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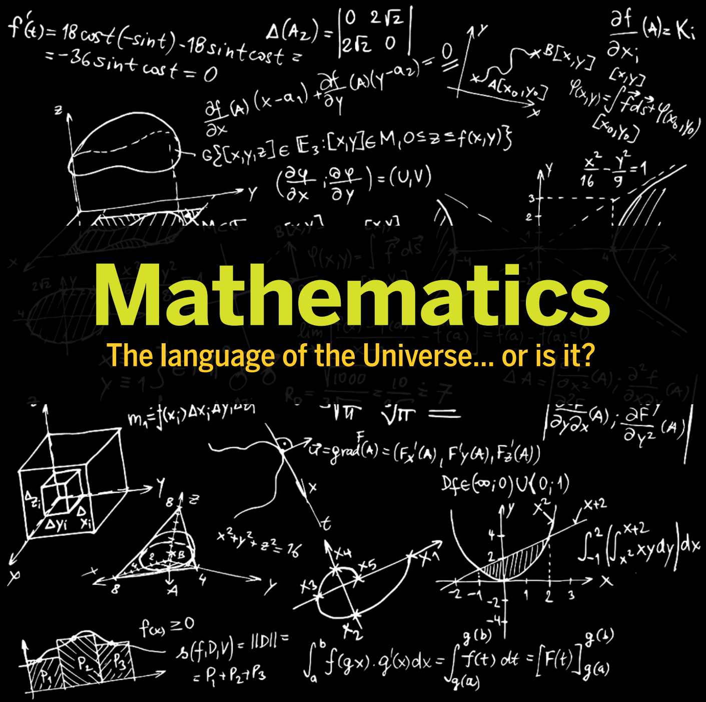
The Small Book of Big Thoughts



Mathematics

The language of the Universe... or is it?

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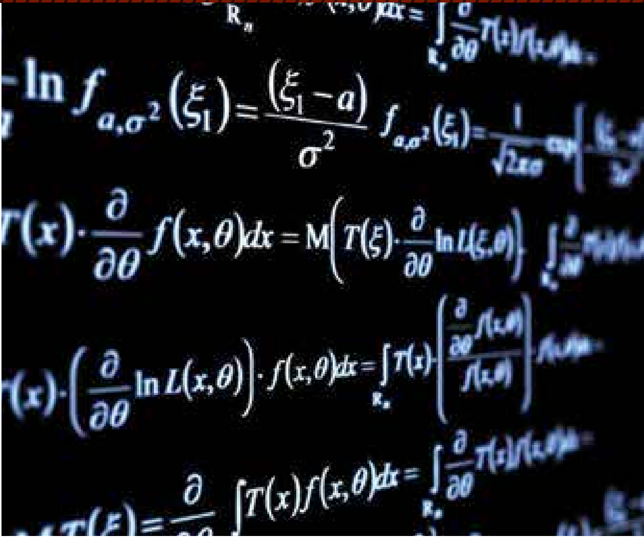

Understand the origins of mathematics, and whether it's human created, or not

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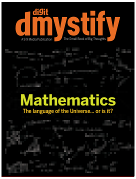
INSIDE



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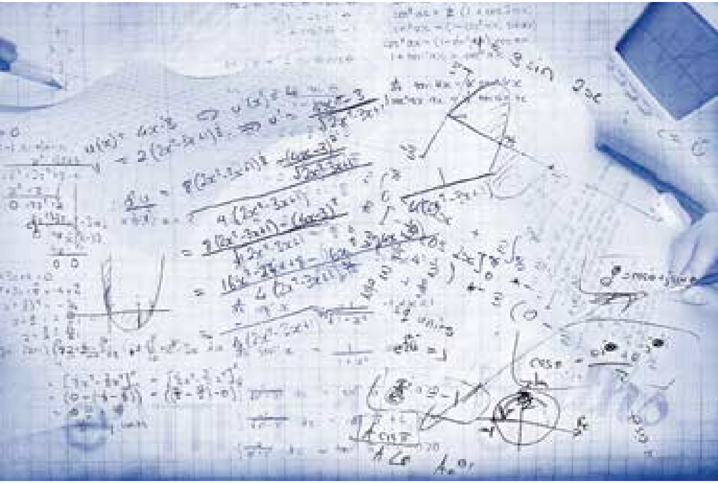
Mathematics

Break down the universe and you end up with mathematics at the very core... or that's what some say.

There is no doubt that mathematics (math from here on, for simplicity) is much, much older than human history itself. Granted, it wasn't called math back then, and it was fairly rudimentary, but some would argue that life itself depends on math. The first cells had to divide (or multiply, whichever way you choose to say it) to create "more" cells than already existed. This in itself was a kind of instinctive math, which has been passed down across billions of years of evolution.

It's generally agreed upon that cellular life did exist on Earth at least 2.5 billion years ago but educated estimates suggest it could have been as early as 3.5 billion years ago that life began on Earth. Either way, math has played a part over billions of years in shaping this planet.

However, this book isn't going to just stick to accepting that math actually is the language of the universe. We're also going to delve into some interesting ideas that suggest that math really is a philosophy, and isn't actually real at all. It's just in our heads!



Just as you can make up stuff by looking at the clouds – you see an elephant, when it's obviously just a cloud, your imagination is what makes it look like an elephant – people believe that math is our observational, pattern-making, biased view of the universe...

But enough philosophising for now, let's get started with the book. Remember, as always, to drop us a line of feedback at editor@digit.in, with the word "dmystify" in the subject to give us feedback and help us improve this book. ■

Math before humans

Here's a look at how math may have existed even before there was anyone to do any counting or calculations

As we mentioned in the introduction, math predates human history. Some would go as far as to suggest that math predates the universe, or at the very least, math is exactly as old as the universe.

You see, when our universe came into being (as per the Big Bang theory), and by the magic that is physics, "something" came out of "nothing", and that something already had a quantity. Scientists say that space and time came into existence at the moment of the big bang, and this means that finally there was a measurement criteria, even if there was no one around to measure anything.

For example, as the universe expanded, it filled "more" space, or rather, space stretched out to add "more" volume to the universe. Time also is mathematical, because there is a before and after, or a past and future, which is nothing but subtracting or adding – this time yesterday was 24 hours ago, or now minus 24 hours, and this time tomorrow will be now plus 24 hours. Everything's relative, and relating things is basically math.

