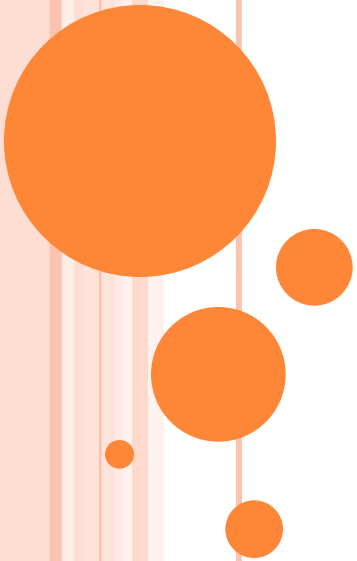
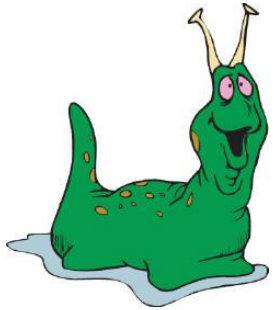


FUZZY LOGIC

WHAT IS FUZZY LOGIC?

- Definition of fuzzy
  - Fuzzy – “not clear, distinct, or precise; blurred”
- Definition of fuzzy logic
  - A form of knowledge representation suitable for notions that cannot be defined precisely, but which depend upon their contexts.

TRADITIONAL REPRESENTATION OF LOGIC



Slow

Fast

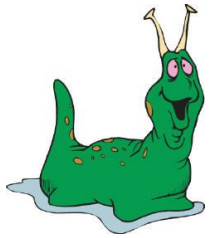
Speed = 0

Speed = 1

```
bool speed;
get the speed
if ( speed == 0) {
//  speed is slow
}
else {
//  speed is fast
}
```

FUZZY LOGIC REPRESENTATION

- For every problem must represent in terms of fuzzy sets.
- What are fuzzy sets?



Slowest

[0.0 – 0.25]

Slow

[0.25 – 0.50]

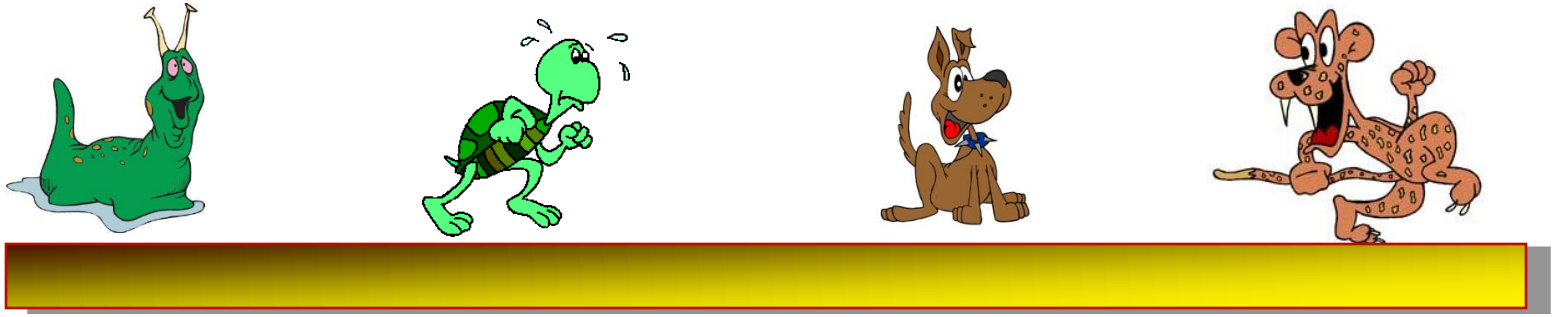
Fast

[0.50 – 0.75]

Fastest

[0.75 – 1.00]

FUZZY LOGIC REPRESENTATION CONT.



