fb74cc4c748>
```



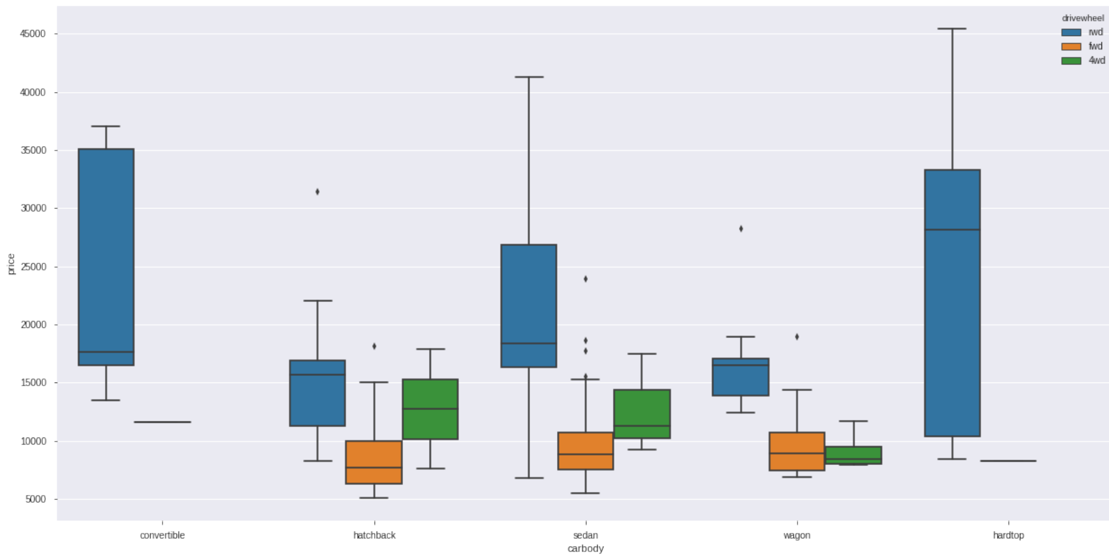

```
In [26]: plt.figure(figsize=(20,10))
         sns.boxplot(x='enginetype', y='price', hue = 'cylindernumber', data=car)
```

```
Out[26]: <matplotlib.axes._subplots.AxesSubplot at 0x7fb74c68def0>
```



```
In [27]: plt.figure(figsize=(20,10))
         sns.boxplot(x='carbody', y='price', hue = 'drivewheel', data=car)
```

```
Out[27]: <matplotlib.axes._subplots.AxesSubplot at 0x7fb74c5d19b0>
```



1.1.1 Outlier treatment

It is extremely necessary that we treat the outliers before modeling as Linear Regression is sensitive to outliers.

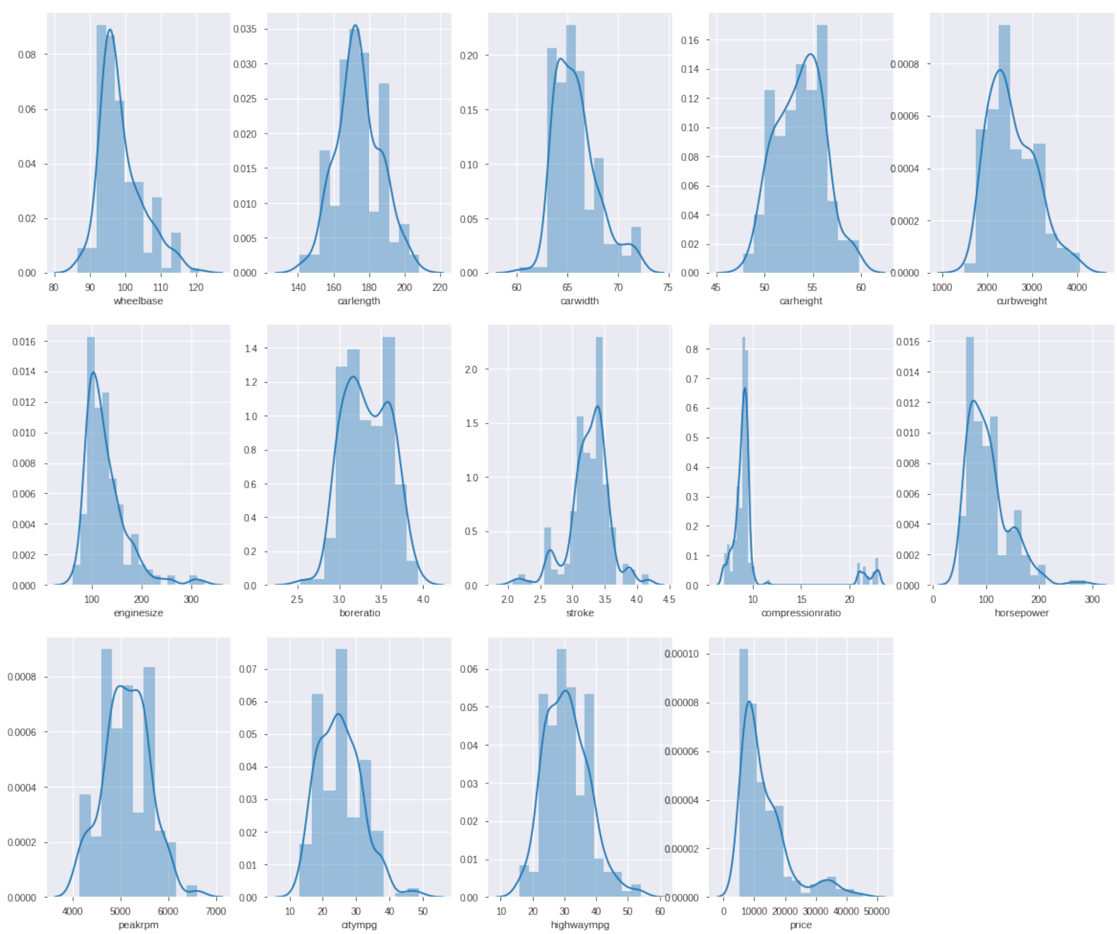
```
In [28]: numeric_variables = list(car.columns[car.dtypes != 'object'])
         print(numeric_variables)
```

```
['wheelbase', 'carlength', 'carwidth', 'carheight', 'curbweight', 'enginesize', 'boreratio', 'st
```

```
In [29]: plt.figure(figsize=(20, 17))
         plt.subplot(3,5,1)
         sns.distplot(car['wheelbase'])
         plt.subplot(3,5,2)
         sns.distplot(car['carlength'])
         plt.subplot(3,5,3)
         sns.distplot(car['carwidth'])
         plt.subplot(3,5,4)
         sns.distplot(car['carheight'])
         plt.subplot(3,5,5)
```

```
sns.distplot(car['curbweight'])
plt.subplot(3,5,6)
sns.distplot(car['enginesize'])
plt.subplot(3,5,7)
sns.distplot(car['boreratio'])
plt.subplot(3,5,8)
sns.distplot(car['stroke'])
plt.subplot(3,5,9)
sns.distplot(car['compressionratio'])
plt.subplot(3,5,10)
sns.distplot(car['horsepower'])
plt.subplot(3,5,11)
sns.distplot(car['peakrpm'])
plt.subplot(3,5,12)
sns.distplot(car['citympg'])
plt.subplot(3,5,13)
sns.distplot(car['highwaympg'])
plt.subplot(3,5,14)
sns.distplot(car['price'])
```

Out[29]: <matplotlib.axes._subplots.AxesSubplot at 0x7fb74c265e80>



INSIGHT:

We notice that enginesize, horsepower and compression ratio variables tend to have a right skewed distribution which is probably due to the presence of the outliers

```
In [32]: print(car[['horsepower','curbweight','enginesize']].quantile([0.01, .96]))
         print(car[['compressionratio']].quantile([0.01, .90]))
```

```
      horsepower  curbweight  enginesize
0.01       52.12     1819.72        70.0
0.96      182.00     3657.80       209.0
      compressionratio
0.01              7.00
0.90             10.94
```

I decide to treat the outliers by clipping the variables curbweight, horsepower, enginesize at 96 precentile value. And clip compressionratio at 90 percentile value

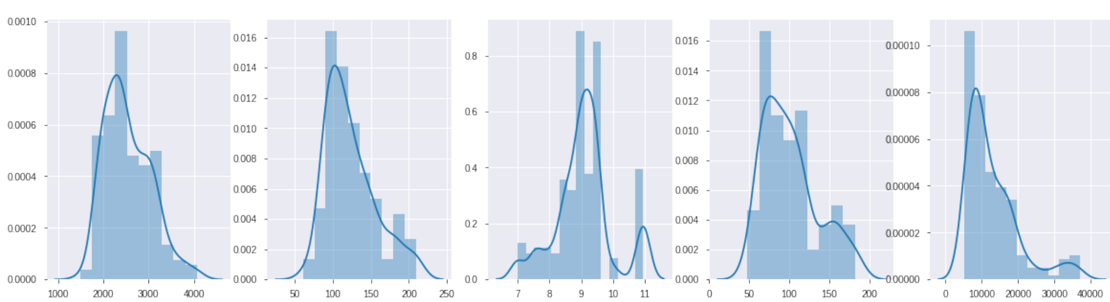
```
In [33]: # Outliers in the price of cars
         car['horsepower'][np.abs(car['horsepower'] > 182.00)]= 182.00
         car['horsepower'][np.abs(car['horsepower'] > 3657.80)]= 3657.80
         car['enginesize'][np.abs(car['enginesize'] > 209.00)]= 209.00
         car['compressionratio'][np.abs(car['compressionratio'] > 10.94)]= 10.94
```

Let us preserve the data for which the price is less than 3 standard deviation.