The universe also became "less" dense and colder (or "less" hot), as it aged, which is, again, a mathematical observation. Once the force of gravity took over, things started binding together. One atom attracted another, and soon there were two, then "more". A few atoms at a time over billions of years stars formed, then exploded, then formed again, then exploded again. Planets formed, then finally we came along trying to understand our origins, and are now sitting here pondering the very nature of math and the universe.

If the Big Bang is too far back in history for you, let's fast forward about 11 or so billion years to when the Earth's oceans were teeming with life. Just as fish today do, sea creatures, we think, were drawn together into schools (or whatever you call a group of ancient sea creatures). Predators perhaps not, but the creatures that were food learnt the strength of numbers. Even today, fish and many other animals display this knowledge of probability of survival being higher in groups.

Of course, evolution explains this, because "loner" fish or animals would be killed or eaten, thus allowing the ones with

Fish swiming in a large group makes the group look like a huge animal from afar, and even up close, they've got this shark confused. Evolution!

Lions are instinctively able to tell when another pride of lions is larger, and when the odds are not in their favour

herd mentality to survive and reproduce. However, it still doesn't explain how smaller herds of animals know when to avoid a larger rival herd, and know when the numbers are in their favour and when they're not – quite accurately at that.

An example here are the many prides of lions in Africa, who instinctively know when another group of lions is smaller or larger than their own. To understand what this means, imagine this: you're with a group of people, and you come across another

Image: Victoria University of Wellington

Kevin C Burns

group, and instinctively, you know that the other group is more numerous, and thus you become edgy, as they're rivals, and being out-numbered in a hostile situation makes us all nervous and tense. However, you have mathematics, and thus you know, say, that you are a group of 10, and you know that their group looks larger. It could be 15, or 20, or 30 even. If it was 11, you'd be counting them, or not as intimidated at first glance, perhaps.

Of course, complications always creep in. A group of 10 muscle-bound bikers wouldn't be intimidated by a group of 20 teenage boys and girls... so math isn't everything, not to humans, but to lions in Africa, it is important for their very survival...

Time to look at a study:

http://dgit.in/RobinExpr

In New Zealand, researchers (Kevin Burns, Alexis Garland and Jason Low) extensively experimented with North Island Robins. Simply put, what they did was put a different amount of worms inside two containers, and checked to see if the robins went for the containers with the larger amount of worms. The result? Robins did go for the containers with more worms more often than not. In fact, if the researchers put in a lot worms into a container, and sneakily hid some, the robins would in fact keep searching for more worms even though they could visually see that there were none. This is kind of like showing a magic trick to a child – put five balls into a hat, quietly remove three, and the child will keep searching for the missing balls, because it doesn't understand the concept of trickery. If five went in, only two came out, more should be in the hat. Right?

One, two, many

The child and the robins from the previous example, don't need to be able to count in order to know the concept of missing numbers. Throughout history, we've found that the concept of counting is usually limited to one, two, and many. By default, living things seems to be able to tell the difference between one and two, and two and many, but somehow, subconsciously, the difference between many and twice as many is also easy for most creatures to judge.

The robins of the previous example also showed that the bigger the difference in worms put into the containers, and the smaller the numbers, the more likely they would be to choose

Image: Università di Padova

Rosa Rugani discovered that chicks, like humans, read from left to right – or at least do so for numbers.

the container with a larger number of worms. So, they were less likely to get the right answer when the containers contained 2 vs 8 worms, and more likely to when the containers had 1 vs 4 worms. So it's not just ratios, it's also actual numbers.

Of course, animals have no "language" to describe this counting. It's just an instinctive behaviour on their part, which again, can be used to form the argument that math is essentially intrinsic to our universe – no matter whether you use a language to describe it or not.

In other experiments, researchers have used baby chickens, who are imprinted on a group of inanimate objects (of differing sizes). Imprinting means the baby chicks believe that the inanimate objects are their parents, or “mother”. When researchers subtracted objects from the groups, the baby chicks would either run to their “mother” if the remainder was more than the subtracted amount, or else go in search of their mother, because they sensed that there was more missing than what was here. So if “mum” was imprinted as a group of five objects, and three were subtracted, the chicks knew that there was more of their mother missing than present, and continued searching.

This wasn’t a size or area preference either. The research also showed that it was the same behaviour pattern no matter what the size of the missing objects – the chicks would continue searching if the count was less than half of the original “mother”, regardless of the sizes of the missing pieces. Thus, they demonstrated that they were able to count... or at the very least know what less than half or more than half of their “mother” looked or felt like.

The same researcher (Rosa Rugani from Italy) was also involved in a study that proves that baby chicks also associate


Image: Università di Padova

Rosa Rugani and her team

the number line from left to right – that is, just as we write the numbers 1, 2, 3, etc, and how 1 is less than 2, is less than 3, etc, (increasing order of magnitude from left to right), chickens also respond in the same way, and perceive left to be of lower value and right to be higher.

After looking at all the evidence from the natural (non-human affected) world, it seems pretty obvious that math, in some form or the other, really does play a part in making our world the way it is. It has for billions of years, and will do so for billions of years after we're gone, until our universe itself changes state into whatever it will eventually become. However, that's enough for the pre-human era, now it's time to look at the math of our ancestors. ■

Ancient human math

Math as we know it today, is really a human invention (or language, or interpretation), and it started a lot earlier than you'd think

When we came down from the trees, started hunting and gathering, and became pack animals, we really started using math. Just as the lions in the Serengeti do, our African ancestors also needed to have a general feeling of numbers. The evolution of the fight-or-flight instinct comes from math, and especially from knowing when you're outnumbered.

That's still not math as we know it, to be honest, but it's important to remember that math predates languages.

The earliest actual proof of humanoids doing calculations dates back to a 20,000 year old "tally stick". A tally stick was basically used by humans to keep score, or "tally" numbers. You borrowed two sheep from me, and had borrowed two before that, so how many sheep do you owe me? Of course, 20,000 years ago, there may have been no word or number that defined “two”, and thus was just a bunch of notches used to “tally” notches to sheep. It could also be used by warriors or hunters, rather morbidly, to represent kills. Like prehistoric gamers keeping score in a killing competition.

The Ishango bone, which is the fibula of a baboon, has been radio-carbon dated to around 20,000 years old, and scientists are still divided over whether it’s a simple tally stick, or a more complex measuring standard, however, they’re all in agreement that this is the oldest confirmed usage of rudimentary “written” math. The “written” bit being notches inscribed in a dead baboon’s bone.

There are older notched-bones, the most famous being the Lebombo bone, which is as much as 44,000 years old, and has 29 notch-markings on it. However, scientists have disregarded it as a math tool because each notch seems to have been made by a different tool, which would be unlikely if it was designed at once as a tally stick or calculator of some sort. The popular belief is that it was used for rituals that were spaced out in time (quarterly, yearly, etc.) which would explain why the tool used to notch it kept changing. There are even older notched bones that we’ve found, (as much as 80,000 years old), but there is

A 20,000 year old baboon bone, the Ishango bone
is perhaps the earliest proof of simple math

no evidence of them being even remotely math-related.

Thus, we're left with the 20,000-year old Ishango bone, which was found at what is today the border of Uganda and Congo. A large settlement existed at this site until 20,000 years ago, when it was tragically ended by a volcanic eruption. Thankfully for us, however, being buried by a volcano ended up preserving the bone for us to be able to study it.

Babylonian math

Sumer was a region of Mesopotamia (in what is now called Iraq). Known by many as the "Cradle of Civilisation", the Sumerians are credited as the ones who started a writing system, the first to do agriculture, and they even invented the wheel! Another Mesopotamian region, Babylonia, is credited with coming up with something that's more akin to the math we have today.

Thanks to the methods they used for recording everything – they would mark clay tablets while they were still wet, and then bake them into hard clay so as to preserve the markings. This preserved Babylonian tablets a lot longer than, say, the papyrus that was later used by Egyptians.

The earliest evidence of written math dates back to the Sumerians – around 3,000 to 2,500 BCE – but many historians think that the knowledge has to be a lot older, and thus we get

A bill of sale found amongst the ruins of ancient Sumer speculative ranges of their math being developed sometime between 5,000 to 3,000 BCE.

Babylonia was the home of knowledge in ancient Mesopotamia. From around 2500 BCE, there were many scribe schools in Babylon, where people sent some lucky children to be educated, who would then become the scribes of the future. The job of a scribe was prestigious and important in an empire that was vast and thriving. A scribe kept tally of money, taxes, land measurements, records, and was obviously a very important man.

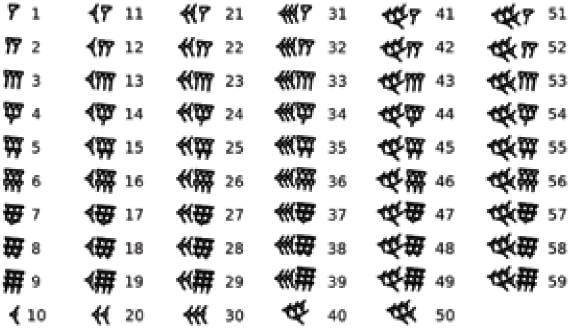
Most of the tablets that have been preserved and uncovered by modern historians and archeologists are thought to be children's homework and schoolwork, from the many aforementioned scribe schools that existed over 4000 years ago! It's rather like some archaeologist 4500 years in the future, finding today's children's paintings from a classroom, or uncovering the artwork that parents stick on their refrigerators, and trying to piece together how advanced we were based on those.

Babylonian math was all about practicality. Weighing and measuring, and doing math for everyday problems. They started, like most humans do even today, by counting on their hands. However, instead of using 10 fingers (eight fingers and two thumbs for the pedantic), to arrive at a base 10 for their counting system, they used the thumb on the left hand to indicate one of the 12


The ruins of Babylon, photographed in 1938

phalanges (joints) of the other four fingers, and the five fingers on the right hand to be able to count to 60 on your hands.

Mathematically, using base 60 (sexagesimal) to count is quite ingenious, as 60 is a very composite number with 12 factors (perfectly divisible by 12 numbers), i.e. 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30 and 60. As you can tell from the factors above, 60 is the smallest number that's divisible by all the numbers from 1 to 6. Perfect for use in a market, for example.



The Babylonian numbers to 50

Think about buying something simple, such as eggs – we still buy them by the dozen, even today – if you use a base 10 system, you can calculate nice round figures for wholesalers such as 100 or 50, or 1,000, but what do you do when a customer wants 3 eggs? Knowing the price of 100 eggs or 10 doesn't make it easy to calculate the cost of three.

Everyday, average people did not want to deal with fractions, so base 60 just made life easier for everyone involved. A wholesaler could sell, say, 300 eggs. Once you know the price of 60, you can calculate for multiples of it for wholesalers. And for customers, you can easily arrive at rates of any number of eggs. Want seven eggs? That's the price of six plus the price of one. Their money was also calculated at base 60, which made such calculations very easy.

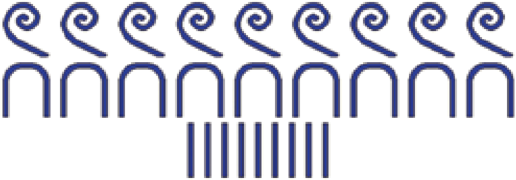
The most important tablet found from Babylon however is Tablet YBC 7289, which shows that the Babylonians knew the value of the square root of 2 to six decimal places.

Egyptian math

Similar to the Babylonian math, Egyptian math was also very practical. Measurements were arrived at by using the human body as a guide –

They were the first to have notations for fractions. The Egyptians, like we do today, used the base 10 system to calculate. Of course, since they wrote with hieroglyphs, everything had a symbol, not a number. They had symbols for 1, 10, 100, 1,000, 10,000, 100,000 and 1,000,000. However, they had no symbol for the numbers between 1 and 10, so nine was just the symbol for 1 marked out 9 times. Thus, the number 999 would have 27 symbols to denote it – nine 100s, nine 10s and nine 1s. This method may seem very limiting to us, but when you look at all that the Egyptians accomplished, it was really remarkable for their time.

What we've discovered about the Egyptians that is truly



Here's how you write 999 in Egyptian

remarkable for those of us interested in computers, is that the Egyptians used to use a binary method to calculate. Yes, you read that right, binary!

They used pre-written multiplication tables to help calculate even very large numbers. For example, they would write down the table for 2 as follows:

2	1	2
2	2	4
2	4	8
2	8	16
2	16	32
2	32	64
2	64	128
2	128	256

Now, if you wanted to know what is 24 times 2, you just add the answers of 2 x 16 and 2 x 8. 160 times 2 is 128 x 2 + 32 x 2, or 256 + 64 = 320.

However, while binary as we know it is base 2 for everything, the Egyptians used the same system for all numbers. Let's illustrate with 3 in the table below:

3	1	3
3	2	6
3	4	12
3	8	24
3	16	48
3	32	96
3	64	192
3	128	384

Now if you wanted to multiply 3 by 150, you would break 150 up into 128 + 16 + 4 + 2. Basically, 3 x 150 = 384 + 48 + 12 + 6 = 450.