Slowest Slow Fast Fastest

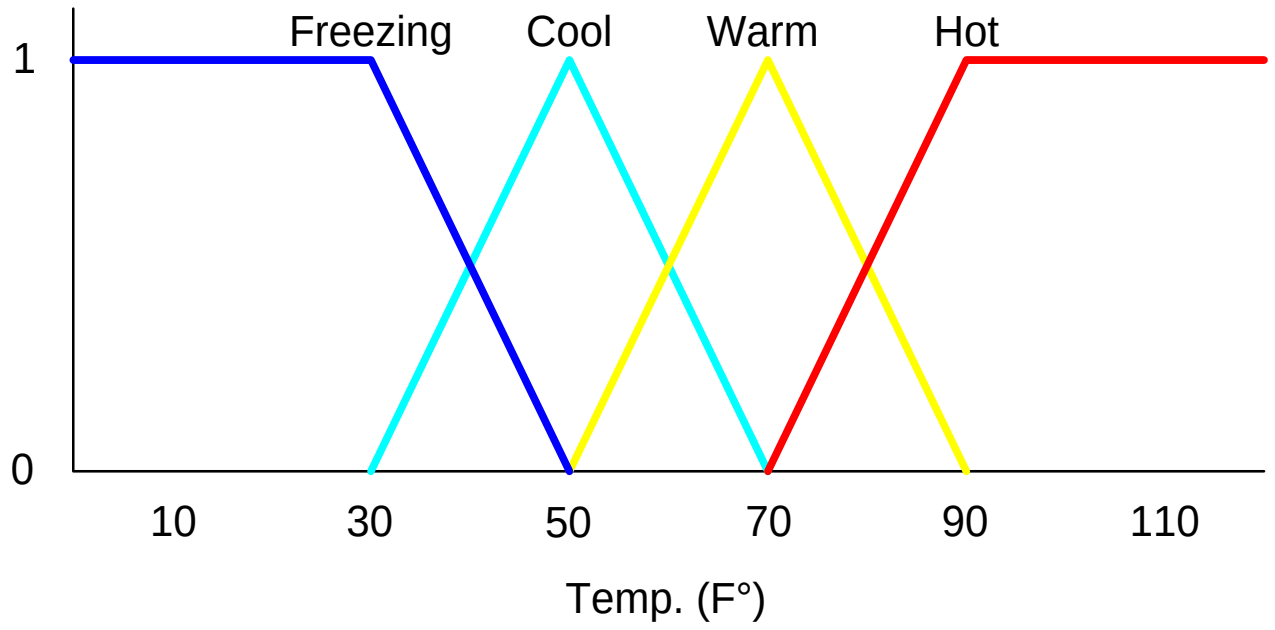
```
float speed;
get the speed
if ((speed >= 0.0)&&(speed < 0.25)) {
//  speed is slowest
}
else if ((speed >= 0.25)&&(speed < 0.5))
{
//  speed is slow
}
else if ((speed >= 0.5)&&(speed < 0.75))
{
//  speed is fast
}
else // speed >= 0.75 && speed < 1.0
{
//  speed is fastest
}
```

ORIGINS OF FUZZY LOGIC

- Traces back to Ancient Greece
- Lotfi Asker Zadeh (1965)
  - First to publish ideas of fuzzy logic.
- Professor Toshire Terano (1972)
  - Organized the world's first working group on fuzzy systems.
- F.L. Smidth & Co. (1980)
  - First to market fuzzy expert systems.

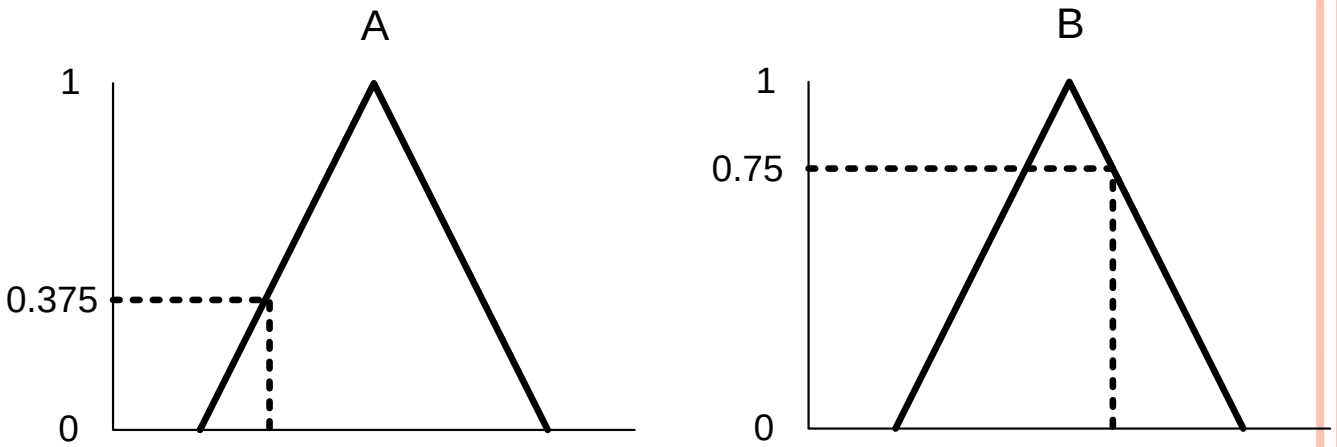
Membership Functions

- Temp: {Freezing, Cool, Warm, Hot}
- Degree of Truth or "Membership"



