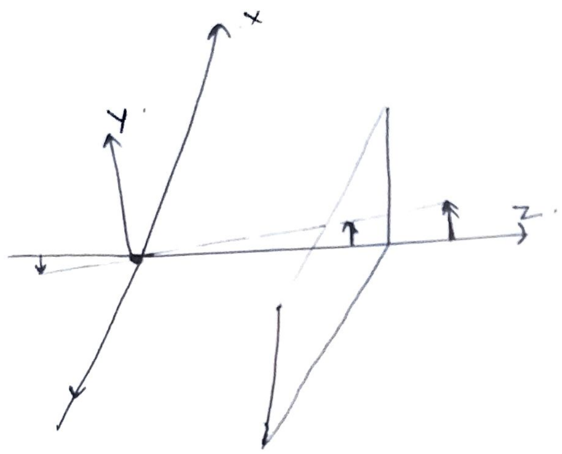
Non Homogenous:

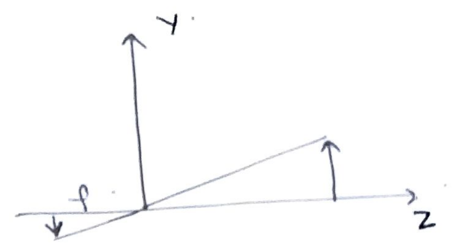
$AX = b \qquad A^T A X = A^T b \qquad X = (A^T A)^{-1} A^T b$

Homogenous

$AX = 0 \Rightarrow SVD(A)$

$U \Sigma V^T \Rightarrow$ Answer is the last column of V^T





$\frac{y}{f} = \frac{Y}{Z} \qquad y = f\frac{Y}{Z}$

$\frac{x}{f} = \frac{X}{Z} \qquad x = f\frac{X}{Z}$

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} f & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

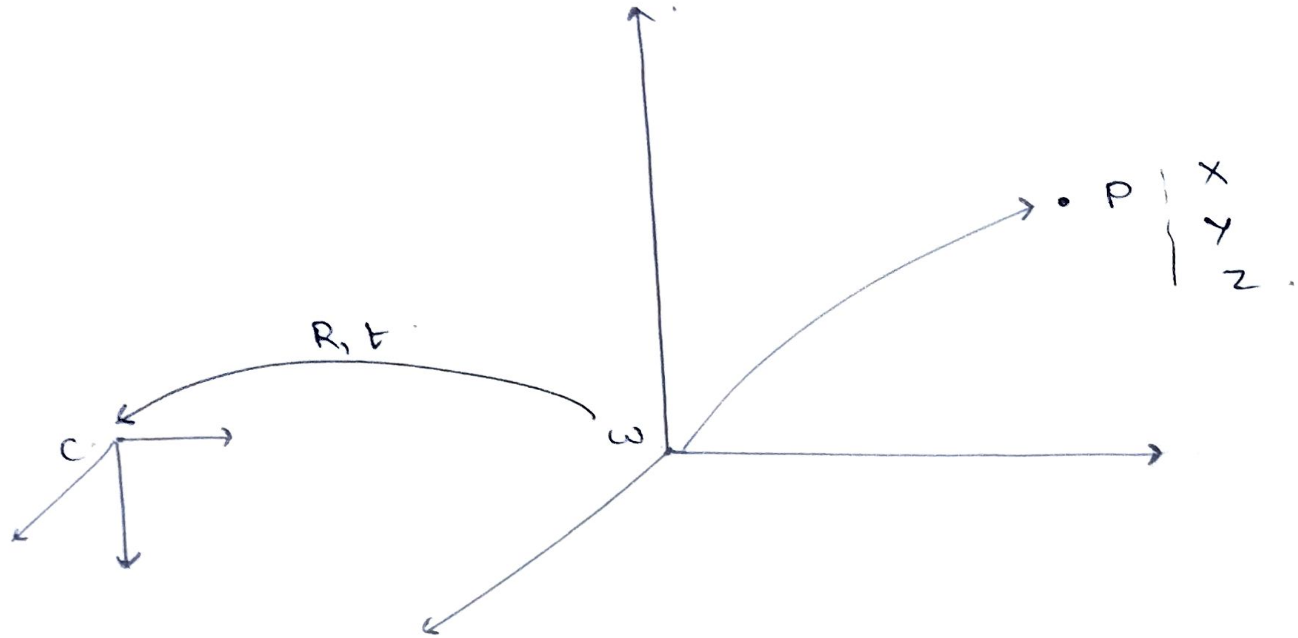
$x = x'/w' \qquad y = y'/w'$

Homogeneous

$$\begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

$\rightarrow X_{cam}$ = position of point in camera frame.

Projection Matrix

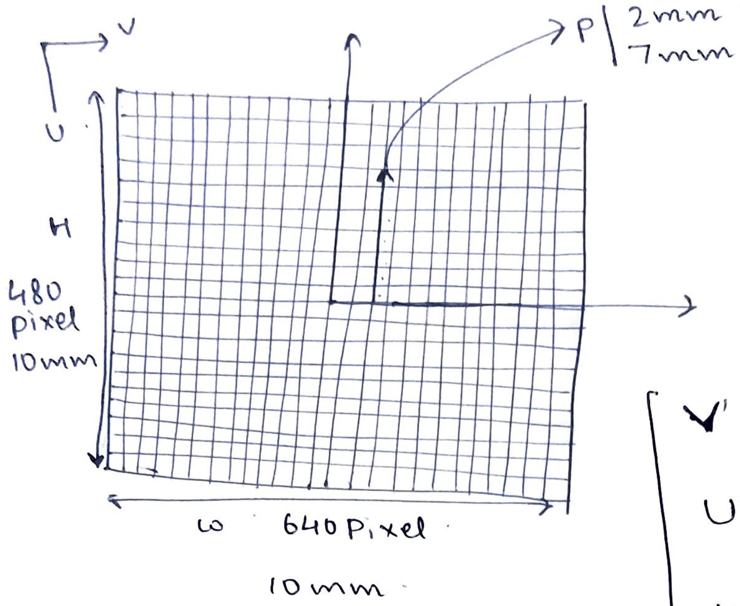


$$X_{cam} = R * X + t \xRightarrow{\text{Homogeneous}} X_{cam} = \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix}_{4\times 4} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}$$

image plane

$$x = \underbrace{K\,[I \mid 0]_{3\times 4} \begin{bmatrix} R & t \\ 0 & 1 \end{bmatrix}_{4\times 4}} \begin{bmatrix} X \\ Y \\ Z \\ 1 \end{bmatrix}_{\omega}$$

$$P = K\,[R \mid t] \qquad \text{——— (1)}$$



$$V = \frac{2 \times 640}{10} + \underbrace{\frac{640}{2}}_{V_0}$$

$$U = \frac{7 \times 480}{2} + \underbrace{\frac{480}{2}}_{V_0}$$

$$\begin{bmatrix} V' \\ U' \\ W' \end{bmatrix} = \underbrace{\begin{bmatrix} f * m_x & 0 & V_0 & 0 \\ 0 & f * m_y & V_0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{K} = K[I | 0]$$

$$V = \frac{V'}{W}$$

$$U = \frac{U'}{W}$$

Projection Matrix:

$X_{cam} = R(X_w - C)$ → coordinate of camera

Homogeneous Form.

$$\begin{bmatrix} R & -RC \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$x = K[I | 0]_{3 \times 4} \begin{bmatrix} R & -RC \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

image plane

$$P = K[R | -RC]$$

$$P = KR[I | -C] \quad \text{---- (2)}$$

Combining (1) and (2) ----

$$\boxed{t = -RC}$$

Camera Calibration DLT:

$$X = P X_{4\times1} \qquad \begin{bmatrix} u' \\ v' \\ w' \end{bmatrix} = P_{3\times4} \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \qquad P = \begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

$$P_1 = [P_{11} \; P_{12} \; P_{13} \; P_{14}]$$

$$u' = P_1 X \qquad u = \frac{u'}{w'} \Rightarrow u w' - u' = 0$$

$$v' = P_2 X$$

$$w' = P_3 X \qquad v = \frac{v'}{w'} \Rightarrow v w' - v' = 0$$

$$\begin{cases} u P_3 X - P_1 X = 0 \\ v P_3 X - P_2 X = 0 \end{cases} \Rightarrow \begin{bmatrix} -X & 0 & uX \\ 0 & -X & vX \end{bmatrix} \begin{bmatrix} P_1^T \\ P_2^T \\ P_3^T \end{bmatrix}_{12\times1} = 0$$

we need at least 6points to solve this.

$$\begin{bmatrix} -x & -y & -z & -1 & 0 & 0 & 0 & 0 & ux & uy & uz & u \\ 0 & 0 & 0 & 0 & -x & -y & -z & -1 & vx & vy & vz & v \end{bmatrix} \begin{bmatrix} P_1^T \\ P_2^T \\ P_3^T \end{bmatrix} 0$$

$$A_{12\times12} \; P_{12\times1} = 0$$

SVD (A)

$U \; \Sigma \; V^T \rightarrow$ Last column is the answer to our equation

$$H = V^T(\; , 12)$$

Decomposing Projection Matrix

$$P = KR[I \mid -C] = H = [\underset{3\times3}{H_1} \mid \underset{3\times1}{H_2}]$$

$$KR = H_1 \Rightarrow \left| \begin{array}{l} H_1^{-1} = R^{-1}K^{-1} \\ H_1^{-1} = QR \end{array} \right. \Rightarrow \boxed{\begin{array}{l} R^{-1} = Q \\ K^{-1} = R \end{array}}$$

$$-KRC = H_2 \Rightarrow C = (-KR)^{-1} H_2$$

$$C = -H_1^{-1} H_2$$