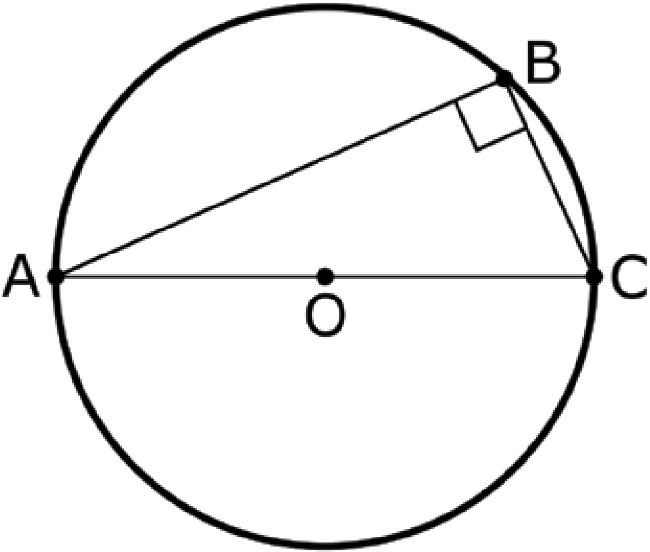
This may seem like a much longer way of multiplying, and even children can multiply 3 x 150 in their heads these days, but you have to remember that we're talking about math from 4,000 years ago. This method made it easier for those who had no understanding of math to be able to calculate – a sort of rote,

The slab that's placed on Egyptian princess' Nefrititi's tomb machine way of figuring things out. This is something Gottfried Leibniz only introduced to modern mathematics over 3,000 years after the Egyptians, and something all of our computers run on today. Granted, the Egyptians didn't know enough to even call it binary, but the fact that they figured it out way back when is a testament to a great civilisation, where people could sit and learn and improve on math. Besides, one look at the pyramids they built will settle any doubt anyone has about them lacking math skills!

Greek math

There's a lot of speculation in the math world about whether the Greeks really did come up with a lot of the math they are credited for on their own, or did they use a lot of Egyptian and Babylonian knowledge, and then just build upon that. Being so far back in the history of mankind, it's likely that we will never get a concrete answer either way, however, to try and undermine what the Greeks did for mathematics would be idiotic.

From what historical evidence we have managed to piece together, all math before the greeks was inductive in nature. That is, you experiment, make an observation, experiment again, make another observation, rinse and repeat until you have enough to make a rule. The Greeks, however, gave the world deductive



Thale's Theorem: Since AC is the diameter, the angle ABC has to be a right angle reasoning, which is the backbone of modern mathematics, and science in general. Deduction allows you to arrive at a logical conclusion, and then go about trying to prove it correct using mathematics. It's top down logic, and allows you to use smaller rules to arrive at a mathematical conclusion. For example:

- All humans are born
- You are human
- Therefore, you were born

Basically: "If A = B and B = C, then A = C".

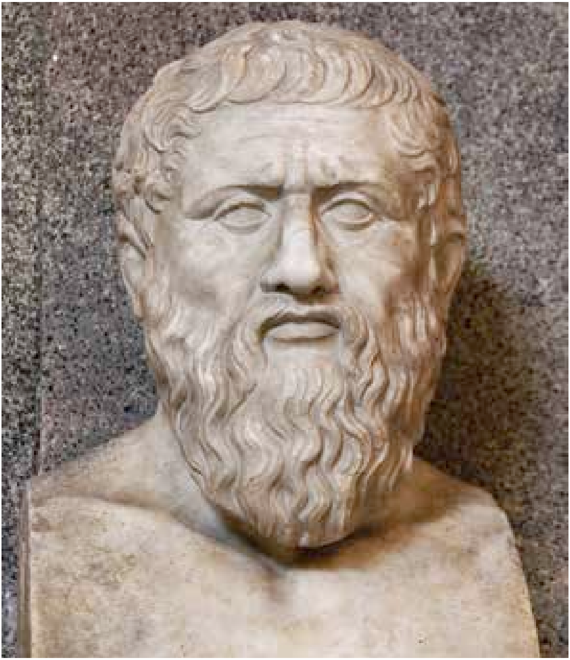
Greek math had its stars, and of those, the first star mathematician was Thales (624 to 546 BCE). Although no direct written work of his has survived, he is mentioned and credited in texts across Greek history. He is the first person, by name, to whom a mathematical discovery is credited. Called Thales' Theorem, he used deductive reasoning to prove that if a triangle ABC has all three points on a circle, and AC is the diameter of the circle, then the angle ABC must be a right angle. Brilliant deductive reasoning for the time, which he then proved to be correct.

One of the biggest math stars from Greece, Pythagoras, didn't just study math, he lived it. He started the Pythagorean school, whose motto was "All is number", or basically, that the universe was run by math. Although there is some conflict over which civilisation (Babylonian, Egyptian or Greek) first discovered what we now know as Pythagoras' Theorem – in a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other sides – Pythagoras is the one credited with proving it. Pythagoras is also credited with inventing the words "philosophy" (meaning: love of wisdom), and "mathematics" (meaning: that which is learned). Those of you who weren't very good at math in school, can now take a moment to curse Pythagoras for inventing the study of mathematics. Pythagoras was also the first to apply mathematics to stringed musical instruments, and first noticed that harmonious sounding musical notes always have whole number ratios.

On a guitar, for example, when you play an open string (without pressing down on it), you get a note. If you measure the string precisely, and press down exactly at the halfway point, you will get the same note, just an octave higher. Now press one third of the way down, and you get a different but harmonious note. This convinced that his philosophy of "All is number" or "God is number" was essentially correct, and that everything could actually be explained mathematically.

Plato (428 to 348 BCE) is thought of as a philosopher these days, but he started out as a mathematician. He setup his famous

Pythagoras is one of the most famous Geek mathematicians



Plato is credited with starting The Academy, which gave the world a lot of Greek mathmaticians

Euclid's "The Elements" is regarded by many as the most influential mathematics publication of all history

Archimedes is famous for more than just running naked and shouting "Eureka! Eureka!"

Academy in 387 BCE, from which a lot of Greek math was done. So much did he love and encourage math, that he became known as the "maker of mathematicians". Those of you who feel that becoming an engineer or doctor requires too much study, should know that the standard course at the Academy was 15 years!

By the third century BCE, when Alexander the Great conquered everything, including Egypt and Mesopotamia, Greek scholars were given insights into everything that the Egyptians and Babylonians had done. Alexandria was the new home of mathematics, and learning, and many Greeks went there to study and teach, including Euclid and Archimedes.

Euclid is thought to have studied at Plato's Academy, but as Alexandria grew to rival it – especially because of the vast and world famous Alexandrian library – he moved there. He is famous for Euclidian Geometry, which is really the result of his book – something that all mathematicians will agree is the most important geometry book every written – The Elements. It was the most comprehensive compilation of 465 theorems and proofs, described exactly as you would expect from a math textbook today. In fact, almost all of it is still taught in schools today, over 2,300 years later, because it is just that elegant and correct.

He had some simple rules that he outlined in the Elements:

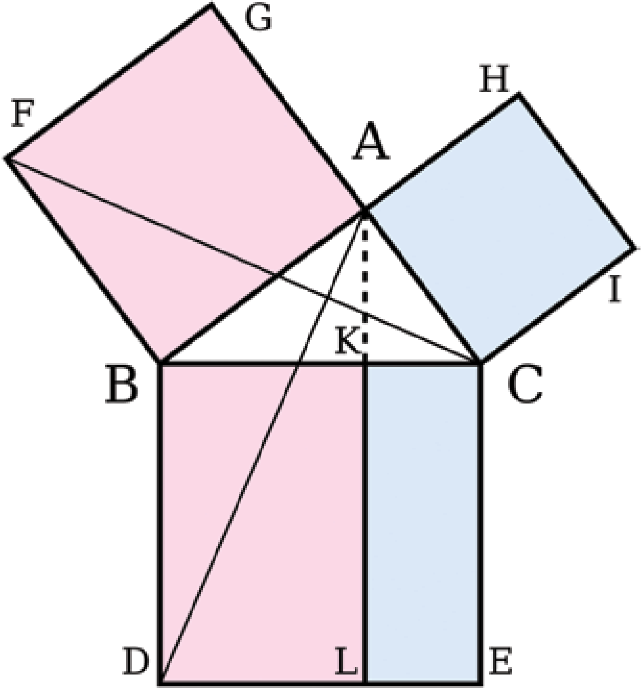
The Five Axioms

- Things equal to the same thing are equal to each other
- If equals are added to equals, the sums are equal
- If equals are subtracted from equals, the remainders are equal
- Things that coincide with one another are equal to one another
- The whole is greater than the part

The Five Postulates

- You can draw a straight line between any two points
- You can extend a straight line between two points indefinitely (in a straight line)
- You can create a circle with any centre and distance
- All right angles are equal
- If a straight line cuts across two other straight lines, and the sum of the angles it makes with those straight lines on one side are less than 180 degrees, then the two lines will meet at some point on that same side.

He was also the first to realise and prove that there are an infinite amount of prime numbers. He identified the first four "perfect numbers". These are numbers whose factors add up to give you the number itself. So, the factors of 6 = 1, 2, 3 and 6 is also = 1 + 2 + 3. 28 = 1 + 2 + 4 + 7 + 14.



Euclid's proof of Pythagoras' equation

Archimedes (287 to 212 BCE) is considered by many to be among the greatest mathematicians of all time. He started off by trying to calculate volumes of complex shapes such as circles and cylinders by using simpler shapes. He used a method of exhaustion to arrive at an approximate area of a circle, for example. He enclosed a circle in a square, and drew another square inside that same circle, and approximated the area of the circle as halfway between the area of the internal square and the external square. Then he repeated the process with a pentagon, then a hexagon, and then with polygons of increasing sides, until he arrived at the conclusion that you could never quite know the exact area of a circle, but you can approximate it to a high accuracy.

He repeated a similar experiment with a sphere, by approximating the volume of the sphere by splicing it into many smaller cylinders. He also realised that for increased accuracy, you had to splice into ever smaller cylinders. This use of infitesimals was a very new idea at the time, and the value of it was only fully understood when Calculus was invented by Newton and Leibniz, much later in the 17th century CE.

Archimedes is probably most remembered by school students as the mad Greek scientist who ran about the streets naked yelling "Eureka! Eureka!", but his mathematical exploits were no less eye catching for his time. He got a close approximation for the square root of 3, and was the first Greek mathematician to accept infinity.

All of these great civilisations added a lot to mathematics as we know it today, but there is one great civilisation we're leaving for the next chapter... ■

Indian and Islamic Math - Zero and Algebra

Now it's time to look closer to home, and understand the math of ancient India and then the Islamic Empire that ruled over much of it

Indian math largely developed independently of Chinese math and there are no links to Babylonian math either. The earliest known proofs for Indian math are the ruins of the Indus Valley Civilisation (3300 to 1700 BCE), where it's been discovered that there was a lot of practical math being used. For example, they used weights in fixed ratios: 500, 200, 100, 50, 20, 10, 5, 2, 1, 1/2, 1/5, 1/10 and 1/20, and they used bricks that were in a 4:2:1 ratio to build their houses.

Even before 1,000 BCE, in the Vedic Period, there were examples of really large numbers being used in the Vedic texts. For example:

"Hail to śata, hail to sahasra, hail to ayuta, hail to niyuta, hail to prayuta, hail to arbuda, hail to nyarbuda, hail to samudra, hail to madhya, hail to anta, hail to parārdha, hail to the us'as (dawn), hail to vyuṣṭi (the twilight), hail to udeṣyat (the one which is going to rise), hail to udyat (the one which is rising), hail to udita (the one which has just risen), hail to svarga (the heaven), hail to martya (the world), hail to all."

Here:

śata = hundred

sahasra = thousand

ayuta = ten thousand

niyuta = hundred thousand

prayuta = million

arbuda = ten million

nyarbuda = hundred million

samudra = billion (also "ocean")

madhya = ten billion (also "middle")

anta = hundred billion (also "end")

parārdha = one trillion (also "beyond parts")

Long before Pythagoras, in the Sulva Sutras (700 BCE),

A statue of Aryabhata, a thinker way before his time, who knew that Pi was irrational there are examples of Pythagorean Triples. These are simple numbers corresponding to the sides of a right-angled triangle. For example, (3, 4, 5) is a Pythagorean Triple, because $3^2 + 4^2 = 5^2$. Other triples include (5, 12, 13), (8, 15, 17), (7, 24, 25) and (12, 35, 37), all of which show up in the Sulva Sutras.