Fuzzy Disjunction

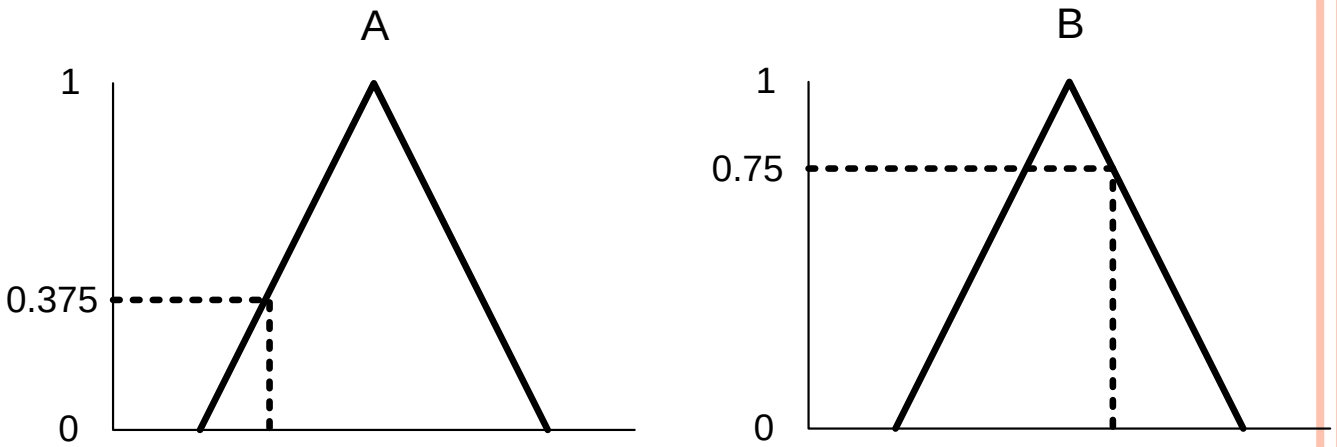
- $A \vee B \triangleq \max(A, B)$
- $A \vee B = C$ "Quality C is the disjunction of Quality A and B"



$$(A \vee B = C) \Rightarrow (C = 0.75)$$

Fuzzy Conjunction

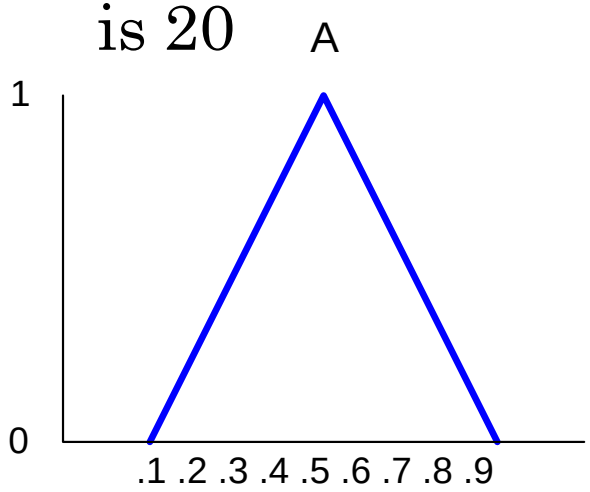
- $A \wedge B \triangleq \min(A, B)$
- $A \wedge B = C$ "Quality C is the conjunction of Quality A and B"

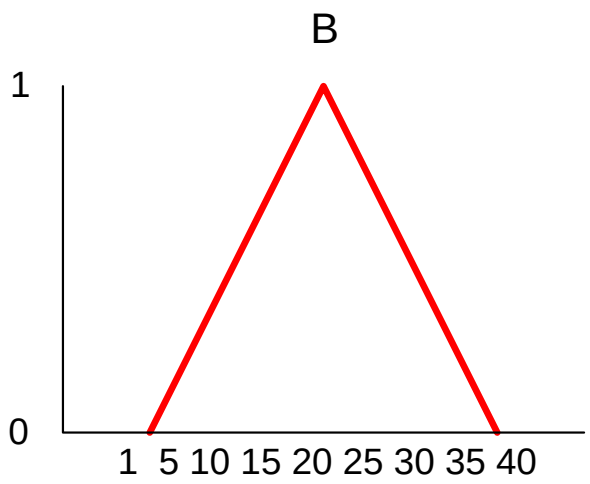


$$(A \wedge B = C) \Rightarrow (C = 0.375)$$

Example: Fuzzy Conjunction

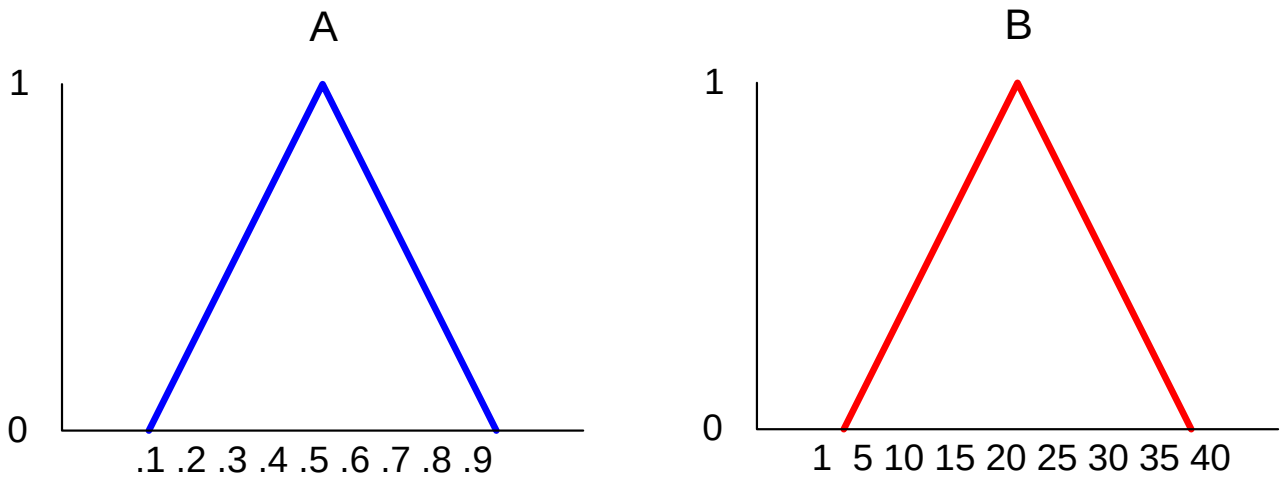
Calculate A∧B given that A is .4 and B is 20





Example: Fuzzy Conjunction

Calculate A∧B given that A is .4 and B is 20



Determine degrees of membership:

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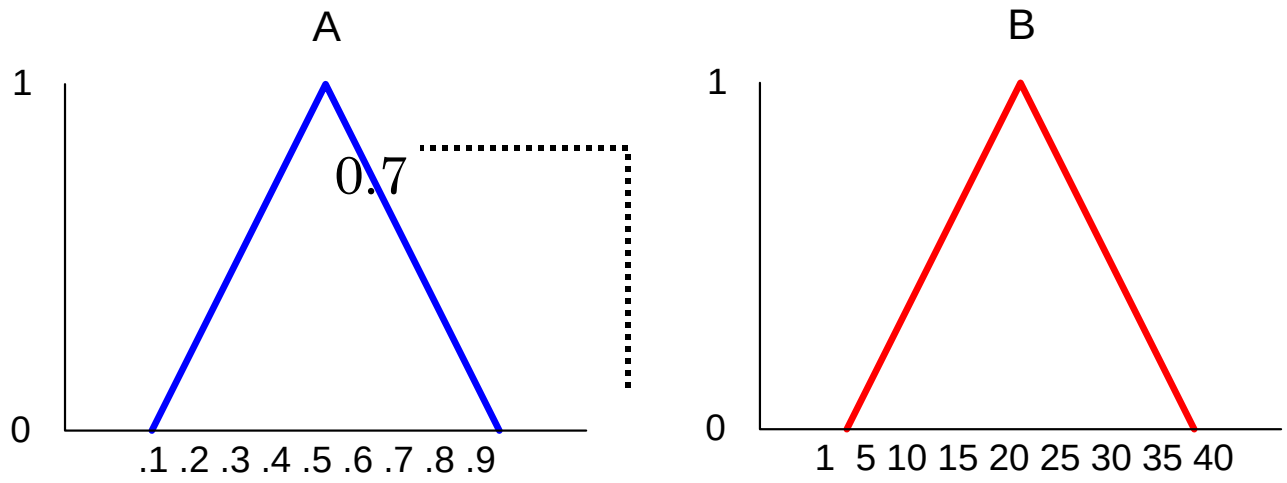
Example: Fuzzy Conjunction