```
In [34]: car = car[np.abs(car.price-car.price.mean()) <= (3*car.price.std())]
```

```
In [35]: plt.figure(figsize=(20, 5))
         plt.subplot(1,5,1)
         sns.distplot(car['curbweight'])
         plt.subplot(1,5,2)
         sns.distplot(car['enginesize'])
         plt.subplot(1,5,3)
         sns.distplot(car['compressionratio'])
         plt.subplot(1,5,4)
         sns.distplot(car['horsepower'])
         plt.subplot(1,5,5)
         sns.distplot(car['price'])
```

```
Out[35]: <matplotlib.axes._subplots.AxesSubplot at 0x7f940721b358>
```



1.1.2 Creating fresh variables

This will be acting as an aid to remove corelated variables.

Earlier, from the scatter plot and the heatmap, we noticed there is a high correlation between carlength, wheelbase, car width, car weight and city/highway mpg's. Let us create new variables from these to try reducing the multicollinearlity.

```
In [36]: car['car_lw_ratio'] = car.carlength/car.carwidth
         car['car_wh_ratio'] = car.carwidth/car.carheight
         car['pw_ratio'] = car.horsepower/car.curbweight
         car['hc_mpg_ratio'] = car.highwaympg/car.citympg

         ## dropping the orignal variables
         car.drop(['carlength','carwidth','carheight','highwaympg','citympg'],axis=1,inplace=Tru
```

We saw that the company brand value also is determinig the price of the car. Let us create a company_seg categorical variable which would tell us that under which segment tier does the car_company comes under. We will segment the car companies based on the mean company price as

- lowtier if company mean price is below 10,000
- midtier if company mean price is above 10,000 and below 20,000
- hightier if company mean price is above 20,000

```
In [37]: car.groupby('car_company').price.mean().sort_values(ascending=False)

Out[37]: car_company
         jaguar         34600.000000
         porsche        31400.500000
         buick          30469.333333
         bmw            23947.857143
         volvo          18063.181818
         audi           17859.166714
         mercury        16503.000000
         alfa-romero    15498.333333
         peugeot        15489.090909
         saab           15223.333333
         mazda          10652.882353
         nissan         10415.666667
         volkswagen     10077.500000
         toyota          9885.812500
         renault         9595.000000
         mitsubishi      9239.769231
         isuzu           8916.500000
         subaru          8541.250000
         honda           8184.692308
```

```
plymouth        7963.428571
dodge           7875.444444
chevrolet       6007.000000
Name: price, dtype: float64
```

In [38]:
```python
company_seg_dict = {
    'cheverolet' : 'lowtier',
    'dodge' : 'lowtier',
    'plymouth' : 'lowtier',
    'honda' : 'lowtier',
    'subaru' : 'lowtier',
    'isuzu' : 'lowtier',
    'mitsubishi' : 'lowtier',
    'renault' : 'lowtier',
    'toyota' : 'lowtier',
    'volkswagen' : 'midtier',
    'nissan' : 'midtier',
    'mazda' : 'midtier',
    'saab' : 'midtier',
    'peugeot' : 'midtier',
    'alfa-romero' : 'midtier',
    'mercury' : 'midtier',
    'audi' : 'midtier',
    'volvo' : 'midtier',
    'bmw' : 'hightier',
    'buick' : 'hightier',
    'porsche' : 'hightier',
    'jaguar' : 'hightier',
    }
car['company_seg'] = car['car_company'].map(company_seg_dict)
# Dropping the orignal car_company variable
car.drop('car_company',axis=1,inplace=True)
car.head()
```

Out[38]:
```
  symboling fueltype aspiration doornumber      carbody drivewheel  \
0     risky      gas        std        two  convertible        rwd
1     risky      gas        std        two  convertible        rwd
2  moderate      gas        std        two    hatchback        rwd
3     risky      gas        std       four        sedan        fwd
4     risky      gas        std       four        sedan        4wd

  enginelocation  wheelbase  curbweight enginetype  ... stroke  \
0          front       88.6        2548       dohc  ...   2.68
1          front       88.6        2548       dohc  ...   2.68
2          front       94.5        2823       ohcv  ...   3.47
3          front       99.8        2337        ohc  ...   3.40
4          front       99.4        2824        ohc  ...   3.40
```

```
   compressionratio horsepower  peakrpm    price  car_lw_ratio  car_wh_ratio  \
0               9.0        111     5000  13495.0      2.633385      1.313525
1               9.0        111     5000  16500.0      2.633385      1.313525
2               9.0        154     5000  16500.0      2.613740      1.250000
3              10.0        102     5500  13950.0      2.667674      1.219153
4               8.0        115     5500  17450.0      2.659639      1.222836

   pw_ratio  hc_mpg_ratio  company_seg
0  0.043564      1.285714      midtier
1  0.043564      1.285714      midtier
2  0.054552      1.368421      midtier
3  0.043646      1.250000      midtier
4  0.040722      1.222222      midtier

[5 rows x 24 columns]
```

In [39]:
```python
car.groupby('company_seg').price.mean()
```

Out[39]:
```
company_seg
hightier    29107.309524
lowtier      8987.369565
midtier     13231.839151
Name: price, dtype: float64
```

Handling Categorical Variable for Linear Regression

In [40]:
```python
# Converting categorical variables with two levels to either 1 or 0
car['fueltype'] = car['fueltype'].map({'gas': 1, 'diesel': 0})
car['aspiration'] = car['aspiration'].map({'std': 1, 'turbo': 0})
car['doornumber'] = car['doornumber'].map({'two': 1, 'four': 0})
car['enginelocation'] = car['enginelocation'].map({'front': 1, 'rear': 0})
car.head()
```

Out[40]:
```
   symboling  fueltype  aspiration  doornumber      carbody drivewheel  \
0     risky         1           1           1  convertible        rwd
1     risky         1           1           1  convertible        rwd
2  moderate         1           1           1    hatchback        rwd
3     risky         1           1           0        sedan        fwd
4     risky         1           1           0        sedan        4wd

   enginelocation  wheelbase  curbweight enginetype  ... stroke  \
0               1       88.6        2548       dohc  ...   2.68
1               1       88.6        2548       dohc  ...   2.68
2               1       94.5        2823       ohcv  ...   3.47
3               1       99.8        2337        ohc  ...   3.40
4               1       99.4        2824        ohc  ...   3.40

   compressionratio horsepower  peakrpm    price  car_lw_ratio  car_wh_ratio  \
```

```
0               9.0        111     5000  13495.0      2.633385      1.313525
1               9.0        111     5000  16500.0      2.633385      1.313525
2               9.0        154     5000  16500.0      2.613740      1.250000
3              10.0        102     5500  13950.0      2.667674      1.219153
4               8.0        115     5500  17450.0      2.659639      1.222836

   pw_ratio  hc_mpg_ratio  company_seg
0  0.043564      1.285714      midtier
1  0.043564      1.285714      midtier
2  0.054552      1.368421      midtier
3  0.043646      1.250000      midtier
4  0.040722      1.222222      midtier

[5 rows x 24 columns]
```

In [41]:
```
# Creating dummy variables
df = pd.get_dummies(car)

# Droping 1 dummy variable and Keeping n-1 variables for each feature
df.drop(['symboling_risky',
         'carbody_hatchback',
         'drivewheel_4wd',
         'enginetype_l',
         'cylindernumber_three',
         'fuelsystem_1bbl',
         'company_seg_lowtier'],axis=1,inplace=True)
df.columns
```

Out[41]:
```
Index(['fueltype', 'aspiration', 'doornumber', 'enginelocation', 'wheelbase',
       'curbweight', 'enginesize', 'boreratio', 'stroke', 'compressionratio',
       'horsepower', 'peakrpm', 'price', 'car_lw_ratio', 'car_wh_ratio',
       'pw_ratio', 'hc_mpg_ratio', 'symboling_moderate', 'symboling_safe',
       'carbody_convertible', 'carbody_hardtop', 'carbody_sedan',
       'carbody_wagon', 'drivewheel_fwd', 'drivewheel_rwd', 'enginetype_dohc',
       'enginetype_dohcv', 'enginetype_ohc', 'enginetype_ohcf',
       'enginetype_ohcv', 'enginetype_rotor', 'cylindernumber_eight',
       'cylindernumber_five', 'cylindernumber_four', 'cylindernumber_six',
       'cylindernumber_twelve', 'cylindernumber_two', 'fuelsystem_2bbl',
       'fuelsystem_4bbl', 'fuelsystem_idi', 'fuelsystem_mfi',
       'fuelsystem_mpfi', 'fuelsystem_spdi', 'fuelsystem_spfi',
       'company_seg_hightier', 'company_seg_midtier'],
      dtype='object')
```

Splitting Dataset into Training/Test set

In [42]:
```
# Splitting the avilable data into training and testing set.