Apart from other things, there is also evidence of calculations of the squareroot of 2, which they described as 1 + 1/3 + 1/(3x4) - 1/(3x4x34) = 1.414215686274509, which is correct to the true value of root 2 (1.414213562373095), to 5 decimal places. There are also concepts discussed about numbers as high as 10^{421}, which most people would think of as infinity today, given that the total number of atoms in the universe is estimated to be a mere 10^{80}!

Speaking of infinities, the concept of infinity had no problem with ancient Indians. There were no cultural norms preventing people from accepting irrational numbers or infinities. Jain mathematics taught that there were five different types of infinities – perpetual infinity, infinity everywhere (all directions), infinite areas, infinite in two directions and infinite in just one direction. Buddhists felt that numbers were countable, uncountable or infinite.

The ancient Indians also used a decimal system (as did the Chinese), but the Indians gave the world the nine numerals that the arabs would later modify and popularise across the world. The numbers we use almost globally had very Indian beginnings.

The majority of Indian mathematical work was done between the 400 and 1100 CE, and actually predated many similar discoveries in the west, by centuries at times.

For example, the Indians were the first to realise that when the moon was exactly half (half in shadow and half lit up), it meant that the angle between the sun and the moon and the moon and the earth was a right angle, and this means they could use trigonometry to find the ratio of the distance to the moon to the distance of the sun from Earth. Since the angle they perceived was 1/7 of a degree, they estimated that the ratio was 400:1. So the sun was 400 times further away from the Earth than the moon is from the Earth. Today we know that the distance to the moon is about 0.384 million km, and the distance to the sun is about 150 million km, which gives us a ratio of 390:1. Not bad for calculations done over 1,500 years ago!

Sticking with trigonometry, about 1,500 years ago, the most famous Indian mathematician, Aryabhata, came up with proper definitions of sine, cosine, and inverse sine, and made a table of these with great accuracy, to make life simpler to calculate. He also gave an approximation for pi, which was correct to four decimal places. He was also the first to suggest that it was

Brahmagupta was the first to suggest that quadratic equations could have negative number solutions irrational – about 1,200 years before Johann Lambert, who is credited with proving pi to be irrational.

Aryabhata also used pi to estimate the circumference of the Earth, and arrived at a figure that was 99.8 per cent accurate. He also calculated the Earth's rotational speed, and calculated the day to be 23 hours, 56 minutes and 4.1 seconds long – we now know it to be 23:56:4.091, or just 0.009 seconds slower than Aryabhata's calculation. His calculation of the length of a year was also just 3 minutes and 20 seconds off the mark.

He also was the first to explain that a lunar eclipse is when the moon falls in the Earth's shadow, and a solar eclipse is when the Earth goes into the moon's shadow, and also went against the prevailing logic at the time to suggest that planets and the moon merely reflect light, and don't emit light like the sun does.

The biggest contribution to the field of mathematics by Indians, however, was undoubtedly the number zero. It was in India, sometime in the 7th century CE that we find the first usage of "0" to signify zero. We're pretty sure it was Brahmagupta who popularised it by treating it as an actual number, and not as just a placeholder to signify "nothing". However, there are some suggestions that it could also have been Bhaskara I, who first did this.

It's also assumed that the circle "0" was used to denote zero for a much longer period in Indian history before the 7th century. It's only when Brahmagupta (or Bhaskara I) started using it as a number that India's contribution to the math world really took off.

Although even a child today knows that 1 - 0 = 1, 1 + 0 = 1 and 1 x 0 = 0. The logic of why was worked out by Indians 1,500 years ago, and was a huge deal at the time.

Brahmagupta was also the first to suggest rules for dealing with negative numbers, and pointed out that quadratic equations could possibly have both positive and negative number answers. This is basic Algebra, before algebra even existed. Even the Babylonians had similar forays into algebra, but they didn't follow it up to make any rules for it.

His rules were the simple rules we use today: a negative multiplied by a negative is positive, a positive multiplied by a positive is positive, and a negative multiplied by a positive is negative. He also suggested that 0 - (-N) = +N and 0 - +N = -N.

Brahmagupta's only error seems to be that he assumed that 1 divided by 0 = 0. It was in the 1100s that Bhaskara II would theorise that anything divided by 0 = infinity. However, today it is widely accepted that the answer is actually "undefined", and not infinity.

Much later, in medieval India, a mathematician from Kerala called Madhava (1350 to 1425 CE) would again recalculate the value

Muhammad Al-Khwarizmi was the one to adopt the
Hindu numeral system in the Islamic Empire

of pi, and come up with an answer correct to 13 decimal places. An amazing feat for the time. What's even more amazing is that he did this using a method that involved infinity. He calculated pi by alternately adding and subtracting odd fractions of 4. So, 4/1 - 4/3 + 4/5 - 4/7 + 4/9 - 4/11 + 4/13 ... to infinity = pi. This is a method that was only rediscovered by Leibniz over 200 years later.

His work in trigonometry to more accurately calculate sine, cosine, tangent and arctangent, and his work on infinite series, coupled with the later work done by his students at his school in Kerala, surely laid the foundation for the Europeans to, much later, develop calculus. Historians suggest that since the port of Kochi was along various European trade routes, it seems logical that Madhava's teachings would certainly have reached Europe, and been at least the inspiration for the development of calculus.

Islamic Math

When the Islamic Empire started, literally, taking over the world, starting somewhere in the 8th century CE, there was a lot of information sharing that happened. A lot of the learnings from Babylon, Egypt and India were put together to give Islamic scholars a foundation like no one had ever had before.

It starts with their love for geometry, which is attributed by some to the prohibition by the Prophet Mohammed of depicting any human shape religiously. This meant that shapes were all that could be used to adorn mosques, and since symmetry was obviously sought, Islamic architecture gives us some really unique patterns.

In the year 810, on the banks of the river Tigris, in the city of Baghdad, the Islamic Empire set up the House of Wisdom. Here, almost all learnings from Greek, Babylonian and Indian civilisations were being translated into Arabic. It was one of the directors of the House of Wisdom of the ninth century CE, Muhammad Al-Khwarizmi, who first recognised the value of the Indian numerical system (0 to 9), and encouraged its adoption by the whole of the Islamic world. Today the whole world uses the same numeral system because of the Indians, and Al-Khwarizmi.

Al-Khwarizmi also gave the world Algebra – originally: Al-Jabr (which in Arabic means: reunion of broken parts). Algebra was a critical step forwards in mathematics, because until now, the more popular Greek method was to look at everything geometrically. Algebra turned everything around by allowing one field of mathematics to work with both rational and irrational numbers, as well as geometry. This opened up math to many more facets of what we call "science" today.

