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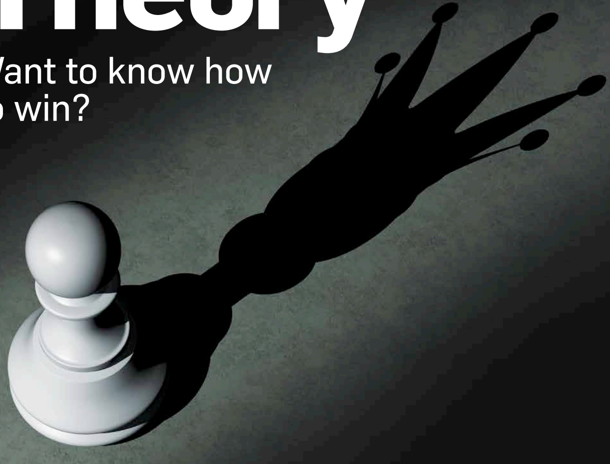
The Small Book of Big Thoughts

Game Theory

Want to know how to win?



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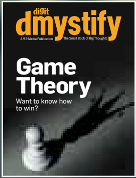
October 2017

Playing to Win

"Victorious warriors win first and then go to war, while defeated warriors go to war first and then seek to win"

– Sun Tzu, The Art of War

INSIDE



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Published by 9.9 Mediaworx

October 2017
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Pick a number...

Picture this – It's the first day of school. Your mathematics teacher walks in. He asks you to settle down, since he wants to play a game with you. He urges all 100 students to choose a number between 1 and 100. No fractions and no decimals are allowed. Simple enough? Now this is where he throws in the catch. For your number to win, you need to choose a number that is closest to 2/3rd of the average of all numbers. Go ahead, think! We will wait for you to hash it out. Still confused? Let's work it out step-by-step.

Hypothetically, imagine a scenario where there are only 4 students in the class and they choose the numbers 15, 25, 45 and

60. The average of these numbers is 36 and 2/3rd of 36 would be 24. In this case, the student who chose 25 would win. Your class, however, has 100 students and 100 probabilities. So what then is the best number for you to choose in order to increase your chances of winning? The most common assumption at this point is that the average would be 50, 2/3rd of which is 33. Isn't that what you thought? You're absolutely wrong. It stands to reason that other 99 people also used the same logic and chose 33. Then the winning number would be 5 or 6. Reiterating that logic you finally arrive at 1. Yep, believe it or not 1 is the optimal number to choose. You assumed that the other 99 people with you are also rational and logical and you changed your choice based on theirs and kept iterating this methodology.

This is the logic that underlies Game Theory. ■

What is Game Theory

Know the game, play the game, win the game

Game theory always comes into play (pun intended) in every strategic situation. In every game – Chess, Othello, Abalone, Risk, or even computer games such as Age of Empires, Smite, Counter Strike, etc. – there is negotiation or bargaining. Whether it's a competition with a classmate for first place, or with a colleague for a promotion... If you have ever been involved in such a situation, you have unwittingly applied or been subject to Game Theory. It's not even a new concept, it's been around for centuries. It has been used ever since the first war was waged, or the first negotiation took place. It gained mainstay prominence post the award winning biopic A Beautiful Mind based on the life of the celebrated John Nash.

Game theory is used not only in sports and war but also in politics, business strategy and economics, mathematics as well as developing sophisticated AI. The books we read, the movies and TV shows we watch are replete with examples of game theory. When we said TV shows the first names that popped into your head were House of Cards and Game of Thrones correct? You are right but we bet that the popular sitcom Friends wasn't a part of your list, was it? More on that in the last chapter. Game theory is all around us, either we are playing it or watching it from the sidelines or reading about it. It's a part of our daily social interactions. So what games do we play? How do we play? And how can we get the most benefit out of them? Let's find out.

Games people play

Game theory is a mathematical theory of studying strategic situations. A strategic situation is any situation that involves 2 or more people competing for a desirable outcome, following certain rules. Such a situation is called a *game* in game theory. The participants are called *players,* the possible outcomes are called *payoffs* and the steps taken to achieve the outcome is called a *strategy*.

Interestingly, the objective of game theory is not to *win* (i.e. achieve the absolutely desired outcome), but rather to decide the optimal strategy. The optimal strategy achieves the payoff, or determines a set of payoffs, which would leave you better off than if you had followed any other strategy. Consider this example. A and B are caught for some crime. Both are being interrogated separately with no means of communication between one another. Both are given two options – either stay silent or betray the other person. Depending on the option that both choose, you will either be punished or set free. The rules are as follows:

If both A and B stay silent, both go to jail for 1 year

If both A and B betray each other, both go to jail for 2 years

If A stays silent and B betrays A, A goes to jail for 3 years and B goes free and vice versa.

Confusing? Let's make it pictoral.

		B	
		Stays silent	**Betrays**
A	Stays silent	1 year 1 year	3 years Goes free
	Betrays	Goes free 3 years	2 years 2 years

So breaking down this example, A and B are the players, no communication are the rules, the actions that A and B take are the strategies adopted by them individually and the matrix shown above is the payoff matrix.

Hannu Rajaniemi wrote The Quantum Thief, and used the Prisoner's Dilemma in the opening chapter

What would you do if you were A? Yes, always betray. Simply because when you betray, no matter what B does, you are better off than the alternative. Say both of you stay silent, both go to jail for 1 year but if you stay silent and B betrays you, you go to jail for 3 years. On the other hand, when you betray, you either go free (if B stays silent) or you go to jail for 2 years (if B also betrays you). So the payoffs between staying silent and betraying are 1 year jail vs 3 years jail for staying silent and no jail time vs 2 years jail time for betraying. So you always betray. This example is called as the Prisoner's Dilemma and is one of the building blocks of game theory.

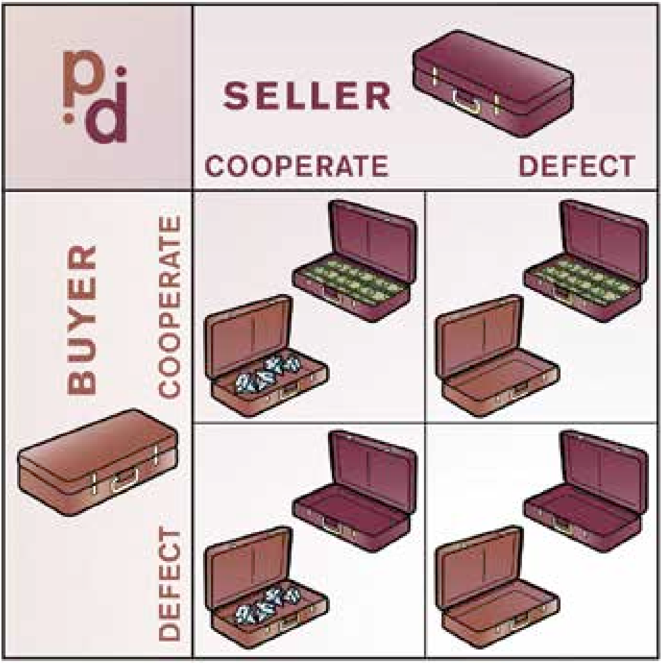
Now let's revisit our earlier statement – the optimal strategy achieves the payoff or determines a set of payoffs which would leave you better off than if you had followed any other strategy. In this example, your desired payoff would be least jail time or no jail time. When we choose the betrayal strategy, we are choosing between no jail time and 2 years and forgoing the 1 year jail time option. But your payoffs are always better with this strategy than with any other strategy.

In this example, choosing to betray always gives the better payoff than choosing to stay silent. Hence it is known as the *Dominant Strategy* and staying silent is the *Dominated Strategy*. Whenever there is clear dominant strategy, you should always go with it (duh!). In most cases, in simulated games and real life, there is not always a clear and constant dominant strategy. Then you need to adopt a *Mixed Strategy*. In a mixed strategy variation, you change strategies based on your opponent and the payoffs you desire to achieve. A game of rock, paper, scissors has no dominant strategy. The winning strategy depends upon the choices of the other players.

To fight or not to fight, that is the question

Consider this example. You are India chief of Uber, and you find out that there is a new entrant in the in the same space in Mumbai only. The new entrant also comes with deep pockets and immense experience in the market. Now you as chief of Uber can either share the market with the new entrant and earn profits, just a lower amount than before. Let's call this strategy "Cooperate".

Alternatively, you can engage in a price war by giving higher incentives to your drivers or customer discounts to retain the larger market share and scare further competition. Let's call this strategy, "Compete". The compete strategy entails an immediate loss but a possible gain in the future. Your competition has the same options as a counter



Another version of Prisoner's Dilemma is the "Closed Bag Exchange". Can you figure out your optimal strategy?

strategy. In this case, there is no dominant strategy. If you choose to cooperate, someone else can enter the market in the future, and eventually it will become unsustainable. You can choose to compete, and either drive out the competition or be driven out yourself (pun intended). If you win the upside is tremendous, if you lose it's tragedy. Thus, your strategy is highly dependant on the response you expect from the competition. If you don't have the funds or resources to compete, you cooperate until you do. If you can compete, you do so for as long as possible. The optimal strategy changes.

Now the question arises how do you determine the optimal strategy? That depends on a few factors. The most important is the dependance on the strategy of the other players and their reevaluation of it. Consider the game of Chess compared to Prisoner's Dilemma. In chess the opponent's strategy determines yours and any change in strategy by the opponent forces you to reevaluate yours. In Prisoner's Dilemma, no matter what strategy B adopts, you adopt the betrayal strategy and stick with it.

Another important factor is communication. Whether you can communicate with the other person or have to operate in a vacuum affects your choices and your strategy. Your goal and objective is another factor. If you know your goal, you devise strategies that maximise the probability of achieving the objective. And last but not the least is the *game* iterative with a finite or infinite repetitions. ■

History

There's more to it than just A Beautiful Mind

Ideas don't develop in a vacuum. New breakthroughs are achieved by building upon existing work done by others. Game theory is no exception. The roots of game theory can be traced to probability calculus, since probability is one of the cornerstones of game theory strategies. The beginning of probability calculus is associated with the correspondence of Pierre de Fermat and Blaise Pascal in 1654.

In 1713, James Waldegrave provided the first known mixed strategy solution to a matrix game (one involving a payoff matrix) in a letter to Pierre Remond de Montmort. It was a 2 player card game named *le Her* where the objective is to hold a higher card than the opponent. Every player can change cards exactly once, the second player can exchange with the starting player and the starting player can exchange with the deck. There are a few more rules, but we won't get into them right now. This was the first recorded mathematical solution of adopting a strategy in a game. And of course, this was based on probability.

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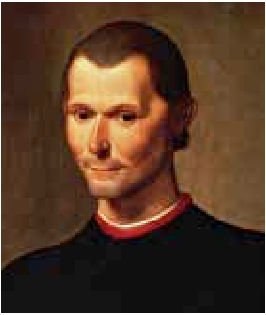
Blaise Pascal and Pierre de Fermat who laid the founding stone of Game Theory by establishing the concept of probability

Until the early 1900s game theory was not codified into a mathematical concept. There were scattered examples, illustrations, theories addressing specific examples, etc., but there was no unifying theory to solve for *all* such examples. Then through 1921–1927 Emile Borel published a series of notes on symmetric two-player zero-sum games with a finite number n of pure strategies of each player. Although he applied mathematics to a special case game only, this was the first recorded instance of using mathematics to solve an entire set of games.

Borel was the first to codify game theory or a part of it in mathematical terms but the founding stone of game theory that we see today was laid down by mathematician John Von Neumann, a child prodigy. In 1928 the existence of the solution of any finite two-player zero-sum game in mixed strategies was proved by Neumann. In his paper he provided a proof of minimax strategy in a 2 player zero sum game. 'Minimax' refers to the strategy of minimising your maximum loss. It took into account all the possible moves of the opponent, while devising the optimal strategy under the conditions of perfect information. This treatise in all of its mathematical glory was the biggest milestone achieved in this field till date and earned Neumann the sobriquet of "Founder of Game Theory".

A zero sum game is where one player's gain is equal to the loss of other players. Poker is an excellent example of zero sum game where the winner takes the pot. Thus the winner's profits (total money in the pot less his own bets) are equal to the losses of the other player or players, ignoring any house percentage.

In 1944, Neumann along with economist Oskar Morgenstern furthered the cause of game theory in their seminal work *Theory of Games and Economic Behaviour*. Most of the basic terminology and problem setup that is used even today can be traced to this volume. This publication established game theory as a full-fledged sub-discipline in the field of mathematics. In this treatise, Neumann



Niccolo Machiavelli wrote "The Prince" in the 16th century. It contains several elements of game theory

and Morgenstern again theorised about zero sum games, but with more than two players and imperfect information. Perfect information is where players know, at each time, all moves that have taken place till that point in time. This book outlined mathematical theories involving a broad range of economical problems. This book propagated the usage of game theory in other disciplines apart from mathematics. This is also known as the Classical Game Theory wherein an individual rational player makes consistent decisions in the face of certain and uncertain alternatives. Such a player does not necessarily assume that other players also act rationally. This was the first stage of game theory.

The monumental work of Neumann and Morgenstern still had a few flaws. Their theory was mainly centered around cooperative game theory, which analyzes optimal strategies for groups of individuals, presuming that they can enforce agreements between them about proper strategies. It did not account for non-cooperative, antagonistic scenarios where players cannot enter agreements and distribute payoffs for the optimal strategy and payoff. This need was addressed by Nobel laureate John Forbes Nash.