Calculate A∧B given that A is .4 and B is 20

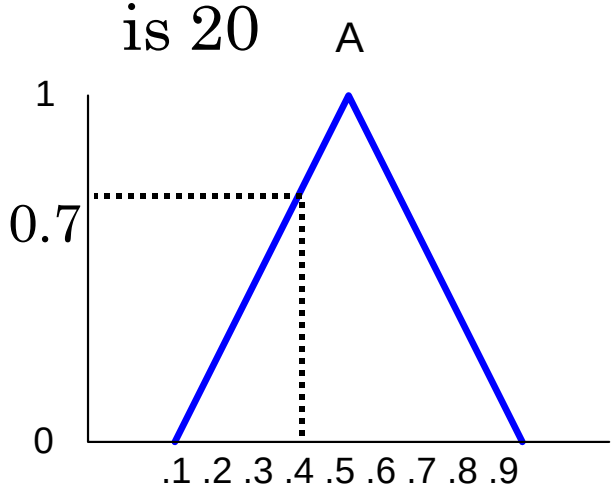


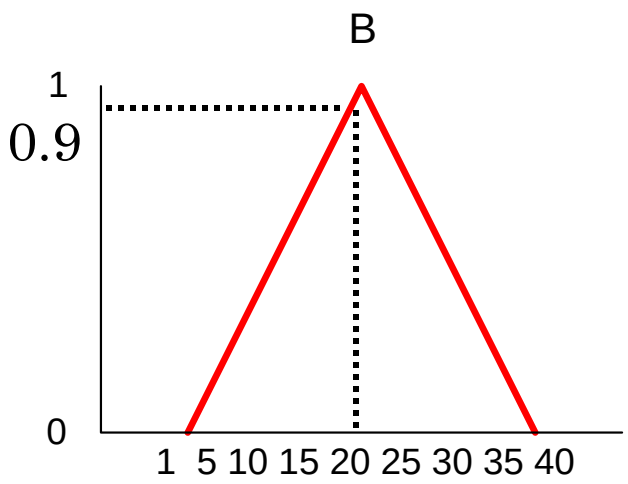
Determine degrees of membership:

A = 0.7

Example: Fuzzy Conjunction

Calculate A∧B given that A is .4 and B is 20





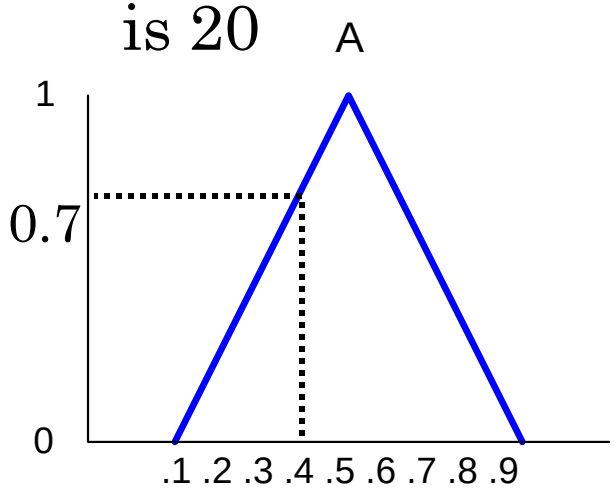
Determine degrees of membership:

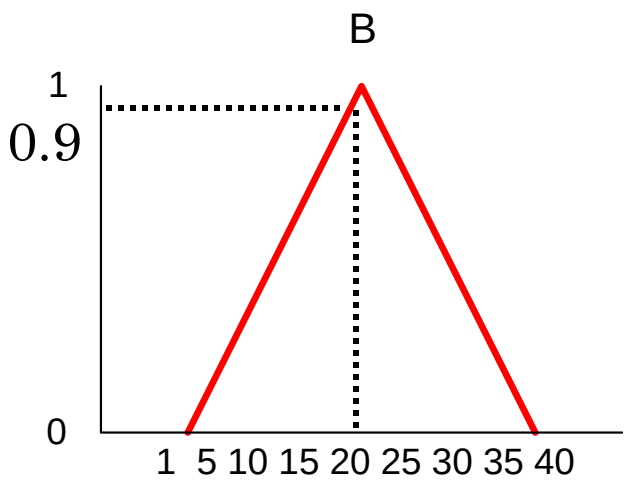
A = 0.7 B = 0.9

Fuzzy Logic

Example: Fuzzy Conjunction

Calculate A∧B given that A is .4 and B is 20





Determine degrees of membership:

A = 0.7 B = 0.9

Apply Fuzzy AND

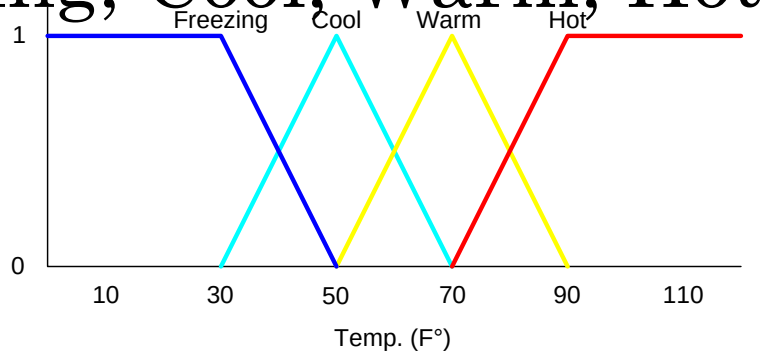
A∧B = min(A, B) = 0.7

Fuzzy Control

- Fuzzy Control combines the use of fuzzy linguistic variables with fuzzy logic
- Example: Speed Control
- How fast am I going to drive today?
- It depends on the weather.
- Disjunction of Conjunctions

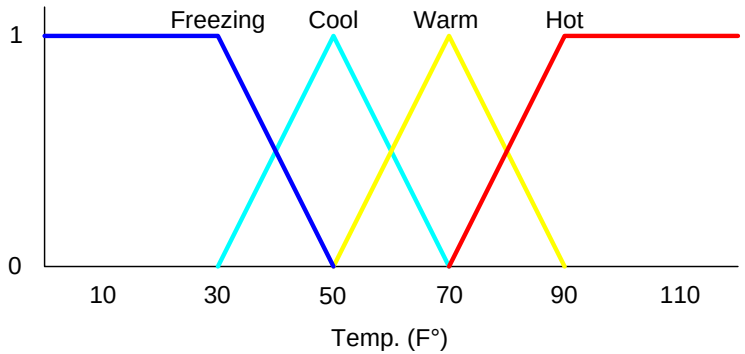
Inputs: Temperature

- Temp: {Freezing, Cool, Warm, Hot}

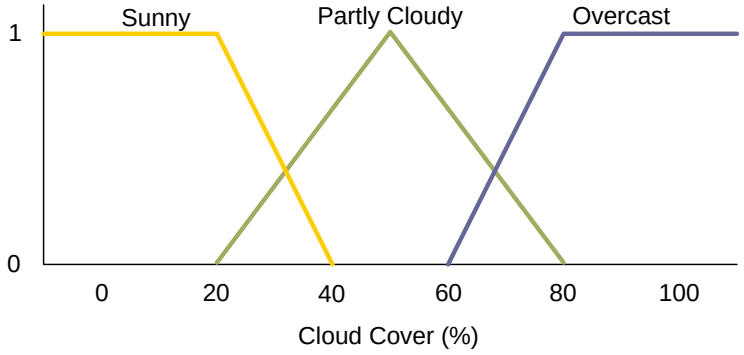


Inputs: Temperature, Cloud Cover

- Temp: {Freezing, Cool, Warm, Hot}



- Cover: {Sunny, Partly, Overcast}



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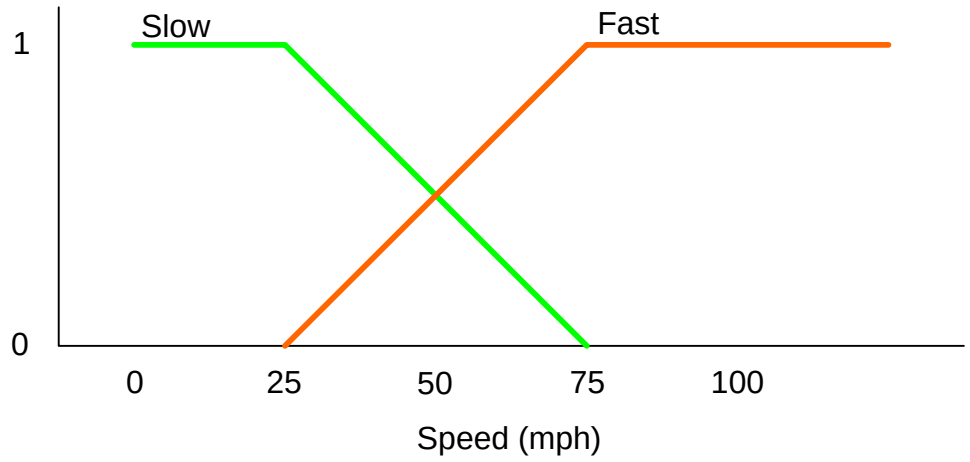
Output: Speed

- Speed: {Slow, Fast}



Rules

- If it's Sunny and Warm, drive Fast

  $\text{Sunny(Cover)} \wedge \text{Warm(Temp)} \Rightarrow \text{Fast(Speed)}$

- If it's Cloudy and Cool, drive Slow

  $\text{Cloudy(Cover)} \wedge \text{Cool(Temp)} \Rightarrow \text{Slow(Speed)}$

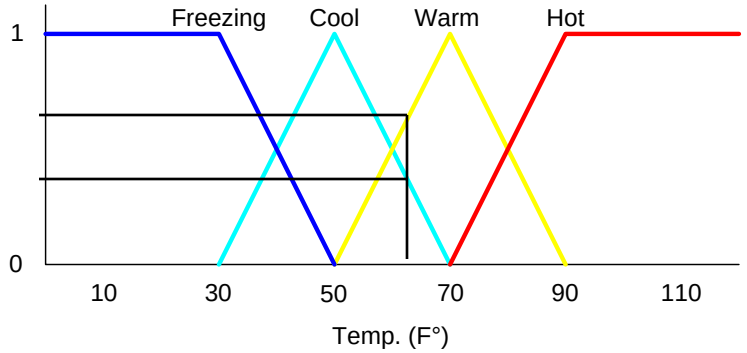
- Driving Speed is the combination of output of these rules…

Example Speed Calculation

- How fast will I go if it is
  - 65 F°
  - 25 % Cloud Cover ?

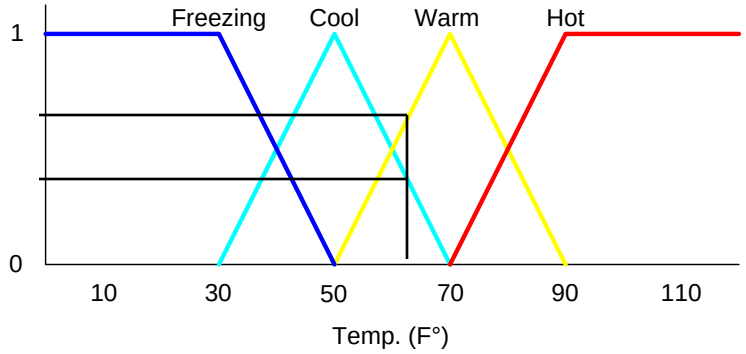
Calculate Input Membership Levels

- 65 F° ⇒ Cool = 0.4, Warm= 0.7

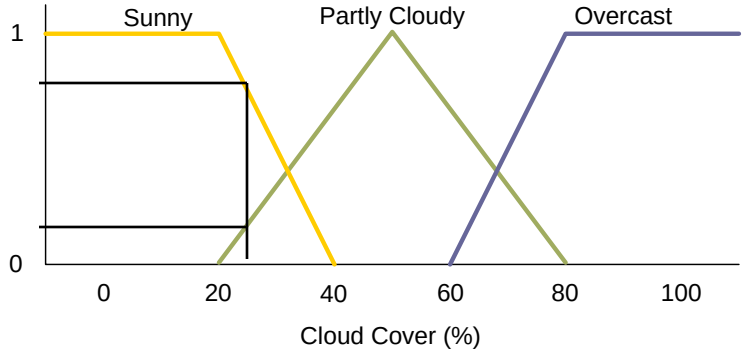


Calculate Input Membership Levels

- 65 F° ⇒ Cool = 0.4, Warm= 0.7



- 25% Cover ⇒Sunny = 0.8, Cloudy = 0.2



...Calculating...

- If it's Sunny and Warm, drive Fast

$$\text{Sunny(Cover)} \wedge \text{Warm(Temp)} \Rightarrow \text{Fast(Speed)}$$

$$0.8 \wedge 0.7 = 0.7$$

$$\Rightarrow \textbf{Fast = 0.7}$$

- If it's Cloudy and Cool, drive Slow

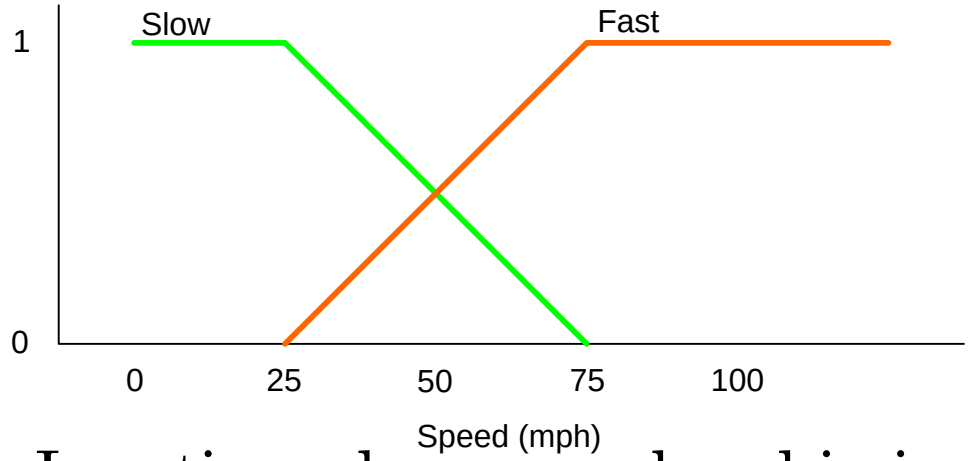
$$\text{Cloudy(Cover)} \wedge \text{Cool(Temp)} \Rightarrow \text{Slow(Speed)}$$