df_train, df_test = train_test_split(df, train_size = 0.7, test_size = 0.3, random_stat
```

Scaling of Features

```
In [43]: # Scaling all the numeric variables in the same scale between 0 and 1.
         scaler = MinMaxScaler()

In [44]: # Applying scaler() to all the columns except the 'yes-no' and 'dummy' variables
         num_vars = ['horsepower','wheelbase','curbweight', 'enginesize', 'boreratio','car_lw_ra

         df_train[num_vars] = scaler.fit_transform(df_train[num_vars])

         df_train.head()

Out[44]:      fueltype  aspiration  doornumber  enginelocation  wheelbase  curbweight  \
         1           1           1           1               1   0.068966    0.411171
         165         1           1           1               1   0.272414    0.301396
         197         1           1           0               1   0.610345    0.602793
         169         1           1           1               1   0.406897    0.412335
         190         1           1           1               1   0.272414    0.284329

              enginesize  boreratio    stroke  compressionratio  ...  \
         1      0.466216   0.664286  0.290476          0.507614  ...
         165    0.250000   0.500000  0.480952          0.609137  ...
         197    0.540541   0.885714  0.514286          0.634518  ...
         169    0.574324   0.771429  0.680952          0.583756  ...
         190    0.324324   0.464286  0.633333          0.380711  ...

              cylindernumber_two  fuelsystem_2bbl  fuelsystem_4bbl  fuelsystem_idi  \
         1                     0                0                0               0
         165                   0                0                0               0
         197                   0                0                0               0
         169                   0                0                0               0
         190                   0                0                0               0

              fuelsystem_mfi  fuelsystem_mpfi  fuelsystem_spdi  fuelsystem_spfi  \
         1                 0                1                0                0
         165               0                1                0                0
         197               0                1                0                0
         169               0                1                0                0
         190               0                1                0                0

              company_seg_hightier  company_seg_midtier
         1                       0                    1
         165                     0                    0
         197                     0                    1
         169                     0                    0
         190                     0                    1

         [5 rows x 46 columns]
```

In [45]: df_train.describe()

Out[45]:

```
          fueltype  aspiration  doornumber  enginelocation   wheelbase  \
count  141.000000  141.000000  141.000000      141.000000  141.000000
mean     0.886525    0.794326    0.439716        0.985816    0.413402
std      0.318304    0.405634    0.498122        0.118672    0.198892
min      0.000000    0.000000    0.000000        0.000000    0.000000
25%      1.000000    1.000000    0.000000        1.000000    0.272414
50%      1.000000    1.000000    0.000000        1.000000    0.358621
75%      1.000000    1.000000    1.000000        1.000000    0.503448
max      1.000000    1.000000    1.000000        1.000000    1.000000

        curbweight  enginesize   boreratio      stroke  compressionratio  ...  \
count  141.000000  141.000000  141.000000  141.000000        141.000000  ...
mean     0.411779    0.420979    0.559980    0.551570          0.536415  ...
std      0.198452    0.225970    0.192528    0.150278          0.229812  ...
min      0.000000    0.000000    0.000000    0.000000          0.000000  ...
25%      0.254849    0.243243    0.421429    0.490476          0.406091  ...
50%      0.359193    0.337838    0.550000    0.571429          0.507614  ...
75%      0.552366    0.540541    0.742857    0.633333          0.609137  ...
max      1.000000    1.000000    1.000000    1.000000          1.000000  ...

        cylindernumber_two  fuelsystem_2bbl  fuelsystem_4bbl  fuelsystem_idi  \
count           141.000000       141.000000       141.000000      141.000000
mean              0.028369         0.333333         0.021277        0.113475
std               0.166616         0.473085         0.144819        0.318304
min               0.000000         0.000000         0.000000        0.000000
25%               0.000000         0.000000         0.000000        0.000000
50%               0.000000         0.000000         0.000000        0.000000
75%               0.000000         1.000000         0.000000        0.000000
max               1.000000         1.000000         1.000000        1.000000

        fuelsystem_mfi  fuelsystem_mpfi  fuelsystem_spdi  fuelsystem_spfi  \
count           141.0       141.000000       141.000000       141.000000
mean              0.0         0.425532         0.042553         0.007092
std               0.0         0.496186         0.202567         0.084215
min               0.0         0.000000         0.000000         0.000000
25%               0.0         0.000000         0.000000         0.000000
50%               0.0         0.000000         0.000000         0.000000
75%               0.0         1.000000         0.000000         0.000000
max               0.0         1.000000         1.000000         1.000000

        company_seg_hightier  company_seg_midtier
count             141.000000           141.000000
mean                0.106383             0.468085
std                 0.309426             0.500759
min                 0.000000             0.000000
25%                 0.000000             0.000000
```

```
50%                 0.000000            0.000000
75%                 0.000000            1.000000
max                 1.000000            1.000000

[8 rows x 46 columns]
```

In [46]:
```
y_train = df_train.pop('price')
X_train = df_train
```

1.1.3 Building our model

Let us follow a mixed approach. 1. Initially, we shall use the LinearRegression function from SciKit Learn for its compatibility with RFE. 2. Then we will be using the statsmodels for statistics analysis of the model

In [47]:
```
# Running RFE
lm = LinearRegression()
lm.fit(X_train, y_train)

rfe = RFE(lm, 15) # running RFE and selecting  best 15 features for describing the pric
rfe = rfe.fit(X_train, y_train)
```

In [48]:
```
list(zip(X_train.columns,rfe.support_,rfe.ranking_))
```

Out[48]:
```
[('fueltype', False, 15),
 ('aspiration', False, 9),
 ('doornumber', False, 24),
 ('enginelocation', True, 1),
 ('wheelbase', True, 1),
 ('curbweight', False, 30),
 ('enginesize', False, 8),
 ('boreratio', False, 6),
 ('stroke', True, 1),
 ('compressionratio', False, 11),
 ('horsepower', True, 1),
 ('peakrpm', True, 1),
 ('car_lw_ratio', True, 1),
 ('car_wh_ratio', True, 1),
 ('pw_ratio', True, 1),
 ('hc_mpg_ratio', False, 22),
 ('symboling_moderate', False, 28),
 ('symboling_safe', False, 27),
 ('carbody_convertible', True, 1),
 ('carbody_hardtop', False, 21),
 ('carbody_sedan', False, 17),
 ('carbody_wagon', False, 25),
 ('drivewheel_fwd', False, 29),
 ('drivewheel_rwd', False, 26),
 ('enginetype_dohc', False, 3),
```

```
 ('enginetype_dohcv', False, 2),
 ('enginetype_ohc', True, 1),
 ('enginetype_ohcf', False, 4),
 ('enginetype_ohcv', False, 12),
 ('enginetype_rotor', False, 16),
 ('cylindernumber_eight', False, 13),
 ('cylindernumber_five', True, 1),
 ('cylindernumber_four', True, 1),
 ('cylindernumber_six', True, 1),
 ('cylindernumber_twelve', False, 7),
 ('cylindernumber_two', False, 18),
 ('fuelsystem_2bbl', False, 19),
 ('fuelsystem_4bbl', True, 1),
 ('fuelsystem_idi', False, 10),
 ('fuelsystem_mfi', False, 31),
 ('fuelsystem_mpfi', False, 23),
 ('fuelsystem_spdi', False, 14),
 ('fuelsystem_spfi', False, 20),
 ('company_seg_hightier', True, 1),
 ('company_seg_midtier', False, 5)]
```

```
In [49]: cols = X_train.columns[rfe.support_]
         cols
```

```
Out[49]: Index(['enginelocation', 'wheelbase', 'stroke', 'horsepower', 'peakrpm',
                'car_lw_ratio', 'car_wh_ratio', 'pw_ratio', 'carbody_convertible',
                'enginetype_ohc', 'cylindernumber_five', 'cylindernumber_four',
                'cylindernumber_six', 'fuelsystem_4bbl', 'company_seg_hightier'],
               dtype='object')
```

```
In [50]: X_train.columns[~rfe.support_]
```

```
Out[50]: Index(['fueltype', 'aspiration', 'doornumber', 'curbweight', 'enginesize',
                'boreratio', 'compressionratio', 'hc_mpg_ratio', 'symboling_moderate',
                'symboling_safe', 'carbody_hardtop', 'carbody_sedan', 'carbody_wagon',
                'drivewheel_fwd', 'drivewheel_rwd', 'enginetype_dohc',
                'enginetype_dohcv', 'enginetype_ohcf', 'enginetype_ohcv',
                'enginetype_rotor', 'cylindernumber_eight', 'cylindernumber_twelve',
                'cylindernumber_two', 'fuelsystem_2bbl', 'fuelsystem_idi',
                'fuelsystem_mfi', 'fuelsystem_mpfi', 'fuelsystem_spdi',
                'fuelsystem_spfi', 'company_seg_midtier'],
               dtype='object')
```

1.1.4 Now, Building model using statsmodels library, for the detailed statistics

```
In [51]: # Creating X_test dataframe with RFE selected variables
         X_train_rfe = X_train[cols]
```

```
In [52]: lm=fit_LR(X_train_rfe)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                  price   R-squared:                       0.954
Model:                            OLS   Adj. R-squared:                  0.949
Method:                 Least Squares   F-statistic:                     173.0
Date:                Mon, 25 Nov 2019   Prob (F-statistic):           6.66e-76
Time:                        01:32:21   Log-Likelihood:                 225.25
No. Observations:                 141   AIC:                            -418.5
Df Residuals:                     125   BIC:                            -371.3
Df Model:                          15
Covariance Type:            nonrobust
========================================================================================
                           coef    std err          t      P>|t|      [0.025      0.975]
----------------------------------------------------------------------------------------
const                    0.5136      0.069      7.457      0.000       0.377       0.650
enginelocation          -0.3728      0.049     -7.680      0.000      -0.469      -0.277
wheelbase                0.2549      0.047      5.402      0.000       0.162       0.348
stroke                  -0.0962      0.037     -2.628      0.010      -0.169      -0.024
horsepower               0.7717      0.078      9.929      0.000       0.618       0.926
peakrpm                  0.0829      0.031      2.706      0.008       0.022       0.144
car_lw_ratio            -0.0557      0.043     -1.306      0.194      -0.140       0.029
car_wh_ratio             0.1182      0.044      2.663      0.009       0.030       0.206
pw_ratio                -0.6346      0.089     -7.162      0.000      -0.810      -0.459
carbody_convertible      0.1551      0.030      5.240      0.000       0.096       0.214
enginetype_ohc           0.0435      0.015      2.957      0.004       0.014       0.073
cylindernumber_five     -0.0517      0.033     -1.562      0.121      -0.117       0.014
cylindernumber_four     -0.1085      0.031     -3.491      0.001      -0.170      -0.047
cylindernumber_six      -0.0511      0.030     -1.711      0.090      -0.110       0.008
fuelsystem_4bbl         -0.0468      0.042     -1.121      0.264      -0.129       0.036
company_seg_hightier     0.2152      0.022      9.628      0.000       0.171       0.259
==============================================================================
Omnibus:                        4.198   Durbin-Watson:                   2.089
Prob(Omnibus):                  0.123   Jarque-Bera (JB):                4.179
Skew:                           0.235   Prob(JB):                        0.124
Kurtosis:                       3.700   Cond. No.                         59.4
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

fuelsystem_4bbl has p-value > 0.05. Let's drop it.

In [53]:
```
X_train1 = X_train_rfe.drop('fuelsystem_4bbl', axis=1)
```

In [54]:
```
lm1=fit_LR(X_train1)
```

```
                            OLS Regression Results
==============================================================================
```

```
Dep. Variable:                  price   R-squared:                       0.954
Model:                            OLS   Adj. R-squared:                  0.948
Method:                 Least Squares   F-statistic:                     184.9
Date:                Mon, 25 Nov 2019   Prob (F-statistic):           8.94e-77
Time:                        01:32:21   Log-Likelihood:                 224.55
No. Observations:                 141   AIC:                            -419.1
Df Residuals:                     126   BIC:                            -374.9
Df Model:                          14
Covariance Type:            nonrobust
=========================================================================================
                            coef    std err          t      P>|t|      [0.025      0.975]
-----------------------------------------------------------------------------------------
const                     0.5018      0.068      7.365      0.000       0.367       0.637
enginelocation           -0.3687      0.048     -7.610      0.000      -0.465      -0.273
wheelbase                 0.2529      0.047      5.359      0.000       0.160       0.346
stroke                   -0.1006      0.036     -2.762      0.007      -0.173      -0.029
horsepower                0.7947      0.075     10.588      0.000       0.646       0.943
peakrpm                   0.0788      0.030      2.588      0.011       0.019       0.139
car_lw_ratio             -0.0668      0.042     -1.608      0.110      -0.149       0.015
car_wh_ratio              0.1102      0.044      2.513      0.013       0.023       0.197
pw_ratio                 -0.6499      0.088     -7.416      0.000      -0.823      -0.476
carbody_convertible       0.1564      0.030      5.286      0.000       0.098       0.215
enginetype_ohc            0.0450      0.015      3.063      0.003       0.016       0.074
cylindernumber_five      -0.0364      0.030     -1.206      0.230      -0.096       0.023
cylindernumber_four      -0.0898      0.026     -3.422      0.001      -0.142      -0.038
cylindernumber_six       -0.0367      0.027     -1.360      0.176      -0.090       0.017
company_seg_hightier      0.2192      0.022      9.928      0.000       0.176       0.263
==============================================================================
Omnibus:                        3.223   Durbin-Watson:                   2.106
Prob(Omnibus):                  0.200   Jarque-Bera (JB):                2.852
Skew:                           0.222   Prob(JB):                        0.240
Kurtosis:                       3.537   Cond. No.                         58.4
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

cylindernumber_five has p-value > 0.05. Let's drop it.

```
In [55]: X_train2 = X_train1.drop('cylindernumber_five', axis=1)
```

```
In [56]: lm2=fit_LR(X_train2)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                  price   R-squared:                       0.953
Model:                            OLS   Adj. R-squared:                  0.948
Method:                 Least Squares   F-statistic:                     198.3
```

```
Date:                    Mon, 25 Nov 2019   Prob (F-statistic):          1.28e-77
Time:                            01:32:22   Log-Likelihood:                223.74
No. Observations:                     141   AIC:                           -419.5
Df Residuals:                         127   BIC:                           -378.2
Df Model:                              13
Covariance Type:                nonrobust
==========================================================================================
                            coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------------------
const                     0.4708      0.063      7.447      0.000       0.346       0.596
enginelocation           -0.3571      0.048     -7.507      0.000      -0.451      -0.263
wheelbase                 0.2434      0.047      5.221      0.000       0.151       0.336
stroke                   -0.0979      0.036     -2.687      0.008      -0.170      -0.026
horsepower                0.7913      0.075     10.531      0.000       0.643       0.940
peakrpm                   0.0865      0.030      2.901      0.004       0.028       0.146
car_lw_ratio             -0.0650      0.042     -1.564      0.120      -0.147       0.017
car_wh_ratio              0.1193      0.043      2.758      0.007       0.034       0.205
pw_ratio                 -0.6544      0.088     -7.460      0.000      -0.828      -0.481
carbody_convertible       0.1541      0.030      5.210      0.000       0.096       0.213
enginetype_ohc            0.0370      0.013      2.818      0.006       0.011       0.063
cylindernumber_four      -0.0664      0.018     -3.754      0.000      -0.101      -0.031
cylindernumber_six       -0.0162      0.021     -0.773      0.441      -0.058       0.025
company_seg_hightier      0.2268      0.021     10.705      0.000       0.185       0.269
==============================================================================
Omnibus:                        2.841   Durbin-Watson:                   2.110
Prob(Omnibus):                  0.242   Jarque-Bera (JB):                2.482
Skew:                           0.187   Prob(JB):                        0.289
Kurtosis:                       3.532   Cond. No.                         58.3
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

cylindernumber_six has p-value > 0.05. Let's drop it.

```
In [57]: X_train3 = X_train2.drop('cylindernumber_six', axis=1)
```

```
In [58]: lm3=fit_LR(X_train3)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                  price   R-squared:                       0.953
Model:                            OLS   Adj. R-squared:                  0.948
Method:                 Least Squares   F-statistic:                     215.5
Date:                Mon, 25 Nov 2019   Prob (F-statistic):           1.14e-78
Time:                        01:32:22   Log-Likelihood:                 223.41
No. Observations:                 141   AIC:                            -420.8
Df Residuals:                     128   BIC:                            -382.5
```

```
Df Model:                          12
Covariance Type:            nonrobust
==========================================================================================
                             coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------------------
const                      0.4596      0.061      7.482      0.000       0.338       0.581
enginelocation            -0.3525      0.047     -7.481      0.000      -0.446      -0.259
wheelbase                  0.2474      0.046      5.348      0.000       0.156       0.339
stroke                    -0.1002      0.036     -2.765      0.007      -0.172      -0.028
horsepower                 0.7904      0.075     10.537      0.000       0.642       0.939
peakrpm                    0.0925      0.029      3.215      0.002       0.036       0.149
car_lw_ratio              -0.0687      0.041     -1.667      0.098      -0.150       0.013
car_wh_ratio               0.1279      0.042      3.064      0.003       0.045       0.211
pw_ratio                  -0.6646      0.087     -7.676      0.000      -0.836      -0.493
carbody_convertible        0.1561      0.029      5.303      0.000       0.098       0.214
enginetype_ohc             0.0379      0.013      2.900      0.004       0.012       0.064
cylindernumber_four       -0.0597      0.015     -3.883      0.000      -0.090      -0.029
company_seg_hightier       0.2272      0.021     10.744      0.000       0.185       0.269
==============================================================================
Omnibus:                        2.679   Durbin-Watson:                   2.097
Prob(Omnibus):                  0.262   Jarque-Bera (JB):                2.419
Skew:                           0.140   Prob(JB):                        0.298
Kurtosis:                       3.577   Cond. No.                         58.2
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

car_lw_ratio is having a high VIF and is highly correlated to wheelbase as well as slightly correlated with horsepower. Let's remove this one too.