In 1949 John Nash wrote his Ph.D. thesis named Non-Cooperative Games, where the concept of equilibrium point was proven. This equilibrium point is known as the Nash Equilibrium and John Nash won the Nobel prize in Mathematics for establishing this theorem. This is the second stage known as the Modern Game Theory where it is assumed that only is the player rational but all the other players are also rational to a high degree which will lead to cooperation to find the most optimal strategy for all thereby leading to a Nash Equilibrium.

In 1953, Lloyd Shapely introduced the concept of payoff distribution in a coalition such that each players receives the optimal payoff according to his or her attributes and powers. This fair distribution of payoff is called the Shapely Value, named in his honour. The impact of Llyod Shapely on the subject of game theory was such that he was regarded by many as the "Father of Game Theory" and the most important or equally important name after Neumann. Shapely was also awarded the nobel prize for his contributions. In 2012, Alvin E. Roth and Lloyd S. Shapley were awarded the Nobel Prize in Economics "for the theory of stable allocations and the practice of market design".

In the 1950's, game theory flourished and was applied to political science for the first time. Again, Llyod Shapely was instrumental in this phenomenon. Martin Shubik and he published a paper applying game theory to political sciences using Shapely value to determine the power of the members of the United Nations Security Council. R. D. Luce along with A. A. Rogow in 1956 and by W. H. Riker in 1959 also applied Game theory to political science.

There were further additions made to the concept of Nash Equlibrium. In 1965, Reinhard Selten introduced his solution concept of subgame perfect equilibria, which further refined the Nash equilibrium. Then In 1967, John Harsanyi developed the concepts of complete information and Bayesian (incomplete information) games. He also contributed extensively to the study of equilibrium selection. Nash, Selten and Harsanyi were jointly awarded the Nobel Awards in Economics in 1994 for their contributions. The Harsyani version was also known as the third stage of game theory called New Game Theory.

John Nash, Jr. *Author: Economicforum*

John Maynard Smith and G. R. Price applied game theory to evolutionary biology in 1973 in their treatise titled *The Logic of Animal Conflict*. This was a transcending moment for the field of game theory as it was proven to be useful not only in social sciences dealing with human behavior but also biological science. Game theory provided the most satisfying explanation of the theory of evolution and the principles of behavior of animals and plants in mutual interactions. This proved another point in game theory. The players, are not necessarily rational, conscious, thinking organisms but the genes in which the instinctive behavior of these organisms is coded. The strategy is then the behavioral phenotype, i.e. the behavior preprogrammed by the genes – the specification of what an individual will do in any situation in which it may find itself; the payoff function is a reproductive fitness, i.e. the measure of the ability of a gene to survive and spread in the genotype of the population in question. This led to the evolutionary stable strategy which if adopted, no mutant strategy can invade the population.

In 1984, Robert Axelrod showed that if a game is repeated, a cooperation strategy based on reciprocity will evolve and sustain. The only condition being that the game is repeated enough times. In such a scenario, a strategy of *do unto others as they do unto you* is the most optimal one. Using a cooperation strategy, the payoffs for all are maximised and the non-cooperative elements are eventually eliminated in the subsequent "rounds".

John Von Neumann, the Founder of Game Theory

Suppose the game is that there are 10 farmers, A to J, all producing a unique crop. Every farmer exchanges one unit of crop with another farmer and receives a unit in return. Assume that the decision to give or not must be taken simultaneously and once taken, cannot be changed. Following the cooperative strategy, every farmer would exchange 1 unit with the other 9 farmers and earn 9 unique units for every single round played with all the farmers. Now suppose there is one "cheater" (J) who simply takes the unit without giving back any. Suppose this cheater cheats on the first round with every farmer, he will earn

Player 1 (10 rounds)	Units earned by player 1	Player 2	Units earned by player 2
A	10	B	10
A	10	C	10
A	10	D	10
A	10	E	10
A	10	F	10
A	10	G	10
A	10	H	10
A	10	I	10
A	0	J	1
Total units earned by A	**80**		

Player 1 (10 rounds)	Units earned by player 1	Player 2	Units earned by player 2
B	10	A	10
B	10	C	10
B	10	D	10
B	10	E	10
B	10	F	10
B	10	G	10
B	10	H	10
B	10	I	10
B	0	J	1
Total units earned by B	**80**		

9 unique units. After the first round, the other farmer will stop cooperating and start cheating. Thus, if every farmer plays 10 rounds with every other farmer, he will have earned 10 units of each unique crop for a total of 80 units. The cheater will have earned a total of 9 units with only 1 unit of every unique crop. Thus, the cheater will be at the bottom of the pack (in terms of unique crops) and will subsequently be eliminated or forced

to change his strategy. This is of course assuming that having unique crops is the winning outcome.

Now let's take a look at the unique units earned by K (the cheater) below. Although the cheater will end up with more overall units of crops (he keeps all of his own 90 units safe by never trading, but earns 9 units as shown below), he will also end up with the least diverse set of crops, which will cause him to fail. ■

Player 1 (10 rounds)	Units earned by player 1	Player 2	Units earned by player 2
J	1	A	0
J	1	B	0
J	1	C	0
J	1	D	0
J	1	E	0
J	1	F	0
J	1	G	0
J	1	H	0
J	1	I	0
Total units earned by J	**9**		

Nash Equilibrium and Shapley Value

The Founding Concept of Game Theory, it's almost Zen

As we saw earlier, not all games have a dominant strategy and in such a situation, the optimal strategy depends upon the other player's choice. If at some point every player chooses the best strategy based on the strategy followed by other players, we have a Nash Equilibrium. Since every player has chosen the optimal strategy, there is no incentive for any player to deviate from the existing strategy.

Let's revisit our earlier example of Uber vs a new entrant. Assume that the goal for both of you is to earn profits and there are conditions on additional financing if there are no profits. Under such circumstances, you and the new entrant will both feel that you will accept a shared marketplace, cooperate and continue earning profits. In such a case, where both of you are earning profits and have no incentive to change the strategy, you are said to be in equilibrium and this point is known as the Nash Equilibrium.

So how does one find the Nash Equilibrium? The most important assumption all players are rational. So who is a rational player? A player is rational if:

- He consistently acts to improve his payoff without the possibility of making mistakes.
- Has full knowledge of other players' intentions and the actions available to them.
- Can calculate all the possible strategies to to find the most optimal one.

If all the players in a game are rational and know that the others are as well, they would stand in the other player's shoes and understand their optimal strategy. Then they would factor in their own strategy and see if it affected the other player's strategy. If not, they have reach equilibrium. If it did affect the other player's strategy, it's just rinse and repeat until equilibrium is achieved. Another method is by constructing a payoff matrix. Assume a payoff matrix between 2 players with payoffs ranging from 0 to 5 for different choices. There are 4 choices in total. It would like this

		B			
		Choice 1	Choice 2	Choice 3	Choice 4
A	Choice 1	4 / 2	2 / 2	0 / 0	4 / 4
	Choice 2	1 / 2	0 / 4	5 / 3	1 / 1
	Choice 3	0 / 3	3 / 0	5 / 0	2 / 2
	Choice 4	4 / 1	2 / 0	3 / 3	0 / 1

Now, in the payoff matrix, underline the highest payoff in the each row for Player A and in each column for Player B. E.g. for A, in row Choice 1, underline 4 as the maximum payoff in that row and for B, in column Choice 1, underline 3 as the maximum payoff in that column. If there is one cell where the payoffs for both the players are underlined, that is the Nash Equilibrium strategy, in this case Choice 1 for A and Choice 4 for B.

In 2006, a Federal judge in the US ordered opposing lawyers to use Rock Paper Scissors to decide a trivial but lengthy matter of debate

		B			
		Choice 1	Choice 2	Choice 3	Choice 4
A	Choice 1	4, 2	2, 2	0, 0	4, 4
	Choice 2	1, 2	0, 4	5, 3	1, 1
	Choice 3	0, 3	3, 0	5, 0	2, 2
	Choice 4	4, 1	2, 0	3, 3	0, 1

Now, is this is the only equilibrium point? What if you decide to keep Uber for the masses and your competition decides to target only the elite luxury sedan crowd? In this case, both of you have captured a different section of the market, are not competing with each other and are making profits. So again, no incentive to change strategy. Thus we see that for a situation, there might exist more than one Nash Equilibrium or none at all.

There is one more twist to the story. Consider the simple game of rock, paper, scissors. Yes the simple game we played as kids has a great impact on Nash Equilibrium and game theory. In this game, if you are playing only against your

Alvin Roth, Nobel Prize laureate who shared the prize with Lloyd Shapely in 2012 for his contributions to game theory. He wrote a paper on National Resident Matching Program which was based on the theoretical work of Shapley.

friend (a 2 player game), Nash Equilibrium exists when you choose rock ⅓ of the time, paper ⅓ of the time and scissor ⅓ of the time. This is also the optimal probability distribution and hence assumed to be the strategy followed by your friend too. Correct? Not according to mathematician Zhijian Wang. He conducted experiments with real people at Zhejiang University in China. He noted that the players who seemed to consistently win had a tendency to stay loyal to their strategy whereas the players who changed strategies often lost. The tendency to stick loyal to their choice is known as "conditional response". Wang found that many people were using the Nash Equilibrium but many were also using the staying loyal strategy or displaying a conditional response. Upon analysis, Wang found that the conditional strategy proved to be 10 percent more reliable for winning than did the Nash Equilibrium.

So we see that Nash Equilibrium can exist, there can be more than one equilibrium point and most importantly, following the equilibrium strategy may not always be the best strategy.

John Nash (extreme left), Lloyd Shapley (right of centre) and Leon Petrosyan (extreme right), who is the Head of the Department of Mathematical Game theory and Statistical Decision Theory at the St. Petersburg University, Russia

Just as the Nash Equilibrium proposes the optimal strategy for an individual player, the Shapley Value proposes the most fair allocation of resources or payoffs to the members of a coalition. Notice the word fair – it does not imply equality. Fairness means that the member holding the most power in a coalition gets the highest proportion of payoffs.

Consider this Voting Game: the shares of a company are held by 4 shareholders A, B, C and D in the distribution of 45%, 25%, 15% and 15% respectively. A resolution to be passed requires 51% of the shareholding. Let's assume that there is some payoff attached to the consenting shareholders who form the majority. In this case, how should the payoff be distributed and between who? The entire payoff will be distributed only between the members of the majority coalition and the other members will get nothing. Obviously no single shareholder holds enough shares to pass the resolution. A coalition of minimum 2 people is required. Let's take a look at the possible coalition

Players	Shareholding
A	45%
B	25%
C	15%
D	15%

The combination of A + X (*any player*) gives the required majority. So B, C and D are replaceable from A's perspective. So under scenario 5, if the payoff was to be 100, half of it will go to A and remaining half

Scenario	Coalition	Total Shareholding
1	B + C + D	55%
2	A + C	60%
3	A + D	60%
4	A + B	70%
5	A + B + C + D	100%

will be split equally between the remaining 3 members B, C and D. So the payoffs are A= 50, B= 50/3= 16.66, C= 50/3= 16.66 and D= 50/3= 16.66. But is this the optimal coalition for all parties involved? You can see that A can break away and form a coalition with any other party and both will enjoy a higher payoff. Similarly for the coalition of B + C + D. let's calculate the payoffs for every coalition.

Scenario	Coalition	Total Share-holding	Payoffs
1	B + C + D	55%	40, 30, 30
2	A + C	60%	75, 25
3	A + D	60%	75, 25
4	A + B	70%	71, 29
5	A + B + C + D	100%	50, 16.66, 16.66, 16.66

Calculation for A + B. A retains the basic 50 (from scenario 5. Since A can get minimum this, there is no incentive for A to break from this grand coalition unless the payoff is increased and similarly for B). In addition to the basic 50, A will also receive the payoff from C & D (16.66 + 16.66) on a pro-rata basis. The pro-rata is calculated as 45/70 (70% is the total shareholding of the coalition out of which 45% is supplied by A). Thus the formula is

Payoff for A = 50 + (16.66+16.66)*45/70

As we can see, if every party demands a fair distribution of payoff to the members of the coalition, the coalition of B, C and D is optimal and can be called as the Nash Equilibrium. In this strategy, no member of the coalition has any incentive to defect from the strategy.

So is there a grand coalition consisting of all members such that no members wish to break away and form a different coalition for a higher payoff? In other words, can the Nash Equilibrium be a strategy that involves all the parties? Yes it

Try to reach the Shapley Value in this Voting Game
https://www.maa.org/sites/default/files/ebooks/Apple-t5Exhausted-Knights.html

can and such a coalition is called a core. This is a stronger version of Nash Equilibrium in the sense that in the Nash Equilibrium, there was no incentive for only one player to deviate from the strategy whereas in this case multiple players don't have any incentive to deviate from the strategy.

If a Shapley Value in the core can be devised for 6 billion people to reap the rewards of protecting the environment, we would soon have the perfect environment possible.

The core does not always exist and is not necessarily unique. Just like Nash Equilibrium, there may not be a core or there can be multiple core values. In the above example, there is no core since members of the grand coalition have an incentive to break away. Also, in the above example there is no single member who is absolutely imperative to be a part of the coalition to succeed. But if the threshold is increased, say to 80%, A & B both become part of the core since they are now indispensable to the coalition. ■

Let the Games Begin!

What are we doing exactly?

Now that we have seen the evolution of game theory and understood the basics, let us turn our attention to the different types of games. Unfortunately this part gets a bit theoretical but we will try to keep it short and sweet.

The simplest way to classify games is by their characteristics (the way we classify anything for that matter). We can classify them as:

1. Number of players. Any game should contain at least 2 players or there could be more than 2 players. The number of players affects the strategies to be employed, the Nash Equilibrium and the Shapley Value as we have seen earlier.
2. Is it a simultaneous or sequential game? Prisoner's Dilemma is a simultaneous game wherein all the players are moving or making their choices at the same time. Political and business scenarios are usually simultaneous games. In sequential games, as the name suggest, you make a move then wait for the other players to make their move. Chess and poker are prime examples of sequential games.