There are many more individual contributors to the sciences, especially chemistry and medicine by Islamic scholars from this

Golden age of Islamic Science. For math, there was a lot of use of Algebra, and a lot of refinement done on more crude calculations done by the Greeks, Babylonians and Indians before. ■

The Fibonacci sequence and Phi

Now it's time for some more fun and some more mystical discoveries that mankind has made

Although the Fibonacci sequence was mentioned in ancient Indian texts, as early as the 6th century CE, it wasn't until the 13th century CE Italian, Leonardo of Pisa (aka Fibonacci) introduced it to Europe that it became popular. Before we go any further, you need to know what the Fibonacci series is. It's really just adding the two numbers in the series before to arrive at the current number. The series looks like this:

0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584, 4181, 6765, etc...

Basically, you start with 0 and 1 as the first two numbers, then you add the last two to get the next number. Simple, right?

Yes, to comprehend the series is child's play, mere addition, hardly a challenge for us math-aholics. However, when really bends your brain is why this sequence keeps showing up in nature, in all sorts of different ways. Before we get to that, however, we need to look at the golden Ratio first.

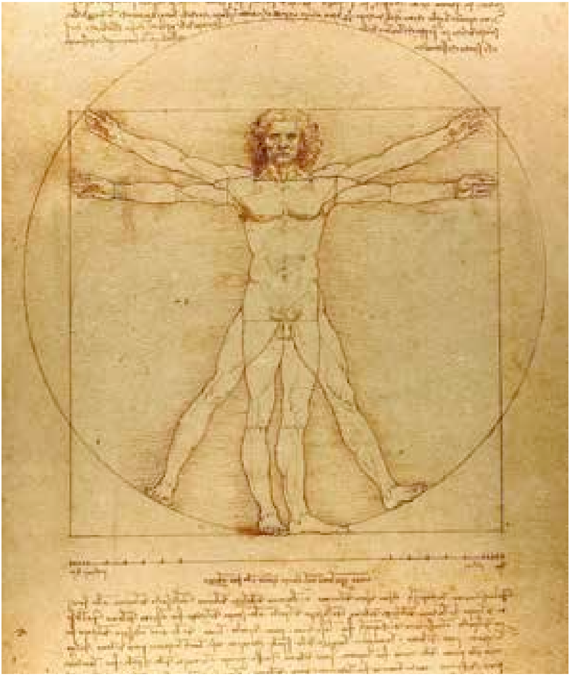
If you take more of the higher numbers in the fibonacci sequence or series, you will find that the ratio they form are 1: 1.6xxx.

5/3 = 1.6666666666667
8/5 = 1.6
13/8 = 1.625
21 /13 = 1.61538
34 /21 = 1.619
...
4181/2584 = 1.61803
6765/4181 = 1.61803

As you can see, this number kind of evens out to about 1.61803398 (actually an irrational number, with no end to the decimal points).

This number has some very interesting qualities itself. Usually denoted by the greek letter phi (φ), there are some interesting things you can do:

$1/\varphi = \varphi - 1$



DaVinci's Virtuvian Man shows that the human body is more than just symmetrical...

$$1/1.61803 = 0.61803 = 1.16803 - 1$$

$$\varphi^2 = \varphi + 1$$

$$1.61803^2 = 2.61803 = 1.61803 + 1$$

The Golden Ratio also becomes a logarithmic spiral when plotted, which is called the Golden Spiral. Basically, the spiral gets wider by φ for every quarter turn.

The Fibonacci series in nature

You would think that nature doesn't need math to work, and especially something as weird as the Fibonacci series, but life has a way of surprising us.

Let's start with flowers. More often than not, the amount of petals a flower has will be a Fibonacci number. Yes, there are flowers that have 4 petals (and 4 is not a Fibonacci number), but they're exceptions. Some go as far as to slot everything in nature into either Fibonacci Numbers, or Lucas Numbers starting with 2. Named after François Lucas, a french mathematician, Lucas numbers or Lucas series are similar to Fibonacci numbers, but don't have to start with 0 or 1.

Consider the Lucas series:

2, 1, 3, 4, 7, 11, 18, 29, 47, 76, 123, 199, 322, 521, etc...

vs the Fibonacci series:

1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, etc...

Most flowers that don't have an amount of petals that match the Fibonacci numbers, usually match the Lucas numbers shown before that. Thus, flowers with 4, 7 or 11 petals might have started from a different base, but still follow the Fibonacci method.

Seeds in flowers such as poppies or sunflowers seem to follow a Fibonacci pattern. The sunflower in particular seems to love the series. Sunflowers have their seeds spirally placed in such a way that if you count the spirals according to the direction they're oriented in, you usually end up with 55 or 89 spirals going one way and 34 or 55 spirals going the other. Even in fruit we see the Fibonacci numbers – a banana has 3 seed chambers and an apple has 5.

If you look at pineapples, pine cones, cauliflowers, cabbage, etc, you find the Fibonacci spirals. In fact, even when you look at the ways plants grow leaves, you find either Fibonacci or Lucas numbers.

You don't have to confine yourself to the Earth either, as we've noticed logarithmic spirals in the arms of entire galaxies in space. Yes, some are golden spirals, and some aren't, but they're following a similar pattern, which leads us to believe that there's something about this that the universe as a whole likes.

For many, the easiest jump to make is to invoke a higher power that has left this "clue" for us mere mortals, and thus we find the "divine" in all of this. However, no good mathematician is ever satisfied with divinity. Instead, we ask ourselves, could it be any other way?

Most sunflower seeds are arranged in a spiral pattern, and the spirals are almost always Fibonacci numbers

As it turns out, evolution is essentially the answer we're looking for. There are two main reasons why the Fibonacci numbers, spirals, the golden ratio, etc are preferred in nature. The first is because it is the best way to cram in as many seeds or leaves into a limited surface area, and the second, because φ is the angular ratio which is best suited for plants to grow leaves that don't overlap.

