HOME WORK - 4

(10.31) Given.

Gamma (5, 1)

$\alpha = 5, \lambda = 1$.

&

$X = 4$ accidents in a month

$$\alpha(x) = \int_0^\infty t^{x-1} e^{-t} \, dt$$

$$\alpha = \sum_{i=1} x_i$$

$$\alpha_x = 5 + 4 = 9.$$

$$\lambda_x = 2.$$

Estimator :-

$$\hat{\theta} = E\{\theta/x\}$$

$$= \frac{\alpha_x}{\lambda_x} = \frac{9}{2} = 4.5.$$

Posterior risk :-

$$E(\hat{\theta}) \ \text{var}\{\theta/x\}$$

$$= \frac{\alpha_x}{\lambda_x^2} = \frac{9}{4} = 2.25.$$

10.37 Given

10 heads straight

$P_1 = 0.99$

$P_2 = 0.01$

P is uniform distribution between $(0, 1)$

$F_H(P) = 1 \quad (0 < P < 1)$

$P(X = 10/P) = P^{10}$

$F_H(P/X = 10) \propto P(X = 10/P)$

$F_H(P)$ for $0 < P < 1$

Now

$$\int_0^1 P\{X = 10/P\} F_H(P)\,dp$$

$$\int_0^1 (P)^{10}\,dp = 1/11$$

PDF of P of heads

$$F_H(P/X = 10) = \frac{P(X = 10/P)\,F_H(P)}{\int_0^1 P(X = 10/P)\,F_H(P)\,dp}$$

$$F_H(P/X = 10) = 11\,P^{10}$$

It is not likely that the coin is fair.

5.1 Given.

$f(x) = 1.5\sqrt{x} \quad 0 < x < 1.$

~~pepos~~ Pdf

$f(x) = P(X < x)$

$= 1.5 \int_0^x \sqrt{x}$

$= 1.5 \left[\frac{x^{3/2}}{3/2} \right]_0^x$

$f(x) = x^{3/2}$

· Then we know

$f(x) = U$

$x = (U)^{2/3}$

cdf.

$F_U(u) = P(U < u)$

$= P(X < F^{-1}(u))$

$= F(F^{-1}(u))$

$F_u(u) = u$

~~Density fo fu~~

$\therefore \quad F_u(u) = F_u^{-1}(u) = 1$

Then:

$U = 0.001$ g random variable x I 0.01

5.6 Given

First mechanic $\lambda = \frac{1}{5}$.

Second mechanic $\lambda = \frac{1}{20}$ is 4 Times faster

~~Pdf~~ $f(x) = \begin{cases} \lambda e^{-\lambda x} & x > 0 \\ 0 & x < 0 \end{cases}$

&

$F(x) = \int f(x)\,dx.$

$= \begin{cases} 1 - e^{-\lambda x} & x > 0 \\ 0 & x < 0 \end{cases}$

Mean $E(X) = \frac{1}{\lambda}$

moment of generation

$\phi(t) = E(e^{tX}) = \frac{\lambda}{\lambda - t}$

$\rightarrow E(x^2) = \frac{2}{\lambda^2}$ & $var(x) = \frac{1}{\lambda^2}$

M1 :-

$f(x) = \begin{cases} 5e^{-5x} \\ 0 \end{cases}$

&

$F(x) = \begin{cases} 1 - e^{-5x} \\ 0 \end{cases}$

$E(x) = \frac{2}{5^2} = 0.08$ $var(x) = \frac{1}{25} = 0.04.$

$\phi(t) = \frac{5}{5-t}$

M2 :-

$$f(x) = \begin{cases} 20e^{-20x} \\ 0 \end{cases}$$

&

$$f(x) = \begin{cases} 1 - e^{-20x} \\ 0 \end{cases}$$

$$E(x^2) = \frac{2}{(20)^2} = 0.005 \cdot \quad var(x) = \frac{1}{(20)^2} = 0.0025$$

$$\phi(t) = \frac{20}{20 - t_2}$$

Now $t_2 = 5t_1$

we get.

$$\phi(t) = \frac{20}{20 - t} = \frac{5}{5 - 5t_1}$$