3. Do players have perfect information? Perfect information is when a player, when about to move, knows all previous moves.
4. Is complete information available? Complete information is different from perfect information. Complete information means that all players know the order in which the players move, all possible moves in each position, and the payoffs for all outcomes. As you can imagine real-world games usually do not have complete information – we don't live in a perfect world! In theory, games with incomplete information are not analysed because it's just harder to do so for obvious reasons.
5. Is it a zero sum game or non zero sum game? A zero sum game is where one player's loss is equal to another player's gain. In theoretical words, the total value of payoffs should be zero. One wins, another loses. A gets +1, B gets -1. Total payoff amounts to zero. On a broader scale, assuming the government as one of the players, you can consider the macroeconomic construct to be a zero sum game where money or wealth does not increase or decrease, it simply changes hands.
6. We also saw that communication is a very important criteria. It can determine your strategies, whether you can cooperate, form a cartel and determine the most commonly acceptable strategy.
7. The logical question after communication is whether the players abide by the agreed strategy? In other words, are the players

cooperative or non-cooperative? This, in turn, will determine the optimal strategy.

It is important to understand that these classifications are not mutually exhaustive and there are several overlaps as you will see. We will examine a few of these games.

1. Simultaneous Games

In a simultaneous game, the first thing we do is design the payoff matrix. This is called the *normal form* of the game. The number of cells in the matrix is dependant upon the number of choices. So for 2 choices in a 2 player game, you have 4 cells (2 x 2) for 3 choices you have 9 cells (3 x 3), for one player with 2 choices and another player with 3 choices, you have 6 cells (2 x 3).

The number of payoffs in the cell are dependant upon number of players and whether it is zero sum or non zero sum game. If it's a zero sum game, there would only be 1 payoff per cell since somebody would win and the other would lose. In a non zero sum game like Prisoner's Dilemma, there would be 2 payoffs, 1 for each player.

Two Player Non-Zero Sum Game (Prisoner's Dilemma)

Two choices per player so 4 cells, non-zero sum game with two players, so 2 payoffs per cell.

		B	
		Stays silent	**Betrays**
You	Stays silent	1 year 1 year	3 years Goes free
	Betrays	Goes free 3 years	2 years 2 years

Two Player Zero Sum Game

Taking the example of the famous Rock, Paper, Scissors, we can construct the payoff matrix without including the opponent (B) since a gain to you is loss to B and vice versa. Where both choose the same option e.g. rock vs rock, it's neutral or no gain no loss for both. So again we have followed the matrix rules given above. Three choices per player so 9 cells and zero sum game so 1 payoff per cell.

You	Rock	Paper	Scissors
Rock	0	-1	1
Paper	1	0	-1
Scissors	-1	1	0

MiniMax Strategy

An important construct in a two player game is a minimax strategy. It refers to minimising the maximum loss, basically putting a floor on the maximum loss you can incur. Hence, this is also known as a security strategy. It can also be defined in terms of minimum payoff, in that case its called as maximin strategy. The payoff matrix looks just like the Prisoner's Dilemma one above.

		B	
		Stays silent	**Betrays**
You	Stays silent	1 year 1 year	3 years Goes free
	Betrays	Goes free 3 years	2 years 2 years

As we already saw, betrayal is the dominant strategy. It limits your maximum loss to 2 years no matter what B does. In this case, betrayal is also the minimax strategy.

Let's look at another example of Odd and Even to illustrate this strategy better. There are two players, you and B. Both of you simultaneously call out a number between 1 and 2. You win if the sum of numbers called out is odd and B wins if the sum is even. So your winning numbers are 1 and 3 and for B its 2 and 4. The amount of gain or loss is equal to the sum of the numbers. If both call 1, your loss is -2. If both call 2, your loss is 4, if you call 1 and B calls 2, you win 3. So the payoff matrix from your perspective would look like:

You	1	2
1	-2	3
2	3	-4

Now let's try and find the minimax solution to this problem. Let's say that p is the probability or the number of times you call 1. So if we remember our probability, (1-p) is the probability that you call 2, since total probability always has to equal 1.

The famous auction house Christie won a game of rock paper scissors with competitor Sotheby to win the rights to an auction that collected $20 million. The winning move was suggested by the 11 year old twin daughters of the director of international arts. Read the bizarre story here:
http://dgit.in/GameRockPaper

So if you call 1 with probability p and B calls 1, you lose 2p. Since you are not always calling 1, your gain has to be adjusted with the probability figure.

So if you call 2 with probability (1-p) and B calls 2, you lose 4(1-p) again adjusted for the probability.

So the possible gains and losses adjusted for probability are as below:

You call	B calls	Your Gain/ Loss
1- probability p	1	-2p
1- probability p	2	3p
2- probability (1-p)	1	3 (1-p)
2- probability (1-p)	2	-4 (1-p)

The minimax or maximin strategy is where you minimise your loss or maximise your gain no matter what the other person does on average in the long run. Which means that no matter what B says, your payoff should remain constant. So we need to formulate an equation where the payoff when both call 1 is equal to the payoff of when both call 2.

So assume that left hand side (LHS) of the equation is where both call 1. From the matrix above, we see the payoffs are -2p + 3(1-p).

So the right hand side (RHS) of the equation is where both call 2. So the payoffs are 3p - 4(1-p).

So we need find p where LHS=RHS.

I.e. $-2p + 3(1-p) = 3p - 4(1-p)$

$-2p + 3x1 - 3p = 3p - 4x1 + 4p$

$-2p + 3 - 3p = 3p - 4 +4p$

$3 - 5p = 7p - 4$

$3 + 4 = 7p + 5p$

$7p + 5p = 3 + 4$

$12p = 7$

$p = 7/12$

$1-p = 1- 7/12 = 5/12$

So if you call 1 with a probability of 7/12 i.e. 7/12 of the time and 2 with a probability of 5/12 i.e. 5/12 of the time your average gain is

$-2 (7/12) + 3 (5/12) = 1/12$ or 8.33%.

This is fixed payoff on average in the long run no matter what B does. Such a strategy is called equalizing strategy. This is the maximin or security value for you. What about B? B can use the same process and keep his maximum loss to 1/12 or 8.33%. In B's case it becomes the minimax strategy.

Can you maximise your gain in this simulated game?
http://ncase.me/trust/

Three or more players

If you have been paying attention and thinking, the burning question in your mind must "what if there are 3 players?" Continuing from our previous examples of shareholders voting for a resolution, let's simplify it to 3 players only, A, B and C.

In this case, you construct 2 matrices each matrix following the rules given above.

Simplifying the example: The shares of a company are held by 3 shareholders A, B and C in the distribution of 45%, 35% and 20% respectively. A resolution to be passed requires 51% of the shareholding. The payoff is +2 for each consenting player if only two players consent and -1 to the dissenting voter, +1 to all if all three players consent, 0 to all if all dissent, if only one player consents, -1 to that player and 0 to the others.

In this case, we would construct 2 payoff matrices, one where A consents and the other where A dissents and the choices of B and C given across the row and columns respectively. The first payoff is for A, the table player, second payoff is for B, the row player and last payoff is for C, the column player.

Note that each individual matrix follows the rules given above- 2 choices so 4 cells, non zero sum game with 3 players so 3 payoffs per cell.

The payoff matrices would look like:

A Consents			
		C	
		Consents	Dissents
B	Consents	1, 1, 1	2, 2, 0
	Dissents	2, 0, 2	-1, 0, 0

A Dissents			
		C	
		Consents	Dissents
B	Consents	0, 2, 2	0, -1, 0
	Dissents	0, 0, -1	0, 0, 0

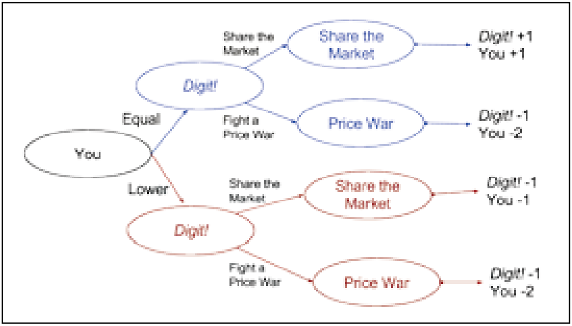
Here's an exercise for you – is there a dominant or Nash Equilibrium strategy in this example and if so, can you find it? Also, are there any players who belong to core? Is there any dominant strategy for any player?

2. Sequential Games

In simultaneous games we constructed a payoff matrix, in sequential games, we construct a decision tree. This decision tree is similar to flowchart and it maps out all the possibilities. Then finally you determine the optimal strategy.

Let's say that you are launching a competitive magazine to *Digit!*. The biggest decision you have to make is on the pricing. You can either keep it at the same pricing as *Digit* or price it lower. You move first, and then *Digit* can choose to ignore and share the market with you or respond with a price war. The decision made by *Digit* will determine the payoffs. We have assumed a payoff variance of +2 to -2 to represent the magnitude of profits and losses.

Let's construct this decision tree.



So now we turn our attention to determining the optimal strategy. We follow a process called "look ahead and reason back". This is also called as backward induction. Almost all sequential games can be solved using backward induction. So we look at the last decision and the payoff it will generate. The last decision lies with *Digit!* and they can choose between a payoff of -1 to +1. So obviously they will choose a payoff of +1. So reasoning back, we see that they will agree to share the market if you enter with equal pricing which give you also a payoff of +1. So win-win for all.

But is this the only decision? We theorized that *Digit!* would make the rational decision and choose a payoff of +1. Its interesting to note that squashing the competition is also a rational decision especially if the company has the means to support a price war. If they engage in a price war, your magnitude of losses are far greater than theirs so theoretically, you won't be able to sustain. If their goal is to drive you out of the market, they will engage in a price war no matter what decision you make. Also this example does not consider several other factors like brand dilution, geographical competition, consumer loyalty, product quality etc., all factors that will contribute to this decision tree. So, the final strategy depends on the goal. In our example, we assumed that profitability was the goal of all players and hence the optimal decision is for you to price it equally and them to share the market.

Does this seem like Nash Equilibrium to you? The optimal strategy has been determined not only for you but for the other player too and neither player has an incentive to deviate from the strategy.

3. Combinatorial Games

Combinatorial games are two person games with perfect information, no chance moves and win or lose outcome. This is usually a sequential game with players alternating moves. Eventually, all possible moves are exhausted and that point is called a *terminal position,* at which point there is one winner and one loser. At the terminal position, if the rule is that the last player to move wins, then it is called the *normal play rule* and if the last player to move loses then it is the misere rule. The game has a finite number of moves before it ends.Such games are further divided into impartial and partizan games. Impartial is where all possible moves are open and same for both players and in partizan games, each player has a different set of moves and has to choose from the set of moves available to him like chess.

There are a few exclusions too. Games involving random chance like rolling of the dice or drawing cards are not allowed in this category. Also, simultaneous games, games with imperfect information, hidden moves and games ending in a draw are also not allowed.

Suppose there are 100 chips; a player can remove 1, 3 or 4 chips in their move and the last to remove the chips wins. You are given the option to move first or second. How do you determine the optimal position to ensure that you win? Let's use backward induction or look ahead and reason back methodology that we learned in sequential games. The final winning position after your move is 0. Let's label it as P. so with 1 chip remaining, there is one possible move to win the game. So we will label this as N. with 2 chips, you can only remove 1 chip allowing the other person to win. So we label this also as P. At 3 and 4 chips, you can remove all chips and win so we label both as N. At 5 you can remove 1, 2 or 3 chips and leave the opponent with 4, 3 or 2 chips. So you would obviously leave him with 2 right? So we label 5 also as N. Let's look at 6 and 7 chips in detail.

6 Chips
Scenario 1
You
Start 6---remove 1----> 5
Opponent
5---remove 1, 3, 4----> 4, 2, 1. Opponent will choose to remove 3 since at the other 2 positions you win.
You
2---remove 1----> 1
Opponent
1---remove 1----> 0, opponent wins.

Scenario 2
You
Start 6---remove 3----> 3
Opponent
3---remove 3----> 0 Opponent wins

Scenario 3
You
Start 6---remove 4----> 2
Opponent
2---remove 1----> 1
You
1---remove 1----> 0 you win.

So 6th chip is labelled as N since there is one scenario where you can force a win.

So 7th chip is labelled as P since there is no scenario where you can force a win.

7 Chips

Scenario 1
You
Start 7---remove 1----> 6
Opponent
6---remove 3----> 2
You
2---remove 1----> 1
Opponent
1---remove 1----> 0, opponent wins.

Scenario 2
You
7---remove 3----> 4
Opponent
4---remove 4 ----> 0, opponent wins

Scenario 3
You
7---remove 4----> 3
Opponent
3---remove 3 ----> 0, opponent wins

So if you are at a situation where you can force a win or directly, it is labelled N like 1, 3, 4, 5, 6. Conversely a situation where you cannot win is labelled as P. So 2 and 7 are P. 0 is P because there are no further moves. So from 0 to 7 the labels read as 0-P, 1-N, 2-P, 3-N, 4-N, 5-N, 6-N, 7-P. This pattern of PNPNNNP repeats continuously. So in a pattern of 7, there are 2 P chips. Dividing 100 by 7, the remainder is 2. So the 100th chip is labelled P. So if you choose to be the second player, you can guarantee a win.