$$0.2 \wedge 0.4 = 0.2$$

$$\Rightarrow \textbf{Slow = 0.2}$$

Defuzzification: Constructing the Output

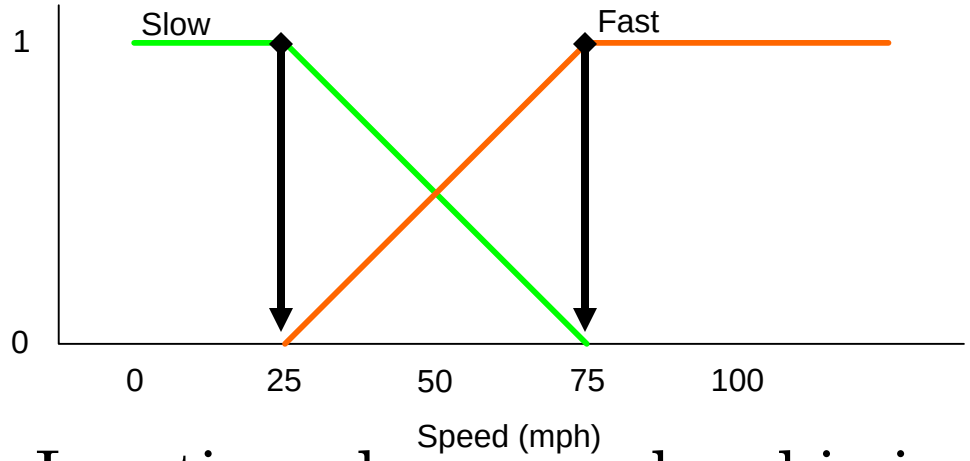
- Speed is 20% Slow and 70% Fast



- Find centroids: Location where membership is 100%

Defuzzification: Constructing the Output

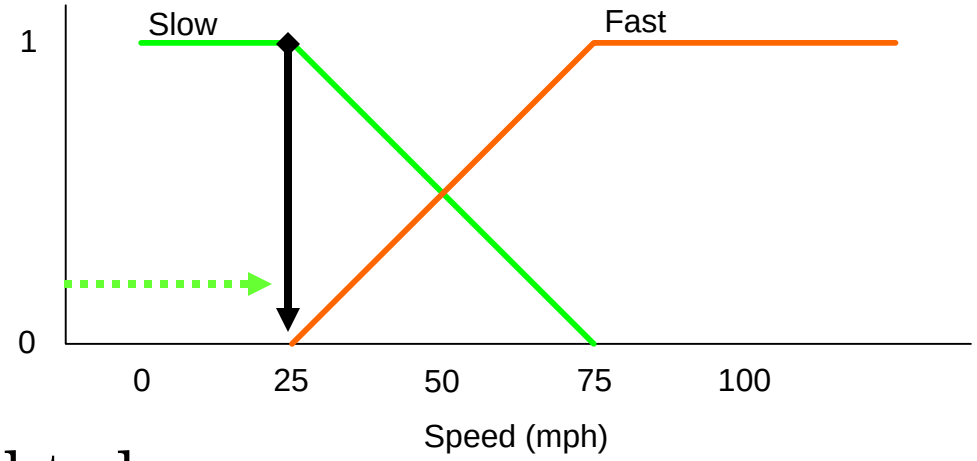
- Speed is 20% Slow and 70% Fast



- Find centroids: Location where membership is 100%

Defuzzification: Constructing the Output

- Speed is 20% Slow and 70% Fast

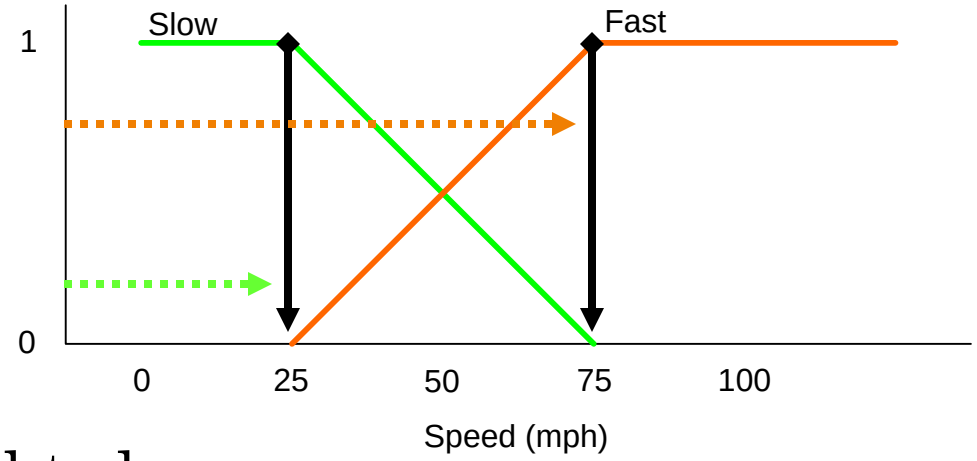


- Speed = weighted mean

  = (2*25+...

Defuzzification: Constructing the Output

- Speed is 20% Slow and 70% Fast



- Speed = weighted mean

  = (2*25+7*75)/(9)

  = 63.8 mph