```
In [59]: X_train4 = X_train3.drop('car_lw_ratio', axis=1)
```

```
In [60]: lm4=fit_LR(X_train4)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                  price   R-squared:                       0.952
Model:                            OLS   Adj. R-squared:                  0.948
Method:                 Least Squares   F-statistic:                     231.6
Date:                Mon, 25 Nov 2019   Prob (F-statistic):           2.88e-79
Time:                        01:32:22   Log-Likelihood:                 221.89
No. Observations:                 141   AIC:                            -419.8
Df Residuals:                     129   BIC:                            -384.4
Df Model:                          11
Covariance Type:            nonrobust
==========================================================================================
                             coef    std err          t      P>|t|      [0.025      0.975]
```

```
----------------------------------------------------------------------------------------
const                   0.4182      0.057      7.392      0.000       0.306       0.530
enginelocation         -0.3441      0.047     -7.294      0.000      -0.437      -0.251
wheelbase               0.2189      0.043      5.057      0.000       0.133       0.305
stroke                 -0.0945      0.036     -2.601      0.010      -0.166      -0.023
horsepower              0.7421      0.070     10.652      0.000       0.604       0.880
peakrpm                 0.0945      0.029      3.266      0.001       0.037       0.152
car_wh_ratio            0.1492      0.040      3.726      0.000       0.070       0.228
pw_ratio               -0.6280      0.084     -7.448      0.000      -0.795      -0.461
carbody_convertible     0.1543      0.030      5.211      0.000       0.096       0.213
enginetype_ohc          0.0358      0.013      2.733      0.007       0.010       0.062
cylindernumber_four    -0.0625      0.015     -4.066      0.000      -0.093      -0.032
company_seg_hightier    0.2339      0.021     11.187      0.000       0.193       0.275
==============================================================================
Omnibus:                        2.888   Durbin-Watson:                   2.057
Prob(Omnibus):                  0.236   Jarque-Bera (JB):                2.844
Skew:                           0.101   Prob(JB):                        0.241
Kurtosis:                       3.666   Cond. No.                         53.1
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

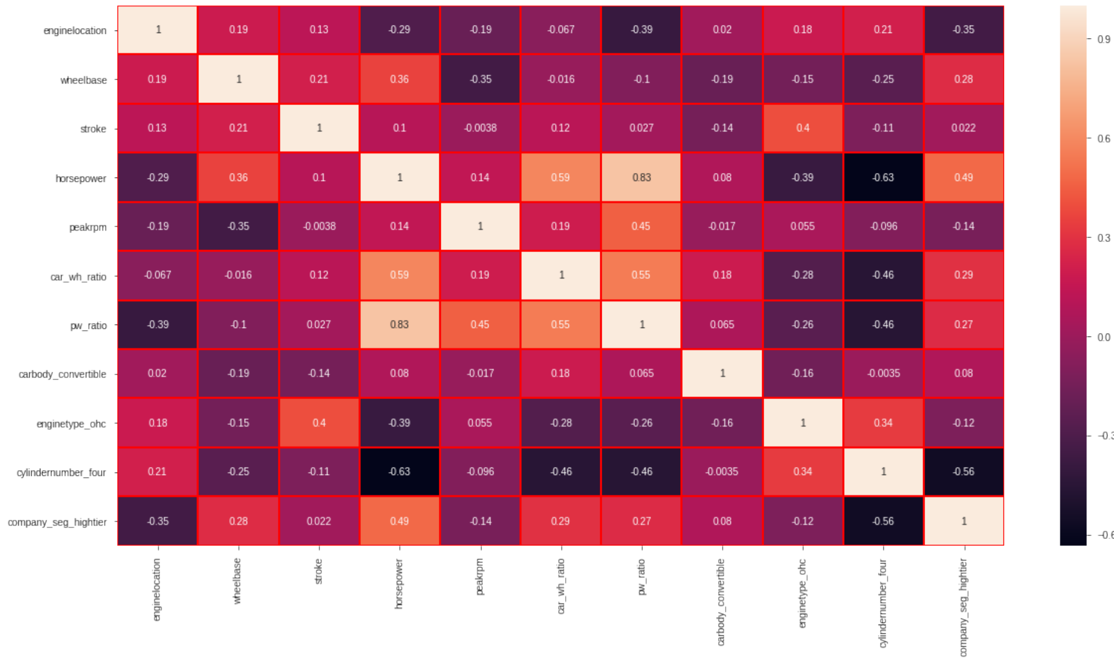
In [61]:
```python
VIF_get(X_train4)
```

Out[61]:

	Features	VIF
6	pw_ratio	64.08
3	horsepower	51.53
0	enginelocation	44.23
2	stroke	20.76
1	wheelbase	19.57
5	car_wh_ratio	13.32
4	peakrpm	8.18
9	cylindernumber_four	8.00
8	enginetype_ohc	5.81
10	company_seg_hightier	1.90
7	carbody_convertible	1.28

In [62]:
```python
plt.figure(figsize = (20,10))
sns.heatmap(X_train4.corr(),annot = True, linewidths=1, linecolor='r')
```

Out[62]: <matplotlib.axes._subplots.AxesSubplot at 0x7f940b06dba8>



- Removal of car_lw_ratio had no impact on the Adjusted R-Squared.
- pw_ratio possesses the highest VIF and is strongly corelated with horsepower, car_wh_ratio and peakrpm. Let's remove this one too.

```
In [63]: X_train5 = X_train4.drop('pw_ratio', axis=1)

In [64]: lm5=fit_LR(X_train5)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                  price   R-squared:                       0.931
Model:                            OLS   Adj. R-squared:                  0.926
Method:                 Least Squares   F-statistic:                     175.7
Date:                Mon, 25 Nov 2019   Prob (F-statistic):           2.00e-70
Time:                        01:32:23   Log-Likelihood:                 196.68
No. Observations:                 141   AIC:                            -371.4
Df Residuals:                     130   BIC:                            -338.9
Df Model:                          10
Covariance Type:            nonrobust
====================================================================================
                       coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------------
const                0.2578      0.062      4.137      0.000       0.134       0.381
enginelocation      -0.3155      0.056     -5.633      0.000      -0.426      -0.205
wheelbase            0.4359      0.038     11.427      0.000       0.360       0.511
stroke              -0.0880      0.043     -2.033      0.044      -0.174      -0.002
```

```
horsepower               0.2766      0.037      7.547      0.000       0.204       0.349
peakrpm                  0.0051      0.031      0.163      0.871      -0.057       0.067
car_wh_ratio             0.1748      0.048      3.678      0.000       0.081       0.269
carbody_convertible      0.2029      0.034      5.895      0.000       0.135       0.271
enginetype_ohc           0.0254      0.016      1.639      0.104      -0.005       0.056
cylindernumber_four     -0.0763      0.018     -4.197      0.000      -0.112      -0.040
company_seg_hightier     0.2597      0.025     10.575      0.000       0.211       0.308
==============================================================================
Omnibus:                        0.202   Durbin-Watson:                   2.201
Prob(Omnibus):                  0.904   Jarque-Bera (JB):                0.315
Skew:                           0.081   Prob(JB):                        0.854
Kurtosis:                       2.836   Cond. No.                         32.2
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

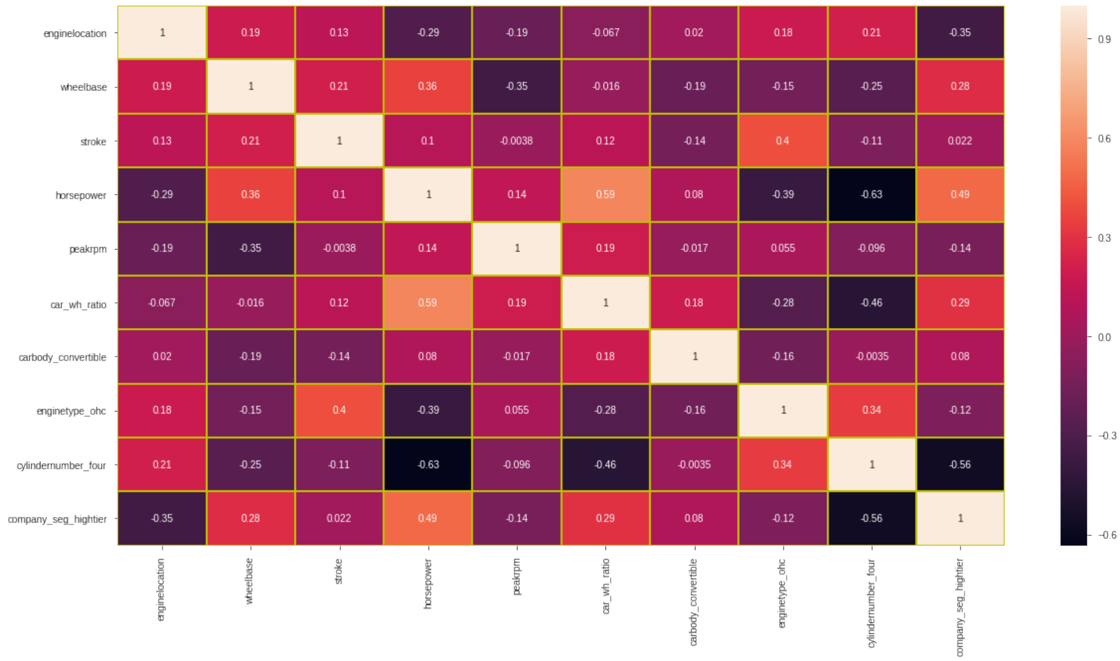
In [65]:
```python
VIF_get(X_train5)
```

Out[65]:

	Features	VIF
0	enginelocation	38.13
2	stroke	20.64
5	car_wh_ratio	13.21
1	wheelbase	10.90
3	horsepower	9.91
8	cylindernumber_four	7.29
4	peakrpm	6.08
7	enginetype_ohc	5.80
9	company_seg_hightier	1.90
6	carbody_convertible	1.20

In [66]:
```python
plt.figure(figsize = (20,10))
sns.heatmap(X_train5.corr(),annot = True,linewidths=1,linecolor='y')
```

Out[66]:
```
<matplotlib.axes._subplots.AxesSubplot at 0x7f940a2159e8>
```



- Removal of pw_ratio reduced the Adjusted R-Squared to 0.926(not much)
- peakrpm is having high p-value. Let us remove this one.

```
In [67]: X_train6 = X_train5.drop('peakrpm', axis=1)
```

```
In [68]: lm6=fit_LR(X_train6)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                  price   R-squared:                       0.931
Model:                            OLS   Adj. R-squared:                  0.926
Method:                 Least Squares   F-statistic:                     196.6
Date:                Mon, 25 Nov 2019   Prob (F-statistic):           1.40e-71
Time:                        01:32:24   Log-Likelihood:                 196.67
No. Observations:                 141   AIC:                            -373.3
Df Residuals:                     131   BIC:                            -343.8
Df Model:                           9
Covariance Type:            nonrobust
========================================================================================
                           coef    std err          t      P>|t|      [0.025      0.975]
----------------------------------------------------------------------------------------
const                    0.2612      0.058      4.478      0.000       0.146       0.377
enginelocation          -0.3165      0.055     -5.713      0.000      -0.426      -0.207
wheelbase                0.4340      0.036     11.996      0.000       0.362       0.506
stroke                  -0.0883      0.043     -2.053      0.042      -0.174      -0.003
horsepower               0.2780      0.036      7.830      0.000       0.208       0.348
```

```
car_wh_ratio             0.1752      0.047      3.709      0.000       0.082       0.269
carbody_convertible      0.2024      0.034      5.924      0.000       0.135       0.270
enginetype_ohc           0.0259      0.015      1.701      0.091      -0.004       0.056
cylindernumber_four     -0.0767      0.018     -4.275      0.000      -0.112      -0.041
company_seg_hightier     0.2587      0.024     10.982      0.000       0.212       0.305
==============================================================================
Omnibus:                        0.217   Durbin-Watson:                   2.197
Prob(Omnibus):                  0.897   Jarque-Bera (JB):                0.338
Skew:                           0.082   Prob(JB):                        0.845
Kurtosis:                       2.825   Cond. No.                         30.9
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

- enginetype_ohc is having p-value > 0.05, making it insignificant in the model. Let's remove this.

In [69]:
```python
X_train7 = X_train6.drop('enginetype_ohc', axis=1)
```

In [70]:
```python
lm7=fit_LR(X_train7)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                  price   R-squared:                       0.930
Model:                            OLS   Adj. R-squared:                  0.925
Method:                 Least Squares   F-statistic:                     217.7
Date:                Mon, 25 Nov 2019   Prob (F-statistic):           3.82e-72
Time:                        01:32:24   Log-Likelihood:                 195.12
No. Observations:                 141   AIC:                            -372.2
Df Residuals:                     132   BIC:                            -345.7
Df Model:                           8
Covariance Type:            nonrobust
========================================================================================
                           coef    std err          t      P>|t|      [0.025      0.975]
----------------------------------------------------------------------------------------
const                    0.2495      0.058      4.277      0.000       0.134       0.365
enginelocation          -0.3001      0.055     -5.462      0.000      -0.409      -0.191
wheelbase                0.4176      0.035     11.891      0.000       0.348       0.487
stroke                  -0.0501      0.037     -1.355      0.178      -0.123       0.023
horsepower               0.2712      0.036      7.633      0.000       0.201       0.342
car_wh_ratio             0.1610      0.047      3.438      0.001       0.068       0.254
carbody_convertible      0.1933      0.034      5.688      0.000       0.126       0.261
cylindernumber_four     -0.0689      0.017     -3.945      0.000      -0.103      -0.034
company_seg_hightier     0.2701      0.023     11.881      0.000       0.225       0.315
==============================================================================
Omnibus:                        0.474   Durbin-Watson:                   2.240
Prob(Omnibus):                  0.789   Jarque-Bera (JB):                0.253
```

```
Skew:                           0.094   Prob(JB):                        0.881
Kurtosis:                       3.088   Cond. No.                         28.2
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

- stroke is having p-value > 0.05 . Let us remove this.

In [71]:
```
X_train8 = X_train7.drop('stroke', axis=1)
```

In [72]:
```
lm8=fit_LR(X_train8)
```

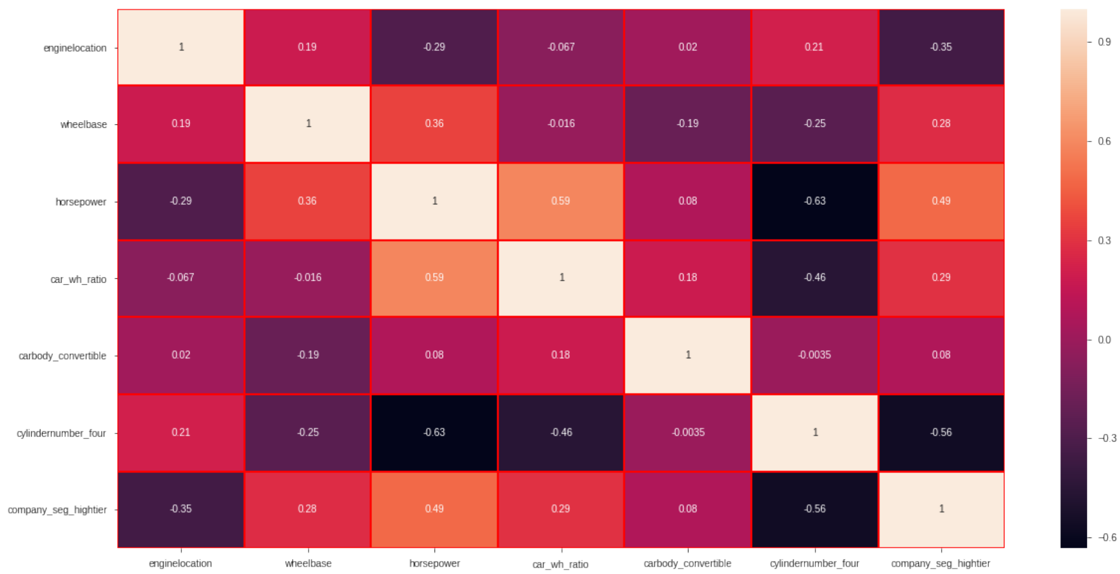
```
                            OLS Regression Results
==============================================================================
Dep. Variable:                  price   R-squared:                       0.929
Model:                            OLS   Adj. R-squared:                  0.925
Method:                 Least Squares   F-statistic:                     247.0
Date:                Mon, 25 Nov 2019   Prob (F-statistic):           5.81e-73
Time:                        01:32:25   Log-Likelihood:                 194.15
No. Observations:                 141   AIC:                            -372.3
Df Residuals:                     133   BIC:                            -348.7
Df Model:                           7
Covariance Type:            nonrobust
========================================================================================
                           coef    std err          t      P>|t|      [0.025      0.975]
----------------------------------------------------------------------------------------
const                    0.2315      0.057      4.063      0.000       0.119       0.344
enginelocation          -0.3057      0.055     -5.561      0.000      -0.414      -0.197
wheelbase                0.4106      0.035     11.783      0.000       0.342       0.480
horsepower               0.2724      0.036      7.645      0.000       0.202       0.343
car_wh_ratio             0.1533      0.047      3.287      0.001       0.061       0.245
carbody_convertible      0.1993      0.034      5.894      0.000       0.132       0.266
cylindernumber_four     -0.0677      0.018     -3.871      0.000      -0.102      -0.033
company_seg_hightier     0.2714      0.023     11.911      0.000       0.226       0.316
==============================================================================
Omnibus:                        0.799   Durbin-Watson:                   2.206
Prob(Omnibus):                  0.671   Jarque-Bera (JB):                0.553
Skew:                           0.147   Prob(JB):                        0.758
Kurtosis:                       3.088   Cond. No.                         26.7
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

In [73]:
```
VIF_get(X_train8)
```

```
Out[73]:               Features    VIF
         0        enginelocation  26.46
         3           car_wh_ratio  12.55
         1              wheelbase   8.74
         2             horsepower   8.37
         5    cylindernumber_four   6.89
         6   company_seg_hightier   1.72
         4    carbody_convertible   1.13
```

```
In [74]: plt.figure(figsize = (20,10))
         sns.heatmap(X_train8.corr(),annot = True, linewidths=1, linecolor='r')
```

```
Out[74]: <matplotlib.axes._subplots.AxesSubplot at 0x7f940ab3e710>
```



- enginelocation is having the highest VIF. Let's remove this one also.

```
In [75]: X_train9 = X_train8.drop('enginelocation', axis=1)
```

```
In [76]: lm9=fit_LR(X_train9)
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                  price   R-squared:                       0.912
Model:                            OLS   Adj. R-squared:                  0.908
Method:                 Least Squares   F-statistic:                     231.4
Date:                Mon, 25 Nov 2019   Prob (F-statistic):           3.82e-68
Time:                        01:32:26   Log-Likelihood:                 179.41
No. Observations:                 141   AIC:                            -344.8
Df Residuals:                     134   BIC:                            -324.2
```

```
Df Model:                           6
Covariance Type:            nonrobust
=============================================================================================
                          coef    std err          t      P>|t|      [0.025      0.975]
---------------------------------------------------------------------------------------------
const                  -0.0431      0.031     -1.370      0.173      -0.105       0.019
wheelbase               0.3216      0.034      9.394      0.000       0.254       0.389
horsepower              0.3450      0.037      9.409      0.000       0.272       0.418
car_wh_ratio            0.0845      0.050      1.699      0.092      -0.014       0.183
carbody_convertible     0.1711      0.037      4.628      0.000       0.098       0.244
cylindernumber_four    -0.0635      0.019     -3.283      0.001      -0.102      -0.025
company_seg_hightier    0.3138      0.024     13.216      0.000       0.267       0.361
==============================================================================
Omnibus:                        7.481   Durbin-Watson:                   2.223
Prob(Omnibus):                  0.024   Jarque-Bera (JB):                8.572
Skew:                           0.366   Prob(JB):                       0.0138
Kurtosis:                       3.962   Cond. No.                         14.4
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

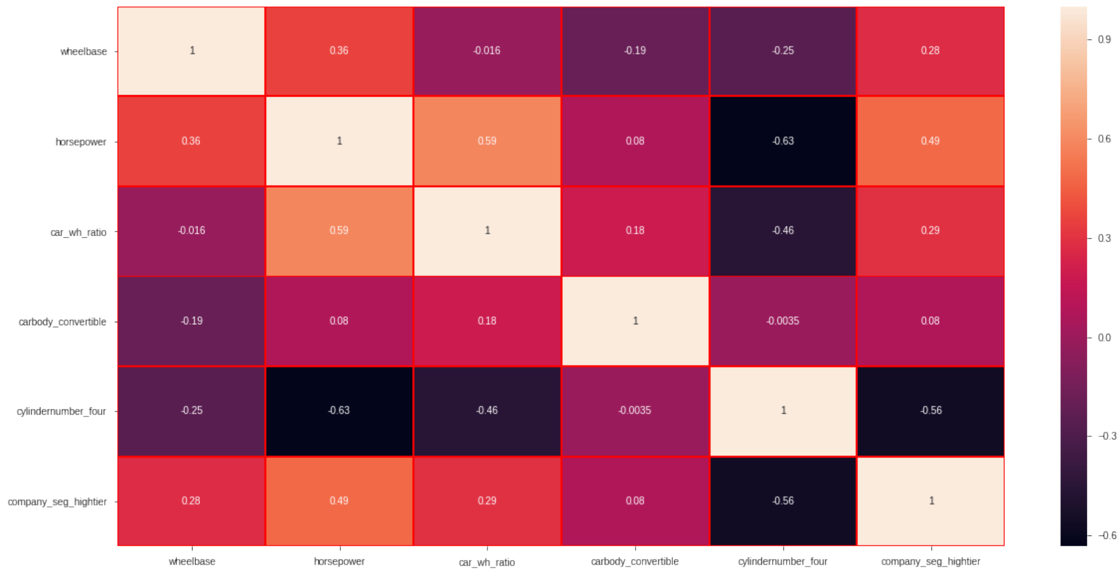
In [77]:
```python
VIF_get(X_train9)
```

Out[77]:

	Features	VIF
1	horsepower	8.35
2	car_wh_ratio	8.05
0	wheelbase	5.76
4	cylindernumber_four	3.63
5	company_seg_hightier	1.72
3	carbody_convertible	1.13

In [78]:
```python
plt.figure(figsize = (20,10))
sns.heatmap(X_train9.corr(),annot = True, linecolor='r', linewidths=1)
```

Out[78]:
```
<matplotlib.axes._subplots.AxesSubplot at 0x7f940ae06470>
```



- removing enginelocation lowers the Adjusted R-Squared to 0.908 - Still a significant fit.
- Horsepower is more business significant in nature than car_wh_ratio which also has a high VIF.

car_wh_ratio is correlated with horsepower. Let's remove car_wh_ratio.

```
In [79]: X_train10 = X_train9.drop('car_wh_ratio', axis=1)
         X_train10 = sm.add_constant(X_train10)
         lm10 = sm.OLS(y_train,X_train10).fit()
         print(lm10.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                  price   R-squared:                       0.910
Model:                            OLS   Adj. R-squared:                  0.907
Method:                 Least Squares   F-statistic:                     273.2
Date:                Mon, 25 Nov 2019   Prob (F-statistic):           9.11e-69
Time:                        01:32:27   Log-Likelihood:                 177.91
No. Observations:                 141   AIC:                            -343.8
Df Residuals:                     135   BIC:                            -326.1
Df Model:                           5
Covariance Type:            nonrobust
========================================================================================
                           coef    std err          t      P>|t|      [0.025      0.975]
----------------------------------------------------------------------------------------
const                   -0.0133      0.026     -0.505      0.614      -0.065       0.039
wheelbase                0.3058      0.033      9.217      0.000       0.240       0.371
horsepower               0.3750      0.032     11.579      0.000       0.311       0.439
```

```
carbody_convertible      0.1785      0.037      4.830      0.000      0.105      0.252
cylindernumber_four     -0.0686      0.019     -3.566      0.001     -0.107     -0.031
company_seg_hightier     0.3129      0.024     13.090      0.000      0.266      0.360
==============================================================================
Omnibus:                        5.403   Durbin-Watson:                   2.212
Prob(Omnibus):                  0.067   Jarque-Bera (JB):                6.579
Skew:                           0.213   Prob(JB):                       0.0373
Kurtosis:                       3.969   Cond. No.                         9.96
==============================================================================