Play the NIM game, a combinatorial game, here- http://www.eprisner.de/MAT109/Applets/AppletNim7.html

4. Symmetric Games

A symmetric game is where both the players have the same options and choices and there is a possibility of a tie between the players. In a symmetric game, the payoff matrix is always square. So again, rock, paper, scissors is a symmetric game. A game of matching pennies where you and B simultaneously choose to show a penny with either the heads or the tails side facing up. If both the pennies match, you win, if they differ B wins. Again a two player zero sum game with a square matrix like this:

You	Heads	Tails
Heads	1	-1
Tails	-1	1

Although this is a square matrix, this is not a symmetric game because there is no zero value or a tie.

Let's consider the Mendelsohn games, (N. S. Mendelsohn (1946). You and B simultaneously choose a positive integer (so 0 is not allowed) between 1 and 100. If both players choose the same number there is a tie and no payoff.

You	1	2	3	4	5	And so on
1	0	-1	2	2	2	2
2	-1	0	1	2	2	2
3	-2	-1	0	1	2	2
4	-2	-2	-1	0	1	2
5	-2	-2	-2	-1	0	1...2....
And so on	-2	-2	-2	-2	1	0...1...2

If you choose a number that is greater by 1 than the number chosen by B, you win 1. If your number is greater by 2 or more than that of B, you lose 2. Since this is a zero sum game, your loss is B's gain and vice versa. Intuitively you would choose 1. Putting this in a matrix,

This is a symmetric game since the matrix is square and there is zero value. Row 1 is the dominant strategy for both as neither one has any incentive to deviate from it – the Nash Equilibrium strategy. ■

Game Theory Applications

Where is it used, and how?

In popular culture

Remember the iconic episode in FRIENDS that we referred to in the beginning of this booklet? That may well be one of the easiest-to-remember example of Game Theory in popular culture.

FRIENDS

In this episode of FRIENDS, Monica and Chandler are trying to hide their budding romance from the rest of their friends. The only unwitting accomplice they have is Joey. When Joey finds out that Rachel and Phoebe also know of the affair; he tries to warn Monica and Chandler; thus hoping to put the cat and mouse game to an end. Much to his chagrin, this is when Monica decides that she wants to have her own fun with the girls. This is when she utters what is one of the most iconic dialogues in FRIENDS ever – "Oh man, they think they are so slick messing with us! But see they don't know that we know that they know!", to which Chandler responds – "Ahh yes, the messers become the messies!" This is a classic case of sequential zero sum game where the actions and assumed logic of Rachel and/or Phoebe affects the strategy adopted by Chandler/ Monica.

Spoiler alert! The truce meeting between Cersei Lannister and Daenerys Targaryen can be interpreted under the lens of game theory. What should have been the optimal strategy?

South Park – Adult Cartoon

In an episode titled 'Toilet Paper', Cartman and the boys keep landing up in detention given their bad attitude in art class. As a result, they decide that the art teacher needs a lesson of her own and decide to TP her house. This episode presents a classic case of the Prisoner's

Dilemma. When the boys are called to Mr. Mackey's office; they plan on letting Cartman take lead and do all the talking. Alas, they are all called in separately and this is when they begin worrying about Kyle ratting them out, given how he is the weakest link in the group. One thing leads to another, and Butters confesses to the crime for reasons best known to him (he was at home with his parents when the crime was committed). The boys decide to confess to their art teacher the next day and let her decide their fates. Much to their surprise, Cartman has already made his confession before Kyle, Stan and Kenny could get to school. As a result, Cartman receives a week's detention; while the others are burdened with two weeks!

Batman Gambit is a plan based upon reading and understanding the other players to a great extent so that you win because they move exactly as you expect them to!! Named after Batman, who through careful analysis can often predict his foes' actions to psychic-like accuracy.

Survivor: Thailand

In the sixth episode of the Thailand leg of the show, there was a mathematical game that was played. Whichever team won the game would in turn win immunity during that episode. The game was simple. There were 21 flags, of which every team could remove either 1, 2 or 3 flags in their turn. The last team to remove a flag was to win immunity. Theoretically speaking, the first team to move should have won. Unfortunately that was not what happened. Sook Jai left Chuay Ghan with only 10 flags; and this was the immediate death of team Sook Jai. What Sook Jai should have done is left Ghan with 12 flags. Let's understand why! The best strategy to apply is to leave the competing team with multiples of the maximum number of flags they can pull out plus one. In this case, the maximum number of flags that could be removed in a single turn was 3. So the winning strategy would be to leave flags in the multiples of 4 for the next team to work with. Here's a breakdown. Imagine you are Team A, and your opponent Team is Team B.

21 flags are what you begin with. For the theory to work, Team A (your team) needs to make the first move.

Team A leaves Team B with 20 flags (remove 1 flag). From here on, regardless of how many flags Team B removes, always leave them with multiples of 4. So in the next round leave 16 flags, 12 in the next round, 8 in the round after; and finally arrive at 4. When there are only 4 flags left for the opponent team to choose from, regardless of their choice, your team will win. This is a master move, and the ultimate winning strategy in a game like this one.

This is a real life example of combinatorial game like the 100 chips one that we studied earlier. You can either create a decision tree or use the terminal value methodology of labelling P and N that we studied earlier. In either case, you should arrive at the same optimal strategy we have given here.

The Godfather by Mario Puzo

Let's look at negotiations from the purview of Game Theory. Most economic relationships are contractual in nature, which means that the parties have come to terms with a state of 'contractual equilibrium' – a state in which the parties involved will negotiate and agree on the issues that are of mutual interest to them. This will automatically denote their intention for the agreement to be enforced. Enforcements, now, can come from two primary sources. Either the contracting parties themselves can turn enforcers for the contract, or they could invite a third party enforcer (an external agent) who is not party to the contract at hand to give an unbiased view of the contract at hand. This third party will also play coordinator to ensure the negotiations go smoothly and the contract is eventually fulfilled.

Keeping this in mind, let's look at the example of the Bocchicchio Family in the book The Godfather. The Bocchicchios are ruthless killers. Their simple code of vengeance is absolute – you harm a

Another trope is Xanatos Gambit which is basically a carefully crafted plan that has every outcome, whether it is winning or losing, benefit the planner in the end.

family member, and they will ensure that revenge follows no matter the personal cost. While this worked for them during their stay in Sicily, it did not help them compete with the Mafia families in America. Their American counterparts were more organized and were successfully running and controlling business structures such as gambling, prostitution and public fraud. Given this schematic of things, the Bocchicchios decided to become negotiators between other warring Mafia families. Let's put this into perspective with an example – Michael Corleone, as the representative of the Corleone Mafia family, has to set up a meeting with rivals Sollozzo and McCluskey. Michael has no way of knowing that he will be safe with them in spite of their promises. Since there is no sense of commitment, the two parties cannot find a mutually acceptable condition under which to meet. This is where the Bocchicchio family steps in. Given that the Bocchicchio family's reputation of vengeance precedes them, they make for excellent third party negotiators. When Michael meets Sollozzo and McCluskey, a member of the

Bocchicchio family's is held hostage by the Corleone family. This hostage will be killed if any harm comes to Michael and the the Bocchicchio family will place the blame on Sollozzo and McCluskey and make them targets. Thus the Bocchicchio family will become the enforcers and all the parties will get the required assurances needed to meet.

The presence of such an enforcer could force a coalition to cooperate and not deviate even if there are better payoffs available.

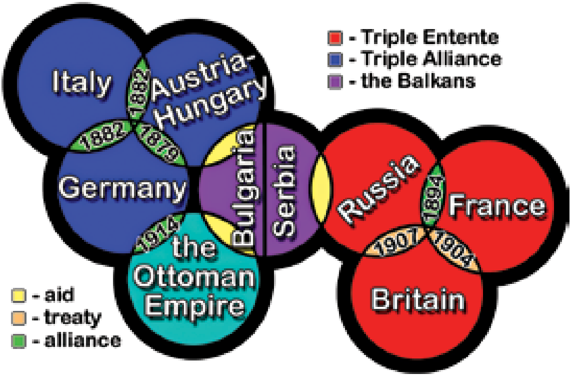
In Business

Auctions

An interesting categorization of auctions could be games with information that is not entirely complete. This is true because in an auction, the bidder will know the personal valuation of an item for him/her. This information, however, is not known to the competing bidders and the seller. In certain other cases, the auction can be attended by proxies, and the outcome of the auction would then not depend on their personal valuation. That being said, one fact remains – auctions are a universally acceptable method of buying or selling any item. Most auctions will involve bids. In standard auctions, the winner will usually be the participant who has bid the highest amount among the pool of participants. There are nonstandard auctions that do not submit to the same prerequisite (example – a lottery). Here's a quick run-through of the different kinds of auctions

Open Bid Auctions

- Ascending Bid – Bidders call out prices higher than the previous bidder. Bidding ends when there is only one bidder who remains, and he/she pays the final price.
- Descending Bid – This one is based on the reserve value of the



A snapshot of alliances and treaties of various countries and empires in history centered around the Balkans

item. The price for the item on bid is lowered until someone accepts the price as being the optimal personal valuation, thus winning the object at the price offered.

Sealed Bid Auctions

- First Price – The highest bid automatically wins
- Second Price – The highest bid will win, but the bidder will pay the amount that was the second highest bid. This method is also called the Vickrey auction. A type of Vickrey auction is employed by Google Adwords where the highest bidder pays the amount of the second highest bidder plus 1 cent.

This is what actually happened at a New Zealand Spectrum Auction in 1990. This auction was designed to be a sealed bid second price auction with no reserve price. The result was jaw dropping; and had game theorists scrambling for days over. The winning bid was NZ $100,000. Given that they had to pay the second highest bid, they ended up winning the contract at a mere NZ $6,000.

Creating Business Strategies

When faced with a problem or issue in business that comes with a fair bit of uncertainty, business owners/ managers should look at Game Theory to help solve their issues. It is a fantastic tool to help by offering perspectives on how various players (in the market) will act under varied circumstances. Knowledge is and has always been a powerful tool. Once you know how your competition is likely to react, it can help you strategize your business plans accordingly thus resulting in the most optimal strategies for your business. A decision tree and payoff matrix that we saw earlier is an excellent tool in deciding the optimal strategies.

For instance, let's look at the deregulation of passenger railways in country X. Suddenly the incumbent (current) rail operator has a series of questions that need answering. Should they engage in a price war? Should they shut down certain routes and provide the attacker (the new entrant rail operator) with a monopoly on those routes? Should they change the frequency of their service or change their schedule? There are a number of options available to both the attackers as well as the incumbents in such a situation. Game theorists have helped chart out a few in a scenario such as this one.

The incumbents could serve as a structure for the attackers to imitate. They could choose to provide services identical to their predecessors. They could take this one step further by adding attractive services into the mix – a rail service that runs only during peak hours, for instance. Or they could come up with an entirely different business model that allows them to operate on a different plain. This would help them have their own piece of the pie without eating into the incumbent's market share. An example of this would be a low cost service provided for those traveling for leisure. This would mean routes that aren't connected to busy business centers, but more scenic low cost routes.

The incumbents, on the other hand, can also choose to respond in multiple ways. They can ignore the entry of the attackers into the market by continuing the way they functioned before the deregulation of the rail network. Or they could choose to counterattack by lowering their prices, or changing their schedules to match those of the attackers. This could, however, lead to an ugly price war. Remember, most price wars are likely to leave both (or all) parties in a loss since it leads to lowering profits for everyone. The only winner in a price war is the consumer.

Both the incumbent and the attacker can help themselves by using the game theory model. They can start out by listing out payoffs for themselves and the other player basis different strategies applied. Once they have this information handy they can make decisions that will in turn depend on whether they are making the first move or their opponents are. Nonetheless, game theory can help them get into the war zone better prepared to walk out with an optimal strategy.

Behavioral Sciences

When game theory is applied to the faculties of economics, finance or business; it operates under the assumption that the players involved in the game are both rational. Behavioral game theory takes this theory one step further and takes into account the players' understanding of the payoffs, their feelings about their opponents' payoffs, limits if any in strategic thinking, and their ability to grasp changing situations and thus shift strategies as and when necessary.

Ultimatum Game

While a lot of theorists used the Ultimatum game to explain game theory, it was this very game that unraveled violations in the standard assumption that all players are rational. In an Ultimatum game experiment, one player (let's call him Player A) is endowed with a sum of money. He is then asked to split this money between himself and another player (Player B). Player B has two options. He can either choose to accept the sum being offered by Player A, in which case they both receive a part of the amount in question. However, if Player B decides to reject A's proposal, they both receive nothing. If rationality were the only basis to be followed here, Player A should offer a small token amount to Player B. Player B in turn should accept this token amount since it is an amount that is coming to him at no cost. However, in most real world scenarios, Player A went on to offer more than a small token amount, some of them even offered an equal split to Player B. In other cases, Player B was also seen to reject the proposal of Player A, showing that he was willing to sacrifice the payoff when he felt that the offer being made to him was unfair.

Global Warming and Climatic Changes

A similar real world example of this would be the current state of affairs when it comes to our environment. As a collective, people are aware of the fact that global warming is a very real problem. All of us need to pull up our socks and take stock. However, taking stock of the issue at hand by reducing your carbon footprint or avoiding the use of plastic bags is not in favor of the individual. The individual then starts thinking for the collective and convinces himself/herself that since it's for the better of everyone, everyone else will work towards it thus giving the individual a chance to cheat. The result – everyone cheats! This is like Prisoner's Dilemma on a global scale. Remember we said earlier that the optimal strategy for the individual is not always the optimal strategy for the collective. We are yet to find a Shapely Value that will induce everyone to think about climate change and global warming. Things remain as they are, and keep deteriorating, and Al Gore is proven right!. ■

http://dgit.in/dmystifyGT

Attribution: Poker Game, and oil painting by Cassius Marcellus Coolidge