Nautilus shells naturally display the Golden Spiral

360 / φ = ~ 222.5, or, 360 - 360/φ = ~ 137.5. Now if you were a plant and grew new leaves at an angle of 137.5 degrees away from the last leaf, you will find that you can get more leaves to grow without overlapping, and thus, all the leaves are able to catch sunlight. In a crowded forest, before land creatures existed and life was mostly in the sea, the plants ruled the land and the struggle was to catch as much sunlight to be able to grow. The plants which were able to use

the golden ratio to their advantage (without knowing it of course), survived, and the ones that didn't, perished. This explains why we see it so often in nature. Of course, there are many plants that don't exhibit this behaviour, and corn, for instance, grows leaves at 180 degrees from one another – which is the angle that ensures overlapping of leaves, not prevents it. There are always

For an excellent and really quick explanation as to why this happens in plants, check out this video: *http://dgit.in/DoodlingMath*

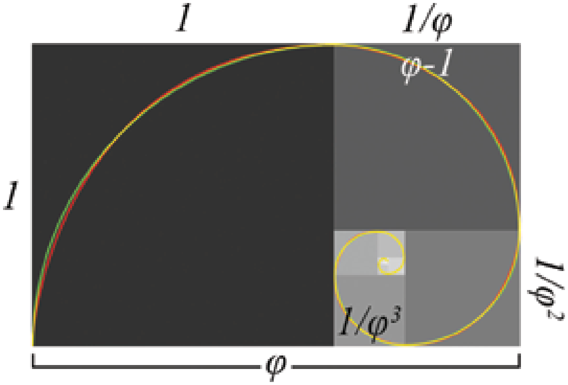
There are three parts, make sure to watch all three.

The golden ratio in us

It's not like we're immune to the Fibonacci numbers or golden ratio ourselves. The ratio of 2:3:5:8 usually applies for the digits on your finger if you look at an xray film. Of course you can also conveniently create a pattern to fit into the numbers: You have 1 thumb on each hand, 1 thumb has 2 bones, 3 bones in the fingers, 5 digits per hand and 8 fingers per human – 1, 1, 2, 3, 5, 8. However, we think this is like spotting an elephant shape in the clouds – you're looking to create a pattern, and not necessarily spot one.

In general, the ratio between the forearm and the hand is also the golden ratio **φ**. Our inner ear is shaped like a golden spiral.

In terms of beauty, artists have always drawn beautiful humans using the golden ratio, and even today professional photoshoppers



The actual golden ratio spiral vs the Fibonacci spiral

use it to touch up images of celebrities. Even dentistry and cosmetic surgeons use it to arrive at what is a generally accepted benchmark of beauty.

For example, the ratio of width in "the perfect smile" of the incisor to premolar is **φ**. The perfect tooth shape in the same smile would sit perfectly in a rectangle whose sides had a ratio of 1:**φ**.

When the ratio of width of your eye to the distance between eyes across your nose (the distance between the whites of the eyeballs, not the actual pupil) is 1:**φ**, you are considered to have beautiful eyes. ■

The Philosophy

Some would say, that mathematics isn't a science, but a philosophy

What is one? Or rather, one what? When we say one, or two, or three, what do we really mean? None, then something, then another, and another... We've given names to numbers, but does that make the numbers real?

This is what the Ancient Greeks under Plato used to philosophise about. Since the Greeks had a geometric view of the world, everything with them was about measurements in geometry. When they said "one" the meant "one unit", and that one unit was not an objective, constant measurement. There was no constant that could be applied, no sacred ruler that one could go and compare lengths to.

When Hippasus, a student of Pythagorus, discovered that root 2 was an irrational number, legend has it that the Pythagoreans were so irritated by the discovery, that they murdered poor Hippasus in order to keep it secret. This is because the discovery proved that numbers do exist after all, and they're not all geometric.

Mathematical Platonism

Stemming from Plato, and the Academy he started 2,500 years ago, this philosophy of math believes that mathematical entities are real, they are abstract, and they exist whether there's anyone around to understand them or not. Thus, one is still one, whether you call it one in English, or Ek in Hindi, etc. Regardless of language, or even intelligent beings around to understand the concepts, mathematics would still exist. It also believes that math is a priori, which means that it is understandable without experience.

For example, if you make the statement: "If I had two oranges and then someone gave me two more, I would have four oranges", this statement is considered a priori true. You didn't need to be there to be able to verify it. No experience of the event described is needed, especially because the "If" in the beginning makes this hypothetical. Thus, mathematics applies, and there are no problems.

However, if you make the statement: "I had two oranges yesterday, and then someone gave me two more, and then I had four oranges." This is a statement of fact that cannot be verified. Mathematics doesn't apply here, what's really missing is proof. Anything which needs proof or experience to be considered true, is what's known as a posteriori.

Logicians

Logicians also believe in a priori knowledge of math, however they make a differentiation from Platonists by suggesting (rather logically) that math is nothing but logic. They feel that it's not special to know or understand math, because it's not special to know or understand logic. Even a child will only burn itself twice (maybe thrice) on a flame before it stops attempting to put its finger in. This is just simple logic at play. – I did it once, I got burned, I did it again, same result, it's logical to assume that I will always get the same painful result, so let's stop putting fingers into flames...

Embodied Minds

There is another school of thought which suggests that mathematics is just something humans made up. It doesn't exist in reality, merely in our brains. It's the living who "count" and come up with the abstract thought of "numbers". Whereas, in actuality, numbers, are just a concept in our heads.

In Fictionalism, for example, mathematicians and philosophers feel that you don't even need numbers to arrive at truths, such as Newtonian physics. This was demonstrated by Harty Field in his book Science Without Numbers (1980), where he used axioms to arrive at Newtonian Mechanics, without using a single number – as the title of his book suggested.

Summing up

Let's be perfectly honest. This book couldn't cover math as a whole. It would take a million books this size to cover every concept of math that human brains have come up with. However, our aim was never to be like a textbook and drill math into your heads, but to try and encourage you to rediscover math, and look at it in a new way, and, for some of you, to try and shake your fear of it.

We hope our little history lesson gave you an insight into how wonderful the human brain really is, and how classical mathematics really transcends time. How else could we explain the ancient Babylonians getting root 2 right to six decimals? How else could we explain Indian mathematicians, who were writing in prose, but pondering ideas that were only rediscovered and rethought of, a thousand years later?

We believe that those who feel that math is something abstract, which doesn't exist, and also those who would have you believe that math is intrinsic to the universe, and is very real, are both correct. We feel that mathematics is a combination of truths and theories, of discoveries and inventions, and as much a part of the universe as it is a human invention. We don't know all that math has to reveal to us, and we may never know, but isn't not knowing, and then trying to find out, the whole point of being human?

Remember to send us feedback and suggestions about this book: write to *editor@digit.in* with the subject "dmystify". ■


http://dgit.in/GrkPhilosphr