Warnings:
[1] Standard Errors assume that the covariance matrix of the errors is correctly specified.
```

In [80]:
```python
VIF_get(X_train10)
```

Out[80]:

	Features	VIF
0	const	19.89
4	cylindernumber_four	1.95
2	horsepower	1.89
5	company_seg_hightier	1.56
1	wheelbase	1.24
3	carbody_convertible	1.08

All the independent variable have considerably low VIF and the **Adj. R-Squared is 0.907**. We will conclude with these variables as the final model predictor variables.

1.2 Residual Analysis of the training dataset

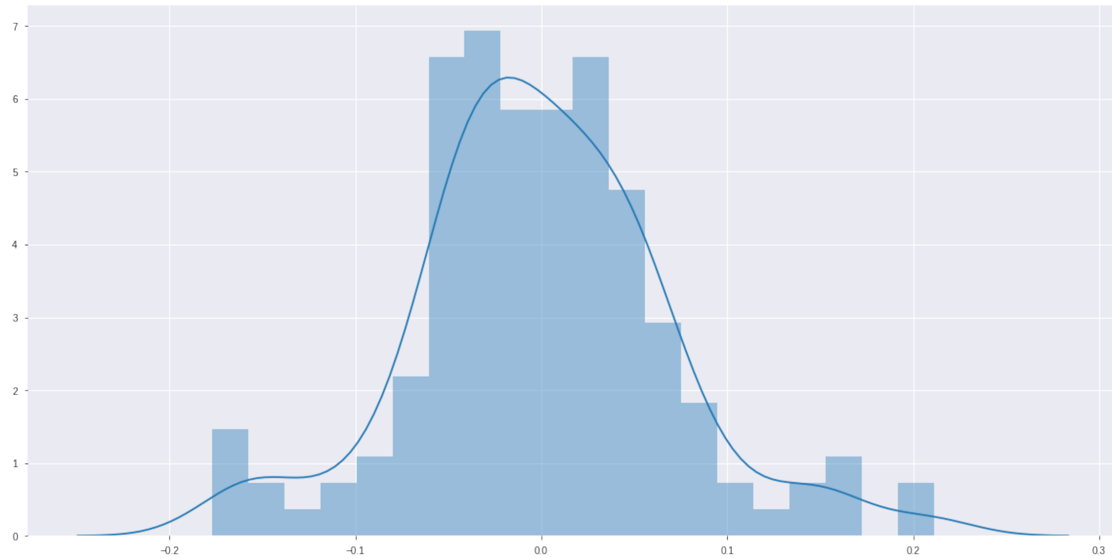
In order to check whether the error terms are also normally distributed (which is one of the major assumptions of linear regression), let us plot the histogram of the error terms and see what it looks like.

In [81]:
```python
y_train_price = lm10.predict(X_train10)
```

In [82]:
```python
# Plot the histogram of the error terms
fig = plt.figure(figsize=(20, 10))
sns.distplot((y_train - y_train_price), bins = 20)
fig.suptitle('Error Terms', fontsize = 20)
plt.xlabel('Errors', fontsize = 18)
```

Out[82]:
```
Text(0.5, 0, 'Errors')
```

Error Terms

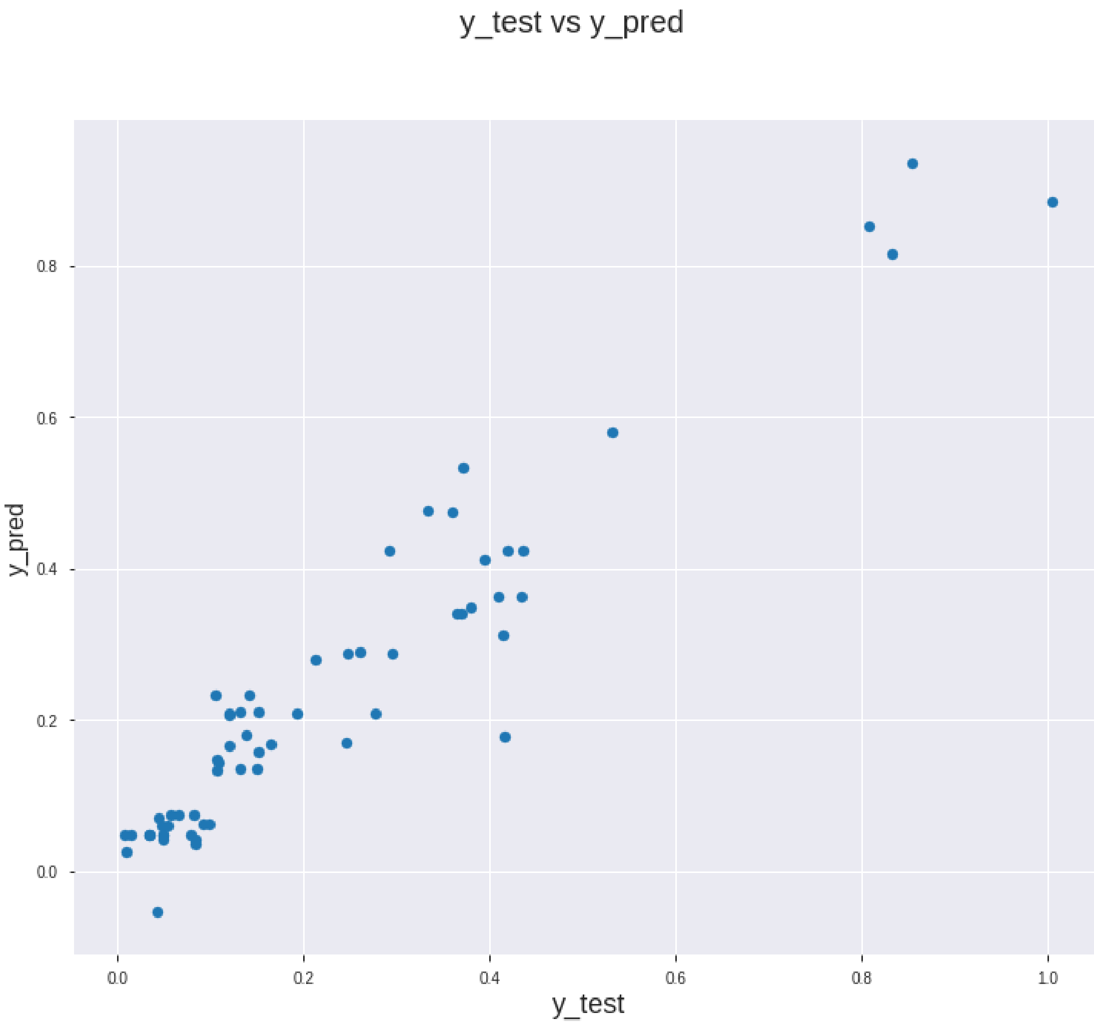


1.3 Making Predictions

Before making any inferences about the equation of the linear regression, let us test it on the test set.

```
In [83]: num_vars=num_vars = ['horsepower','wheelbase','curbweight', 'enginesize',
                             'boreratio','car_lw_ratio','car_wh_ratio','pw_ratio',
                             'hc_mpg_ratio','stroke','compressionratio','peakrpm',
                             'price']
         df_test[num_vars] = scaler.transform(df_test[num_vars])

In [84]: y_test = df_test.pop('price')
         X_test = df_test

In [85]: # Creating X_test_new dataframe by dropping variables from X_test
         X_train10= X_train10.drop(['const'], axis=1)
         X_test_new = X_test[X_train10.columns]

         # Adding a constant variable
         X_test_new = sm.add_constant(X_test_new)

In [86]: y_pred = lm10.predict(X_test_new)

In [91]: fig = plt.figure(figsize=(12,10))
         plt.scatter(y_test,y_pred)
         fig.suptitle('y_test vs y_pred', fontsize=20)
         plt.xlabel('y_test', fontsize=18)
         plt.ylabel('y_pred', fontsize=16)
```

Out[91]: Text(0, 0.5, 'y_pred')



y_test vs y_pred is observed to be almost linear with few variations. Overall, it seems to be a pretty linear spread.

In [88]:
```
from sklearn.metrics import mean_squared_error
from math import sqrt
rmse = sqrt(mean_squared_error(y_test, y_pred))
print('Model RMSE:',rmse)


# ***************************** CALCULATING THE R2 SCORE ****************************
from sklearn.metrics import r2_score
r02=r2_score(y_test, y_pred)
print('Model r2_score:',r2)
# ***************************************************************************************
```

```
Model RMSE: 0.0673924212933167
Model r2_score: 0.9056388935908385
```

r2_score on the test data is very close to the trained Adjusted R-Squared value of the model. We have a significantly high r2_score and a low RMSE of 0.067.

1.3.1 FINAL INFERENCE

Final inference from model evaluation is given as follows:

With a low p-value and low VIF, these variables do describe the price of the automobiles to a good extent.

Final predictors which can be proposed are given as:

Predictor	Coef	p-value
wheelbase	0.3058	0.000
horsepower	0.3750	0.000
carbody_convertible	0.1785	0.000
cylindernumber_four	-0.0686	0.001
cmpany_segment_hightier	0.3129	0.000

Thus, the equation of our best fitted line:

****price = -0.0133 + 0.3058 x wheelbase + 0.3750 x horsepower + 0.1785 x carbody_convertible + -0.0686 x cylindernumber_four + 0.3129 x cmpany_segment_number****

- The price of the car changes by **0.3058** for every unit change in the **wheelbase** dimension if all other variables are held constant. And so is true for all other variables.
- The predictor **carbody_convertible** suggest that the price of car increases by **0.1785** when the car body is convertible.
- The **cylinder_number_four** is the most commonly available feature in **77.6%** of the car data in USA and since having 4 cylinders is attributed with relatively low price, the coefficient for this variable is negative.
- The **company name** is also essential in determining the price of the automobile. High tier companies' names like BMW, Buik, Porsche and Jaguar further add up to the price by **0.3129**.

Overall we have a decent model with the following metrics as show below:

Adjusted R-Squared	r2_score	Prob (F-Statistic)	AIC	BIC	RMSE
0.907	0.905	9.11e-69	-343.8	-326.1	0.067

We have a couple of options:

- choosing another set of variables to get a more normal distribution of error terms or use more useful variables like PWratio inplace of horsepower.
- Build a non-linear model

In [31]:
```
# *******************************************************************
# End of assignment

# Compiled by :
# Prateek Ralhan
# *******************************************************************
```