The **Cahn–Hilliard equation** is an equation of mathematical physics which describes the process of phase separation, by which the two components of a binary fluid spontaneously separate and form domains pure in each component. If is the concentration of the fluid, with indicating domains, then the equation is written as:

$$\frac{\delta c}{\delta t} = -\nabla.J$$

Where,

c=concentration

$$J = -c(1-c)\nabla\mu$$

$$\mu = \chi(1-2c) + \ln(\frac{c}{1-c}) - \kappa\frac{\partial^2 c}{\partial r^2}$$

This is chemical potential.

$$\nabla. = (\frac{\delta}{\delta r} + \frac{A}{r})$$

This is divergence operator with value of A =1 for cylindrical and 2 for spherical coordinate systems.

Boundary conditions(flux)

J at x=0 =0;

J at x= l = -0.6168

Method 1: (Separate variables)

Here the equation is solved by considering μ as separate variable. This converts the problem into 2 separate 2 nd order differential equations. One for c and other for μ .

The equation for c is second order in r and first order in t.to solve it, IMEX time stepping is used. Steps are:

1) Initialize cn values at nodes.
2) Find equation for residue.
3) Convert it to linear equation by finite difference in IMEX with central difference scheme for r.
4) Find values of fictitious nodes initial values using boundary conditions at two ends.
5) Find residue value at each node for next time step.
6) Find Jacobean (K) matrix for residue with corresponding nodes.
7) Find ck values with K and cn. (NR method)
8) Find error value for ck. when error < tolerance, cn=ck
9) Continue the above procedure for given number of time steps.

During above procedure the value of μ is calculated continuously using a function of c.

Due to use of μ as separate function of c, the equation for time steps remain second order and hence **two boundary conditions are sufficient.**

But because the K matrix becoming singular for 0 initial conditions, the validation of solution with exact solution is not possible. Also, as the dependency of μ on c is considered linear, the accuracy of solution is less. But this can provide solution for **non-zero** κ because of consideration in equation of μ.

Method 2: Combined variable

Here the μ is substituted for its function of c so that the residue equation is forth order equation of c and r. This forth order equation converted into linear equation with finite difference with IMEX and central difference requires 4 boundary conditions including 3 rd order variation of c at both ends. Because of **unavailability of these boundary conditions**, we consider only second order terms and hence **neglecting** the effect of κ.

Because of single second order differential equation, the initial conditions of zero concentration can be used and validated with exact solution.

The procedure used is similar as above including residue with IMEX and reducing it with NR method to finally arrive at concentration values for given time step.

The final result comparing exact solution and approximated solution as follows: