Arrays

Instructor: **Dr. Tarunpreet Bhatia**

Assistant Professor, CSED

Thapar Institute of Engineering and Technology

Array ADT

```
float marks[10];
```

- The simplest but useful data structure.
- Assign single name to a homogeneous collection of instances of one abstract data type.
  - All array elements are of same type, so that a pre−defined equal amount of memory is allocated to each one of them.
- Individual elements in the collection have an associated index value that depends on array dimension.

Contd…

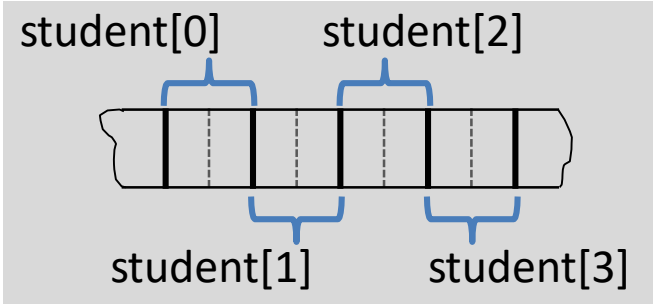
- One-dimensional and two-dimensional arrays are commonly used.
- Multi−dimensional arrays can also be defined.

- Usage:
  - Used frequently to store relatively permanent collections of data.
  - Not suitable if the size of the structure or the data in the structure are constantly changing.

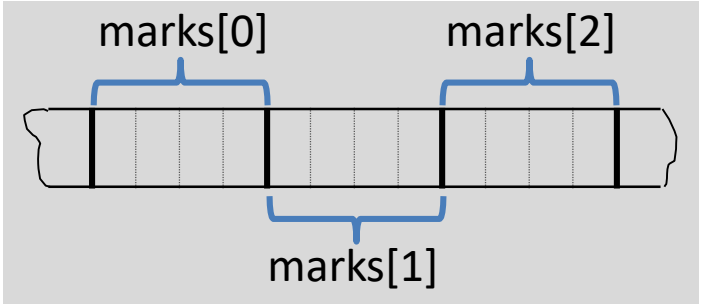
Memory Storage

Memory Storage – One Dimensional Array

```
int student[4];
```




```
float marks[3];
```



Memory Storage – Two Dimensional Array

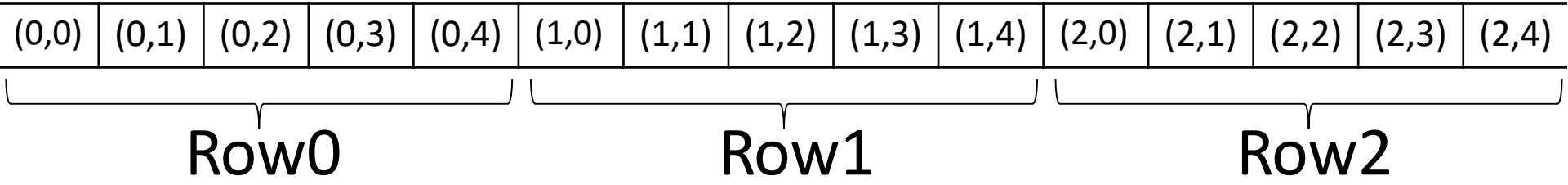
```
int marks[3][5];
```

- Can be visualized in the form of a matrix as

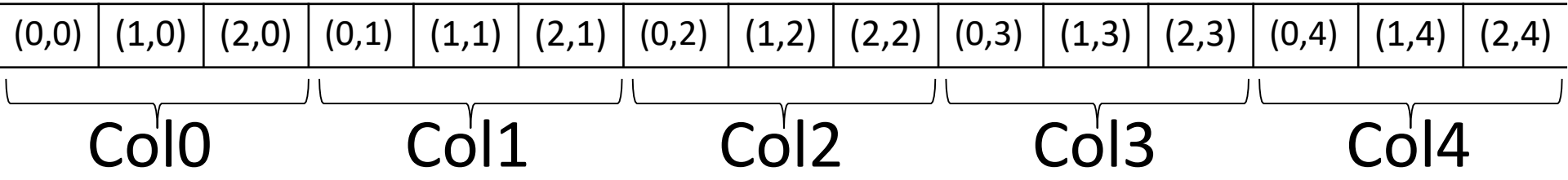
	Col 0	Col 1	Col 2	Col 3	Col 4
Row 0	marks[0][0]	marks[0][1]	marks[0][2]	marks[0][3]	marks[0][4]
Row 1	marks[1][0]	marks[1][1]	marks[1][2]	marks[1][3]	marks[1][4]
Row 2	marks[2][0]	marks[2][1]	marks[2][2]	marks[2][3]	marks[2][4]

Contd…

- Row-major order



- Column-major order



Array Address Computation

1D array – address calculation

- Let A be a one dimensional array.
- Formula to compute the address of the I^{th} element of an array (A[I]) is:

Address of A[I] = B + W * (I – LB)

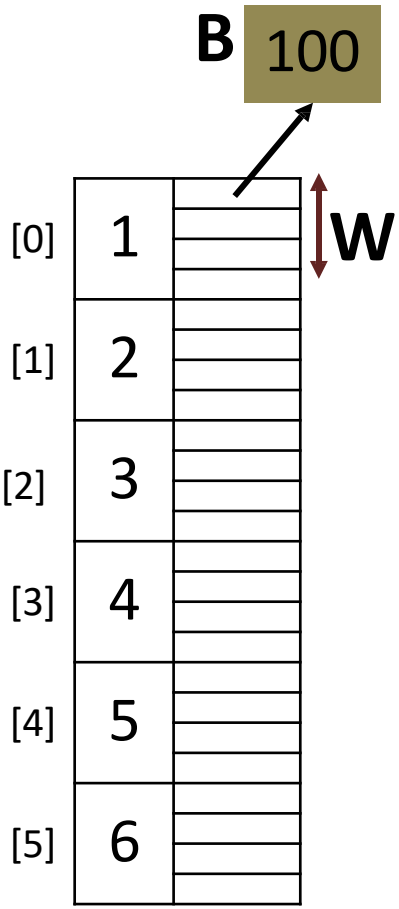
where,

B = Base address/address of first element, i.e. A[LB].

W = Number of bytes used to store a single array element.

I = Subscript of element whose address is to be found.

LB = Lower limit / Lower Bound of subscript, if not specified assume 0 (zero).



1D array – address calculation

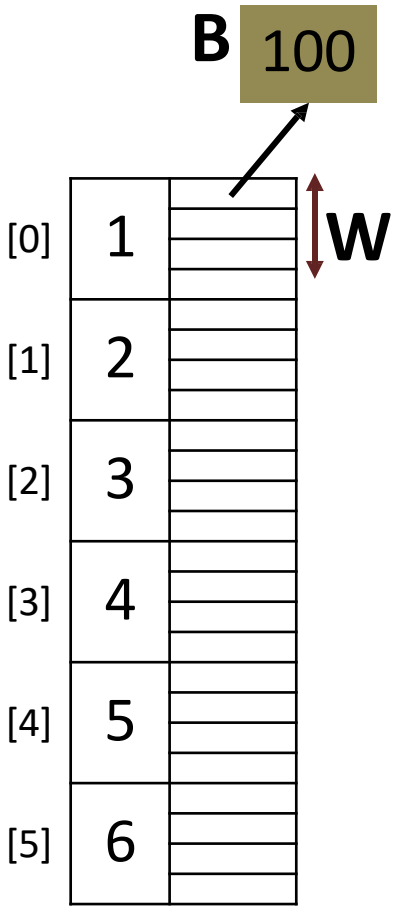
- Let A be a one dimensional array.
- Formula to compute the address of the I^{th} element of an array (A[I]) is:

Address of A[I] = B + W * (I – LB)

Given:

B = 100, W = 4, and LB = 0

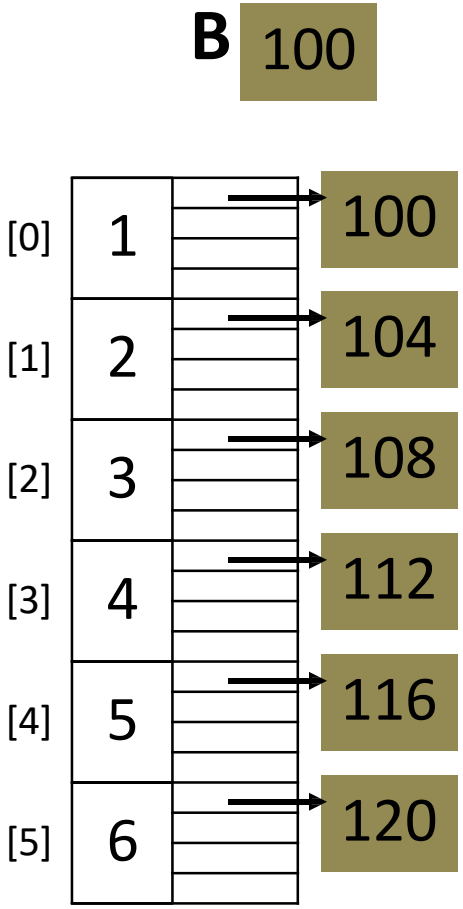
A[0] = 100 + 4 * (0 – 0) = 100



1D array – address calculation

- Let A be a one dimensional array.
- Formula to compute the address of the I^{th} element of an array (A[I]) is:

Address of A[I] = B + W * (I – LB)



Given:

B = 100, W = 4, and LB = 0

A[1] = 100 + 4 * (1 – 0) = 104

A[2] = 100 + 4 * (2 – 0) = 108

A[3] = 100 + 4 * (3 – 0) = 112

A[4] = 100 + 4 * (4 – 0) = 116

A[5] = 100 + 4 * (5 – 0) = 120

Example – 1

- Similarly, for a character array where a single character uses 1 byte of storage.
- If the base address is 1200 then,

Address of A[I] = B + W * (I – LB)

Address of A[0] = 1200 + 1 * (0 – 0) = 1200

Address of A[1] = 1200 + 1 * (1 – 0) = 1201

…

Address of A[10] = 1200 + 1 * (10 – 0) = 1210

Example – 2

- If **LB** = 5, **Loc(A[LB])** = 1200, and **W** = 4.
- Find **Loc(A[8])**.

Address of A[I] = B + W * (I – LB)

$$\begin{aligned} Loc(A[8]) &= Loc(A[5]) + 4 * (8 - 5) \\ &= 1200 + 4 * 3 \\ &= 1200 + 12 \\ &= 1212 \end{aligned}$$

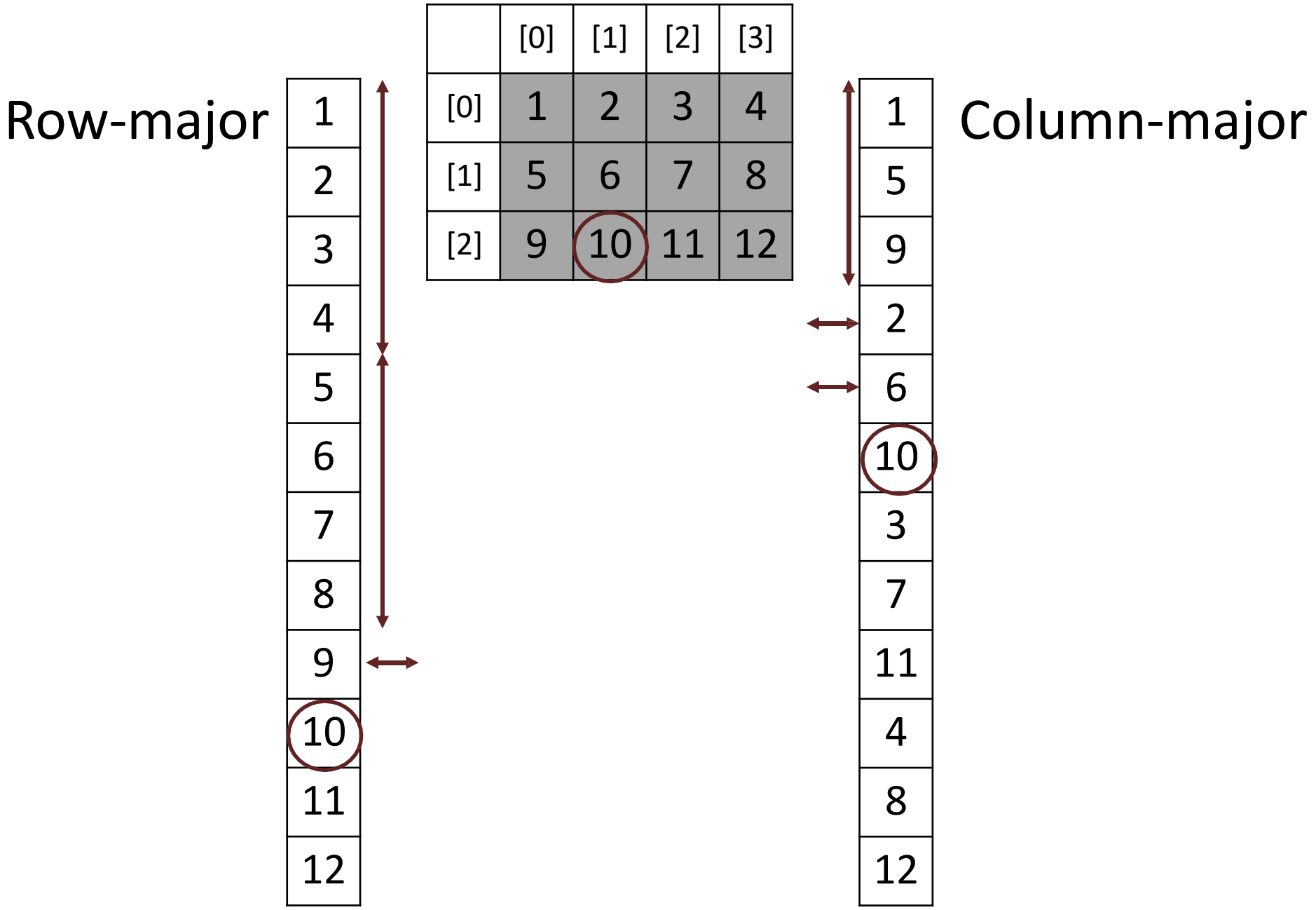
Example – 3

- Base address of an array **B[1300…..1900]** is **1020** and size of each element is 2 bytes in the memory. Find the address of **B[1700]**.

Address of A[I] = B + W * (I – LB)

- Given: **B** = 1020, **W** = 2, **I** = 1700, **LB** = 1300

Address of B[1700] = 1020 + 2 * (1700 – 1300)
= 1020 + 2 * 400
= 1020 + 800
= 1820

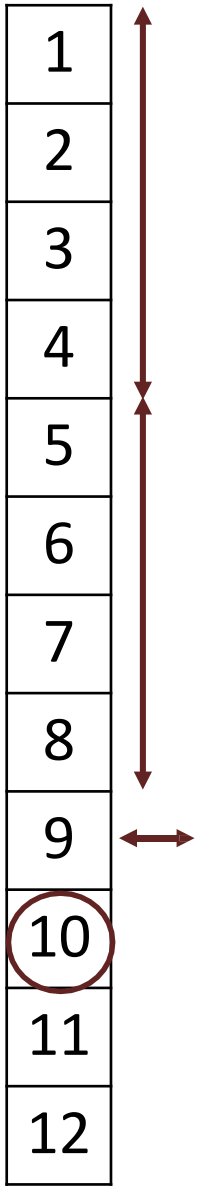

Row-major
Column-major
[0]
[1]
[2]
[3]
1
2
3
4
5
6
7
8
9
10
11
12

2D Array – Address Calculation

- If **A** be a two dimensional array with **M** rows and **N** columns. We can compute the address of an element at **Ith** row and **Jth** column of an array (**A[I][J]**).

  B = Base address/address of first element, i.e. A[LBR][LBC]
  I = Row subscript of element whose address is to be found
  J = Column subscript of element whose address is to be found
  W = Number of bytes used to store a single array element
  LBR = Lower limit of row/start row index of matrix, if not given assume 0
  LBC = Lower limit of column/start column index of matrix, if not given assume 0
  N = Number of column of the given matrix
  M = Number of row of the given matrix

Row-major



	[0]	[1]	[2]	[3]
[0]	1	2	3	4
[1]	5	6	7	8
[2]	9	10	11	12

M = 3

N = 4

Address of A[2][1] =

B + W * (4 * (2 – 0) + (1 – 0))

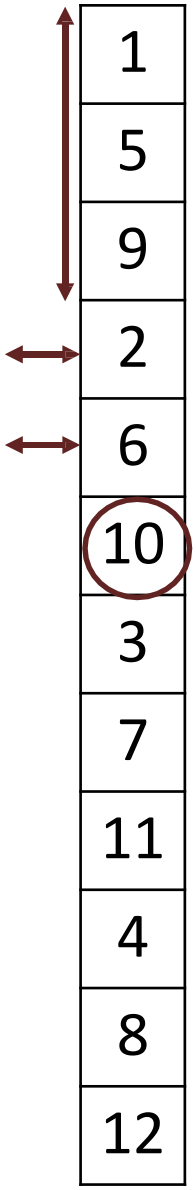
Address of A[I][J] =

B + W * (N * (I – LBR) + (J – LBC))

M = 3

N = 4

	[0]	[1]	[2]	[3]
[0]	1	2	3	4
[1]	5	6	7	8
[2]	9	10	11	12



Column-major

Address of A [2][1] =

B + W * ((2 – 0) + 3 * (1 – 0))

Address of A [I][J] =

B + W * ((I – LBR) + M * (J – LBC))

Contd…

- Row Major

Address of A[I][J] = B + W * (N * (I – LBR) + (J – LBC))

- Column Major

Address of A [I][J] = B + W * ((I – LBR) + M * (J – LBC))

- Note: **A[LBR…UBR, LBC…UBC]**

M = (UBR – LBR) + 1

N = (UBC – LBC) + 1

Example – 4

- Suppose elements of array **A[5][5]** occupies **4** bytes, and the address of the first element is **49**. Find the address of the element **A[4][3]** when the storage is row major.

Address of A[I][J] = B + W * (N * (I – LBR) + (J – LBC))

- Given: **B** = 49, **W** = 4, **M** = 5, **N** = 5, **I** = 4, **J** = 3, **LBR** = 0, **LBC** = 0.

Address of A[4][3] = 49 + 4 * (5 * (4 – 0) + (3 - 0))

= 49 + 4 * (23)

= 49 + 92

= 141

Example – 5

- An array **X [-15…10, 15…40]** requires **one** byte of storage. If beginning location is **1500** determine the location of **X [0][20]** in column major.

  Address of A[I][J] = B + W * [(I – LBR) + M * (J – LBC)]

- Number or rows (**M**) = **(UBR – LBR) + 1** = [10 – (- 15)] +1 = 26
- Given: **B** = 1500, **W** = 1, **I** = 0, **J** = 20, **LBR** = -15, **LBC** = 15, **M** = 26

Address of X[0][20] = 1500 + 1 * [(0 – (-15)) + 26 * (20 – 15)]

= 1500 + 1 * [15 + 26 * 5]

= 1500 + 1 * [145]

= 1645

Example – 6

- A two-dimensional array defined as **A [-4 … 6] [-2 … 12]** requires **2 bytes** of storage for each element. If the array is stored in row major order form with the address **A[4][8]** as **4142**. Compute the address of **A[0][0].**

  Address of A[I][J] = B + W (N (I – LBR) + (J – LBC))

- **Given:**

  W = 2, LBR = –4, LBC = –2

  #rows = M = 6 + 4 + 1 = 11 #columns = N = 12 + 2 + 1 = 15

  Address of A[4][8] = 4142

- **Address of A[4][8] = B + 2 (15 (4 – (–4)) + (8 – (–2)))**

  4142 = B + 2 (15 (4 + 4) + (8 + 2)) = B + 2 (15 (8) + 10) = B + 2 (120 + 10)

  4142 = B + 260

  Thus, B = 4142 – 260 = 3882

- **Now, Address of A[0][0] = 3882 + 2 (15 (0 – (–4)) + (0 – (–2)))**

  = 3882 + 2 (15(4) + 2) = 3882 + 2 (62)

  = 3882 + 124

  = 4006

Array Basic Operations

Operations on Linear Data Structures

- Traversal
- Insertion
- Deletion
- Search – Linear and Binary.
- Sorting – Different algorithms are there.
- Merging – During the discussion of Merge Sort.

TRAVERSAL

Processing each element in the array.

Example – Print all the array elements.

Algorithm arrayTraverse(A,n)

Input: An array **A** containing **n** integers.

Output: All the elements in **A** get printed.

1. for i = 0 to n-1 do
2. Print A[i]

```
1.int arrayTraverse(int arr[], int n)
2.  {
3.     for (int i = 0; i < n; i++)
4.          cout << "\n" << arr[i];
5.  }
```

Example – Find minimum element in the array.

Algorithm arrayMinElement(A,n)
Input: An array **A** containing **n** integers.
Output: The minimum element in **A**.

1. min = 0
2. for i = 1 to n-1 do
3. if A[min] > A[i]
4. min = i
5. return A[min]

```
1. int arrayMinElement(int arr[], int n)
2. {  int min = 0;
3.    for (int i = 1; i < n; i++)
4.    {   if (arr[i] < arr[min])
5.min = i; 6.     }
7.    return arr[min];
8. }
```

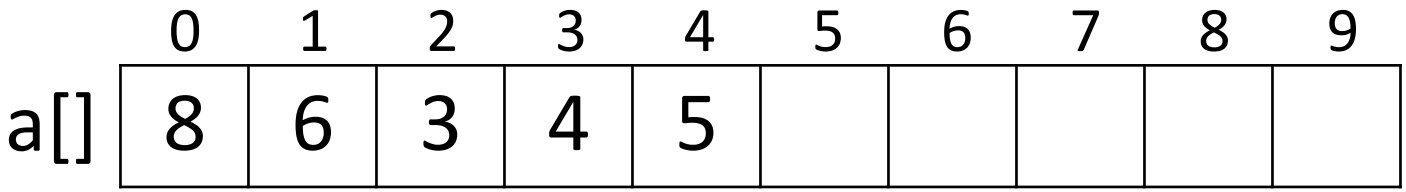
Insertion

Insert an element in the array

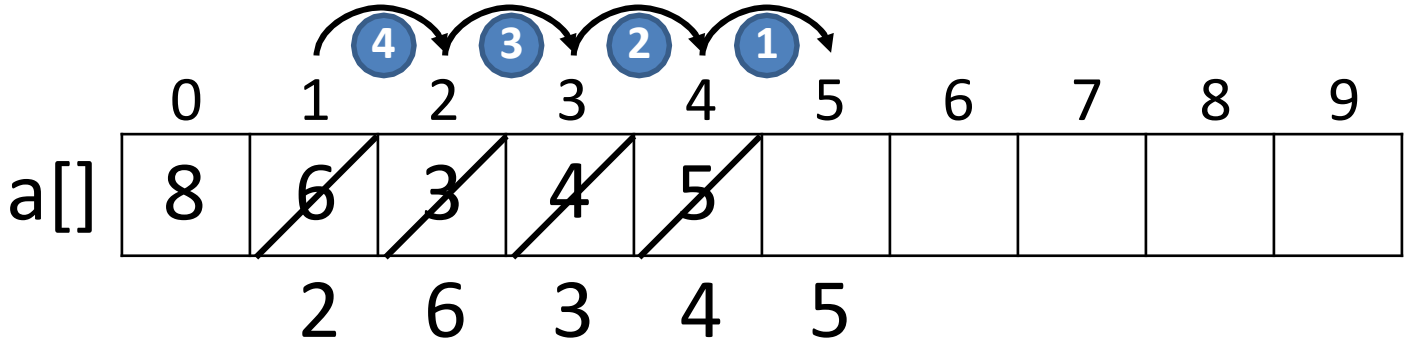
Deletion

Delete an element from the array

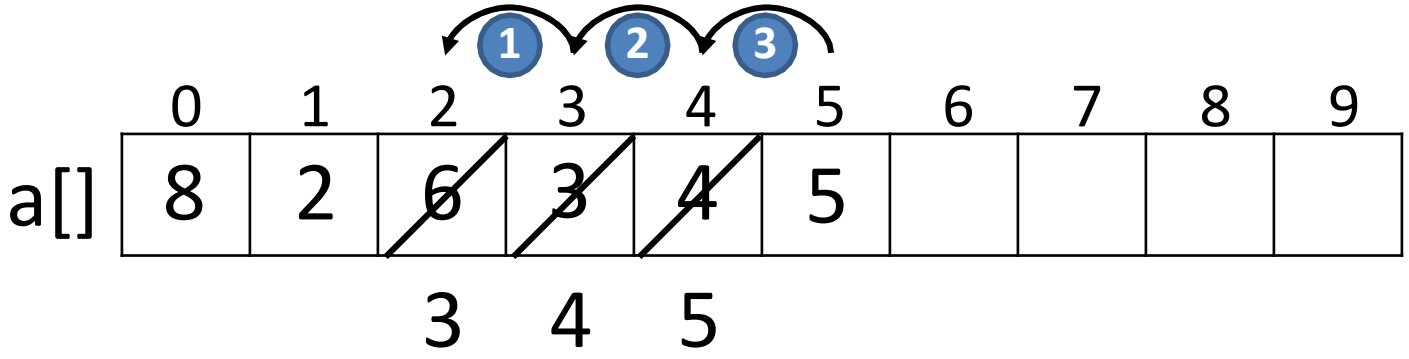
Insertion and Deletion



- Insert 2 at index 1 or position 2



- Delete the value at index 2 or position 3



Algorithm – Insertion

Algorithm insertElement(A,n,num,indx)
Input: An array **A** containing **n** integers and the number **num** to be inserted at index **indx**.
Output: Successful insertion of **num** at **indx**.

1. for i = n – 1 to indx do
2. A[i + 1] = A[i]
3. A[indx] = num
4. n = n + 1

```
1.  void insert(int a[], int num, int pos)
2.{ for(int i = n-1; i >= pos; i--)
3.      a[i+1] = a[i];
4.      a[pos] = num;
5.      n++;
6.  }
```

Algorithm – Deletion

Algorithm deleteElement(A,n,indx)
Input: An array **A** containing **n** integers and the index **indx** whose value is to be deleted.
Output: Deleted value stored initially at **indx**.

1. temp = A[indx]
2. for i = indx to n – 2 do
3. A[i] = A[i + 1]
4. n = n – 1
5. return temp

```
1. int deleteElement(int a[], int indx)
2.  {  int temp = a[indx];
3.     for(int i = indx; i <= n-2; i++)
4.         a[i] = a[i+1];
5.     n--;
6.     return temp;
7.  }
```

Insert at any position pos = index-1

```
// shift elements forward
for (i = size-1; i >= pos-1; i--)
    arr[i+1] = arr[i];

arr[pos - 1] = value;
size++;
```

Delete an element at any position pos = index-1

```
//Deletion: shift elements backwards by one position
for(i = pos-1; i < size - 1; i++)
        arr[i] = arr[i+1];
size--;
```

Deletion of a particular element

Algorithm

- Find the index of the element to be deleted in the array.
- If the element is found,
    - Shift all elements after the position of the element by 1 position.
    - Decrement array size by 1.
- If the element is not found: Print “Element Not Found”

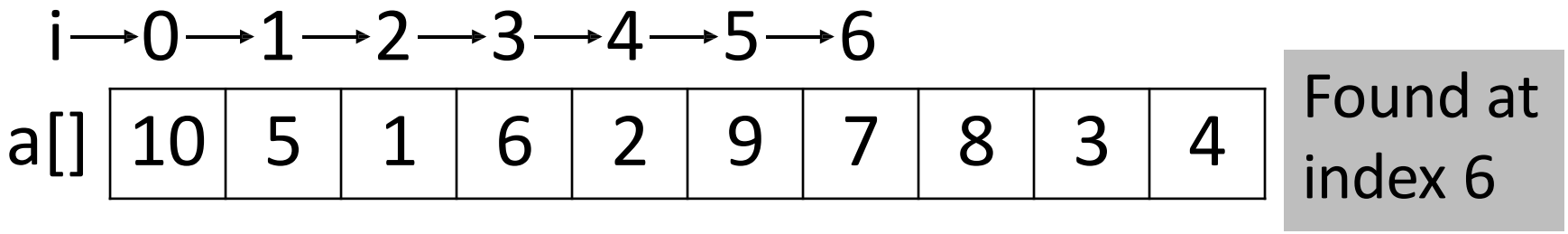
Search

Find the location of the element with a given value.

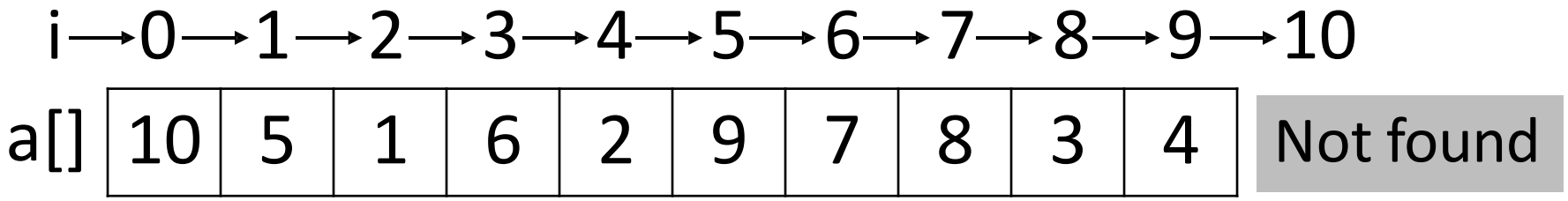
Linear Search

- Used if the array is unsorted.
- Example:

Search 7 in the following array



Search 11 in the following array



Contd…

Algorithm linearSearch(A,n,num)
Input: An array **A** containing **n** integers and number **num** to be searched.
Output: Index of **num** if found, otherwise -1.

1. for i = 0 to n-1 do
2. if A[i] == num
3. return i
4. return -1

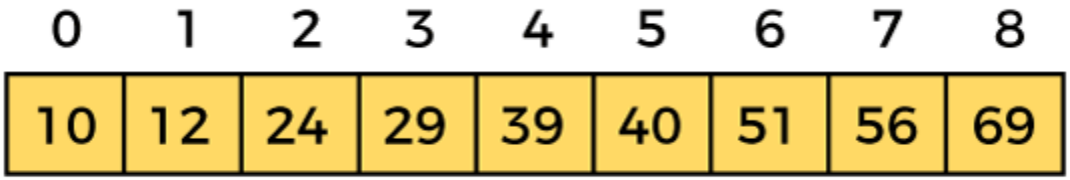
```
1. int linearSearch(int a[], int n, int num)
2. {  for (int i = 0; i < n; i++)
3.       if (a[i] == num)
4.             return i;
5.return -1;
6.  }
```

Binary search

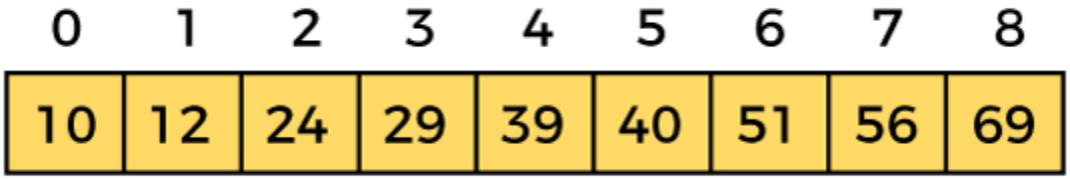
- Binary Search is defined as a searching algorithm used in a sorted array by repeatedly dividing the search interval in half.
- Algorithm
  1. Divide the search space into two halves by **finding the middle index "mid"**.
  2. Compare the middle element of the search space with the key.
  3. If the key is found at middle element, the process is terminated.
  4. If the key is not found at middle element, choose which half will be used as the next search space.
     a. If the key is smaller than the middle element, then the left side is used for next search.
     b. If the key is larger than the middle element, then the right side is used for next search.

  5. This process is continued until the key is found or the total search space is exhausted.

Example (Successful search)



Example (Unsuccessful search)



Binary Search Iterative Algorithm

Algorithm binarySearch(A,n,num)

Input: Sorted array **A** containing **n** integers and number **num** to be searched.

Output: Index of **num** if found, otherwise -1.

```
1.  binarySearch(int arr[], int l, int r, int x){
2.      while (l <= r) {
3.          int mid = (l + r) / 2;
4.       if (arr[mid] == x)
5.              return mid;
6.       if (arr[mid] < x)
7.              l = mid + 1;
8.       else
9.              r = mid - 1;
10.     }
11.     return -1;}
```

Applications of Binary Search

- Building block for more complex algorithms used in machine learning, such as algorithms for training neural networks or finding the optimal hyperparameters for a model.
- It can be used for searching in computer graphics such as algorithms for ray tracing or texture mapping.
- It can be used for searching a database.

Try this!

Problem Description: Given two integer arrays A[] and B[] of size m and n, respectively. We need to find the intersection of these two arrays. The intersection of two arrays is a list of distinct numbers which are present in both the arrays. The numbers in the intersection can be in any order. You can modify the arrays and repeated elements also exist.

For example

Input: A[] = {1,4,3,2,5, 8,9} , B[] = {6,3,2,7,5}

Output: {3,2,5}

Input : A[] = {3,4,6,7,10, 12, 5}, B[] = {7,11,15, 18}

Output: {7}

Solution 1: Use nested loops

For each element in A[], perform a linear search on B[]. Keep on adding the common elements to a list. To make sure the numbers in the list are unique, either check before adding new numbers to list or remove duplicates from the list after adding all numbers.

Solution 2: Sorting and binary search

1. Sort B[] array in increasing order.
2. Search for each element in A[] in the sorted array B[]
3. If the element if found, add it to the answer list
4. Return answer list

Solution 3 Sorting both array

1. Sort both the arrays in increasing order.
2. Initialize the variables i=0 and j=0 in the sorted array A[] and B[] respectively.
3. Run a loop while i < m and j < n (Why ' **and** ', not ' **or** '?)
    a. If A[i] == B[j], add A[i] to the list and increment both i and j
    b. If A[i] < B[j], increase i by 1
    c. If A[i] > B[j], increase j by 1
4. Return answer list

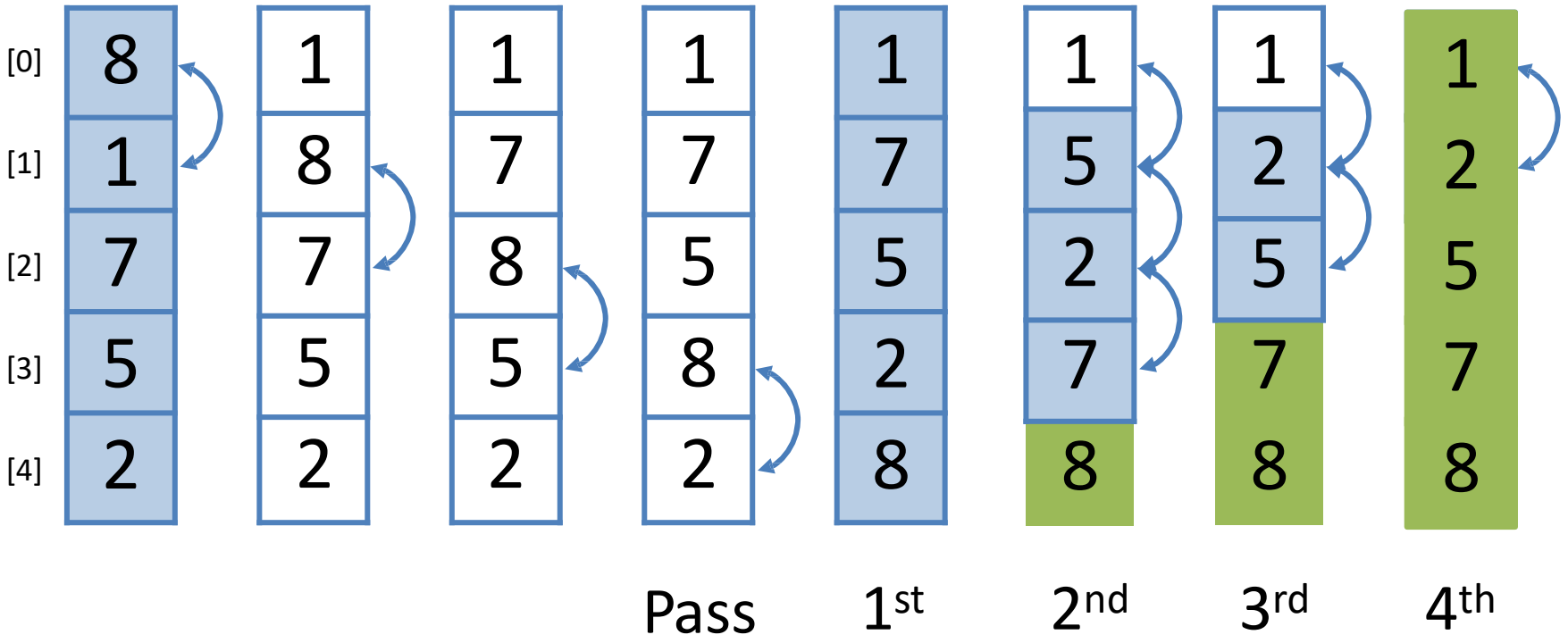
Try this!

Given a positive integer num, write a function which returns True if num is a perfect square else False.

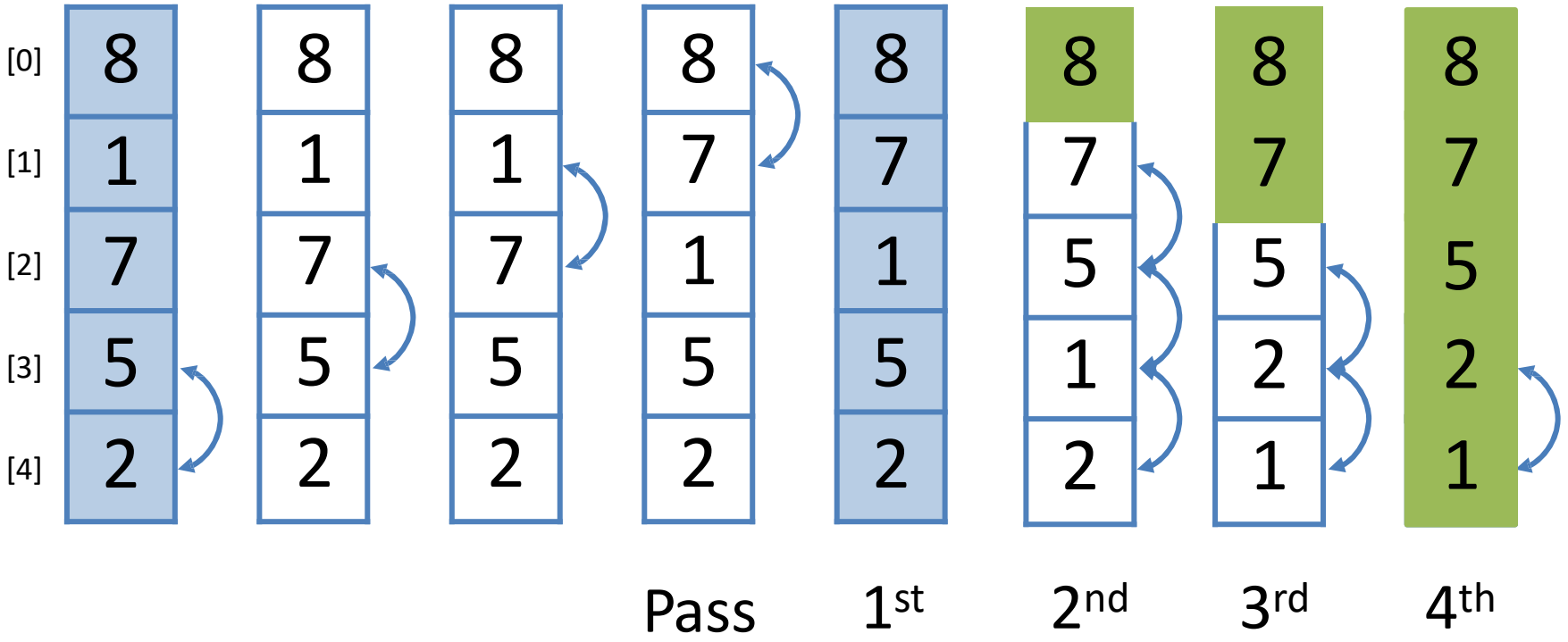
Note: Do not use any built-in library function such as sqrt.

Bubble Sort

Bubble Sort – Ascending



Bubble Sort – Descending



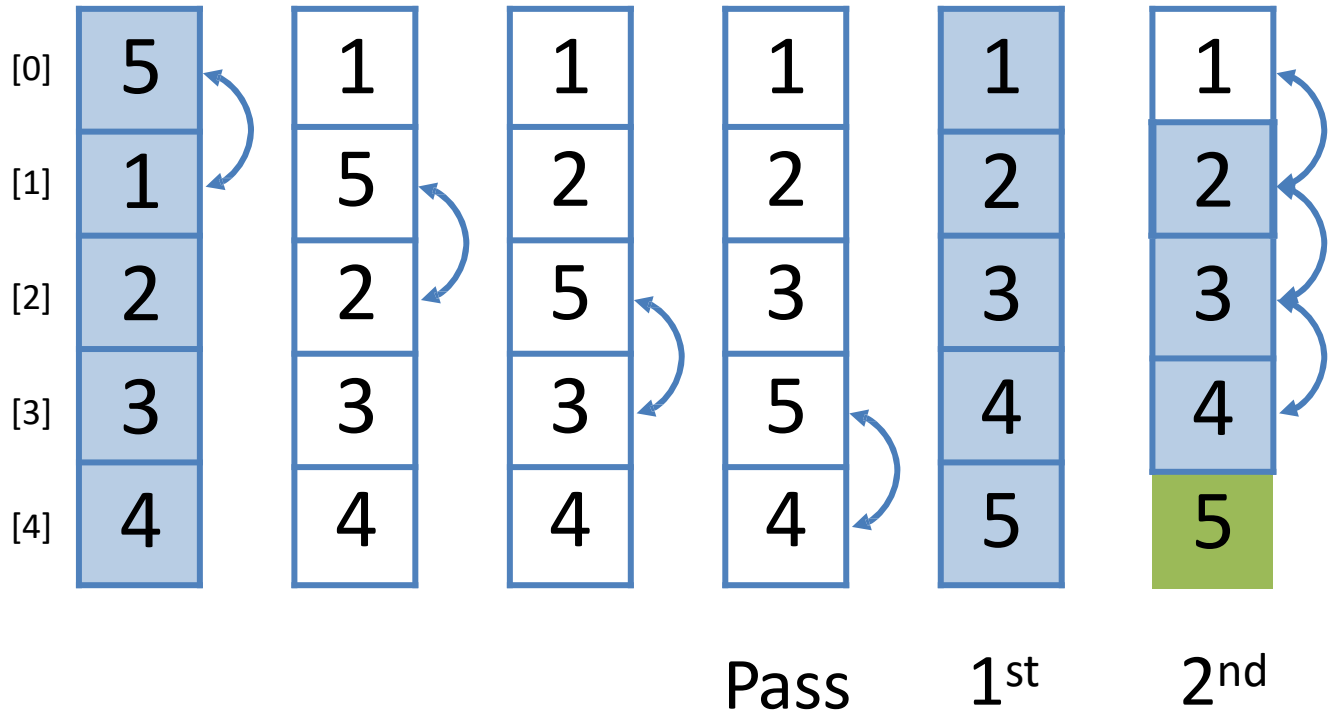
Algorithm – Bubble Sort

Algorithm bubbleSort(A,n)
Input: An array **A** containing **n** integers.
Output: The elements of **A** get sorted in increasing order.

1. **for** i = 1 to n – 1 **do**
2. **for** j = 0 to n – i – 1 **do**
3. **if** A[j] > A[j + 1]
4. Exchange A[j] with A[j+1]

In all the cases, complexity is of the order of n^2.

Optimized Bubble Sort?



Algorithm – Optimized Bubble Sort

Algorithm bubbleSortOpt(A,n)
Input: An array **A** containing **n** integers.
Output: The elements of **A** get sorted in increasing order.

```
1.  for  i = 1  to  n – 1
2.      flag = true
3.      for j = 0  to  n – i – 1  do
4.          if A[j] > A[j + 1]
5.              flag = false
6.              Exchange A[j] with A[j+1]
7.      if  flag == true
8.          break;
```

The best case complexity reduces to the order of n, but the worst and average is still n^2. So, overall the complexity is of the order of n^2 again.

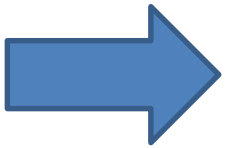
Sparse Matrix

Sparse Matrix

- A matrix is sparse if many of its elements are zero.
- A matrix that is not sparse is dense.
- Two possible representations
  - Array (also known as triplet)
  - Linked list

Array representation

	[0]	[1]	[2]	[3]	[4]	[5]
[0]	15	0	0	22	0	-15
[1]	0	11	3	0	0	0
[2]	0	0	0	-6	0	0
[3]	0	0	0	0	0	0
[4]	91	0	0	0	0	0
[5]	0	0	28	0	0	0



Row	Col	Value
6	6	8
0	0	15
0	3	22
0	5	-15
1	1	11
1	2	3
2	3	-6
4	0	91
5	2	28

Operations

- Transpose
- Addition
- Multiplication

Transpose

	[0]	[1]	[2]	[3]	[4]	[5]
[0]	15	0	0	22	0	-15
[1]	0	11	3	0	0	0
[2]	0	0	0	-6	0	0
[3]	0	0	0	0	0	0
[4]	91	0	0	0	0	0
[5]	0	0	28	0	0	0

Row	Col	Value
6	6	8
0	0	15
0	3	22
0	5	-15
1	1	11
1	2	3
2	3	-6
4	0	91
5	2	28

Original

Row	Col	Value
6	6	8
0	0	15
3	0	22
5	0	-15
1	1	11
2	1	3
3	2	-6
0	4	91
2	5	28

Column Major

	[0]	[1]	[2]	[3]	[4]	[5]
[0]	15	0	0	0	91	0
[1]	0	11	0	0	0	0
[2]	0	3	0	0	0	28
[3]	22	0	-6	0	0	0
[4]	0	0	0	0	0	0
[5]	-15	0	0	0	0	0

Row	Col	Value
6	6	8
0	0	15
0	4	91
1	1	11
2	1	3
2	5	28
3	0	22
3	2	-6
5	0	-15

Row Major

Addition

	[0]	[1]	[2]	[3]	[4]	[5]
[0]	15	0	0	22	0	-15
[1]	0	11	3	0	0	0
[2]	0	0	0	-6	0	0
[3]	0	0	0	0	0	0
[4]	91	0	0	0	0	0
[5]	0	0	28	0	0	0

+

	[0]	[1]	[2]	[3]	[4]	[5]
[0]	15	0	0	0	91	0
[1]	0	11	0	0	0	0
[2]	0	3	0	0	0	28
[3]	22	0	-6	0	0	0
[4]	0	0	0	0	0	0
[5]	-15	0	0	0	0	0

=

	[0]	[1]	[2]	[3]	[4]	[5]
[0]	30	0	0	22	91	-15
[1]	0	22	3	0	0	0
[2]	0	3	0	-6	0	28
[3]	22	0	-6	0	0	0
[4]	91	0	0	0	0	0
[5]	-15	0	28	0	0	0

Addition

Counter = 0

Row	Col	Value
6	6	8
0	0	15
0	3	22
0	5	-15
1	1	11
1	2	3
2	3	-6
4	0	91
5	2	28

Row	Col	Value
6	6	8
0	0	15
0	4	91
1	1	11
2	1	3
2	5	28
3	0	22
3	2	-6
5	0	-15

Row	Col	Value
6	6	14

Addition

Counter = 14

Row	Col	Value
6	6	8
0	0	15
0	3	22
0	5	-15
1	1	11
1	2	3
2	3	-6
4	0	91
5	2	28

Row	Col	Value
6	6	8
0	0	15
0	4	91
1	1	11
2	1	3
2	5	28
3	0	22
3	2	-6
5	0	-15

Row	Col	Value
6	6	14
0	0	30
0	3	22
0	4	91
0	5	-15
1	1	22
1	2	3
2	1	3
2	3	-6
2	5	28
3	0	22
3	2	-6
4	0	91
5	0	-15
5	2	28

Multiplication

Compute A x B

- First take transpose of B.
- Multiply only if the corresponding elements are present and add them for each position in the resultant matrix.

Multiplication

	[0]	[1]	[2]	[3]
[0]	0	10	0	12
[1]	0	0	0	0
[2]	0	0	5	0
[3]	15	12	0	0

×

	[0]	[1]	[2]	[3]
[0]	0	0	8	0
[1]	0	0	0	23
[2]	0	0	9	0
[3]	20	25	0	0

=

	[0]	[1]	[2]	[3]
[0]				
[1]				
[2]				
[3]				

Multiplication

	[0]	[1]	[2]	[3]
[0]	0	10	0	12
[1]	0	0	0	0
[2]	0	0	5	0
[3]	15	12	0	0

×

	[0]	[1]	[2]	[3]
[0]	0	0	8	0
[1]	0	0	0	23
[2]	0	0	9	0
[3]	20	25	0	0

=

	[0]	[1]	[2]	[3]
[0]	240			
[1]				
[2]				
[3]				

Multiplication

	[0]	[1]	[2]	[3]
[0]	0	10	0	12
[1]	0	0	0	0
[2]	0	0	5	0
[3]	15	12	0	0

×

	[0]	[1]	[2]	[3]
[0]	0	0	8	0
[1]	0	0	0	23
[2]	0	0	9	0
[3]	20	25	0	0

=

	[0]	[1]	[2]	[3]
[0]	240	300		
[1]				
[2]				
[3]				

Multiplication

	[0]	[1]	[2]	[3]
[0]	0	10	0	12
[1]	0	0	0	0
[2]	0	0	5	0
[3]	15	12	0	0

×

	[0]	[1]	[2]	[3]
[0]	0	0	8	0
[1]	0	0	0	23
[2]	0	0	9	0
[3]	20	25	0	0

=

	[0]	[1]	[2]	[3]
[0]	240	300	0	
[1]				
[2]				
[3]				

Multiplication

	[0]	[1]	[2]	[3]
[0]	0	10	0	12
[1]	0	0	0	0
[2]	0	0	5	0
[3]	15	12	0	0

×

	[0]	[1]	[2]	[3]
[0]	0	0	8	0
[1]	0	0	0	23
[2]	0	0	9	0
[3]	20	25	0	0

=

	[0]	[1]	[2]	[3]
[0]	240	300	0	230
[1]				
[2]				
[3]				

Multiplication

	[0]	[1]	[2]	[3]
[0]	0	10	0	12
[1]	0	0	0	0
[2]	0	0	5	0
[3]	15	12	0	0

×

	[0]	[1]	[2]	[3]
[0]	0	0	8	0
[1]	0	0	0	23
[2]	0	0	9	0
[3]	20	25	0	0

=

	[0]	[1]	[2]	[3]
[0]	240	300	0	230
[1]	0			
[2]				
[3]				

Multiplication

	[0]	[1]	[2]	[3]
[0]	0	10	0	12
[1]	0	0	0	0
[2]	0	0	5	0
[3]	15	12	0	0

×

	[0]	[1]	[2]	[3]
[0]	0	0	8	0
[1]	0	0	0	23
[2]	0	0	9	0
[3]	20	25	0	0

=

	[0]	[1]	[2]	[3]
[0]	240	300	0	230
[1]	0	0	0	0
[2]	0	0	45	0
[3]	0	0	120	276

	[0]	[1]	[2]	[3]
[0]	0	10	0	12
[1]	0	0	0	0
[2]	0	0	5	0
[3]	15	12	0	0

×

	[0]	[1]	[2]	[3]
[0]	0	0	0	20
[1]	0	0	0	25
[2]	8	0	9	0
[3]	0	23	0	0

=

	[0]	[1]	[2]	[3]
[0]				
[1]				
[2]				
[3]				

Multiplication

	[0]	[1]	[2]	[3]
[0]	0	10	0	12
[1]	0	0	0	0
[2]	0	0	5	0
[3]	15	12	0	0

×

	[0]	[1]	[2]	[3]
[0]	0	0	8	0
[1]	0	0	0	23
[2]	0	0	9	0
[3]	20	25	0	0

=

	[0]	[1]	[2]	[3]
[0]	240	300	0	230
[1]	0	0	0	0
[2]	0	0	45	0
[3]	0	0	120	276

	[0]	[1]	[2]	[3]
[0]	0	10	0	12
[1]	0	0	0	0
[2]	0	0	5	0
[3]	15	12	0	0

×

	[0]	[1]	[2]	[3]
[0]	0	0	0	20
[1]	0	0	0	25
[2]	8	0	9	0
[3]	0	23	0	0

=

	[0]	[1]	[2]	[3]
[0]	240			
[1]				
[2]				
[3]				

Multiplication

	[0]	[1]	[2]	[3]
[0]	0	10	0	12
[1]	0	0	0	0
[2]	0	0	5	0
[3]	15	12	0	0

×

	[0]	[1]	[2]	[3]
[0]	0	0	8	0
[1]	0	0	0	23
[2]	0	0	9	0
[3]	20	25	0	0

=

	[0]	[1]	[2]	[3]
[0]	240	300	0	230
[1]	0	0	0	0
[2]	0	0	45	0
[3]	0	0	120	276

	[0]	[1]	[2]	[3]
[0]	0	10	0	12
[1]	0	0	0	0
[2]	0	0	5	0
[3]	15	12	0	0

×

	[0]	[1]	[2]	[3]
[0]	0	0	0	20
[1]	0	0	0	25
[2]	8	0	9	0
[3]	0	23	0	0

=

	[0]	[1]	[2]	[3]
[0]	240	300		
[1]				
[2]				
[3]				

Multiplication

	[0]	[1]	[2]	[3]
[0]	0	10	0	12
[1]	0	0	0	0
[2]	0	0	5	0
[3]	15	12	0	0

×

	[0]	[1]	[2]	[3]
[0]	0	0	8	0
[1]	0	0	0	23
[2]	0	0	9	0
[3]	20	25	0	0

=

	[0]	[1]	[2]	[3]
[0]	240	300	0	230
[1]	0	0	0	0
[2]	0	0	45	0
[3]	0	0	120	276

	[0]	[1]	[2]	[3]
[0]	0	10	0	12
[1]	0	0	0	0
[2]	0	0	5	0
[3]	15	12	0	0

×

	[0]	[1]	[2]	[3]
[0]	0	0	0	20
[1]	0	0	0	25
[2]	8	0	9	0
[3]	0	23	0	0

=

	[0]	[1]	[2]	[3]
[0]	240	300	0	
[1]				
[2]				
[3]				

Multiplication

	[0]	[1]	[2]	[3]
[0]	0	10	0	12
[1]	0	0	0	0
[2]	0	0	5	0
[3]	15	12	0	0

×

	[0]	[1]	[2]	[3]
[0]	0	0	8	0
[1]	0	0	0	23
[2]	0	0	9	0
[3]	20	25	0	0

=

	[0]	[1]	[2]	[3]
[0]	240	300	0	230
[1]	0	0	0	0
[2]	0	0	45	0
[3]	0	0	120	276

	[0]	[1]	[2]	[3]
[0]	0	10	0	12
[1]	0	0	0	0
[2]	0	0	5	0
[3]	15	12	0	0

×

	[0]	[1]	[2]	[3]
[0]	0	0	0	20
[1]	0	0	0	25
[2]	8	0	9	0
[3]	0	23	0	0

=

	[0]	[1]	[2]	[3]
[0]	240	300	0	230
[1]				
[2]				
[3]				

Multiplication

	[0]	[1]	[2]	[3]
[0]	0	10	0	12
[1]	0	0	0	0
[2]	0	0	5	0
[3]	15	12	0	0

×

	[0]	[1]	[2]	[3]
[0]	0	0	8	0
[1]	0	0	0	23
[2]	0	0	9	0
[3]	20	25	0	0

=

	[0]	[1]	[2]	[3]
[0]	240	300	0	230
[1]	0	0	0	0
[2]	0	0	45	0
[3]	0	0	120	276

	[0]	[1]	[2]	[3]
[0]	0	10	0	12
[1]	0	0	0	0
[2]	0	0	5	0
[3]	15	12	0	0

×

	[0]	[1]	[2]	[3]
[0]	0	0	0	20
[1]	0	0	0	25
[2]	8	0	9	0
[3]	0	23	0	0

=

	[0]	[1]	[2]	[3]
[0]	240	300	0	230
[1]	0			
[2]				
[3]				

Multiplication

	[0]	[1]	[2]	[3]
[0]	0	10	0	12
[1]	0	0	0	0
[2]	0	0	5	0
[3]	15	12	0	0

×

	[0]	[1]	[2]	[3]
[0]	0	0	8	0
[1]	0	0	0	23
[2]	0	0	9	0
[3]	20	25	0	0

=

	[0]	[1]	[2]	[3]
[0]	240	300	0	230
[1]	0	0	0	0
[2]	0	0	45	0
[3]	0	0	120	276

	[0]	[1]	[2]	[3]
[0]	0	10	0	12
[1]	0	0	0	0
[2]	0	0	5	0
[3]	15	12	0	0

×

	[0]	[1]	[2]	[3]
[0]	0	0	0	20
[1]	0	0	0	25
[2]	8	0	9	0
[3]	0	23	0	0

=

	[0]	[1]	[2]	[3]
[0]	240	300	0	230
[1]	0	0	0	0
[2]	0	0	45	0
[3]	0	0	120	276

Multiplication

Row	Col	Value
4	4	5
0	1	10
0	3	12
2	2	5
3	0	15
3	1	12

Counter = 0

Row	Col	Value
4	4	5
0	3	20
1	3	25
2	0	8
2	2	9
3	1	23

Row	Col	Value
4	4	5
0	2	8
1	3	23
2	2	9
3	0	20
3	1	25

Row	Col	Value
4	4	

Multiplication

Row	Col	Value
4	4	5
0	1	10
0	3	12
2	2	5
3	0	15
3	1	12

Counter =

Row	Col	Value
4	4	5
0	3	20
1	3	25
2	0	8
2	2	9
3	1	23

Row	Col	Value
4	4	

Multiplication

Row	Col	Value
4	4	5
0	1	10
0	3	12
2	2	5
3	0	15
3	1	12

Counter =

Row	Col	Value
4	4	5
0	3	20
1	3	25
2	0	8
2	2	9
3	1	23

Row	Col	Value
4	4	
0	0	240

Multiplication

Row	Col	Value
4	4	5
0	1	10
0	3	12
2	2	5
3	0	15
3	1	12

Counter =

Row	Col	Value
4	4	5
0	3	20
1	3	25
2	0	8
2	2	9
3	1	23

Row	Col	Value
4	4	
0	0	240
0	1	300

Multiplication

Row	Col	Value
4	4	5
0	1	10
0	3	12
2	2	5
3	0	15
3	1	12

Counter =

Row	Col	Value
4	4	5
0	3	20
1	3	25
2	0	8
2	2	9
3	1	23

Row	Col	Value
4	4	
0	0	240
0	1	300

Multiplication

Row	Col	Value
4	4	5
0	1	10
0	3	12
2	2	5
3	0	15
3	1	12

Counter =

Row	Col	Value
4	4	5
0	3	20
1	3	25
2	0	8
2	2	9
3	1	23

Row	Col	Value
4	4	
0	0	240
0	1	300
0	3	230

Multiplication

Row	Col	Value
4	4	5
0	1	10
0	3	12
2	2	5
3	0	15
3	1	12

Counter =

Row	Col	Value
4	4	5
0	3	20
1	3	25
2	0	8
2	2	9
3	1	23

Row	Col	Value
4	4	
0	0	240
0	1	300
0	3	230

Multiplication

Row	Col	Value
4	4	5
0	1	10
0	3	12
2	2	5
3	0	15
3	1	12

Counter =

Row	Col	Value
4	4	5
0	3	20
1	3	25
2	0	8
2	2	9
3	1	23

Row	Col	Value
4	4	
0	0	240
0	1	300
0	3	230

Multiplication

Row	Col	Value
4	4	5
0	1	10
0	3	12
2	2	5
3	0	15
3	1	12

Counter =

Row	Col	Value
4	4	5
0	3	20
1	3	25
2	0	8
2	2	9
3	1	23

Row	Col	Value
4	4	
0	0	240
0	1	300
0	3	230
2	2	45

Multiplication

Row	Col	Value
4	4	5
0	1	10
0	3	12
2	2	5
3	0	15
3	1	12

Counter =

Row	Col	Value
4	4	5
0	3	20
1	3	25
2	0	8
2	2	9
3	1	23

Row	Col	Value
4	4	
0	0	240
0	1	300
0	3	230
2	2	45

Multiplication

Row	Col	Value
4	4	5
0	1	10
0	3	12
2	2	5
3	0	15
3	1	12

Counter =

Row	Col	Value
4	4	5
0	3	20
1	3	25
2	0	8
2	2	9
3	1	23

Row	Col	Value
4	4	
0	0	240
0	1	300
0	3	230
2	2	45

Multiplication

Row	Col	Value
4	4	5
0	1	10
0	3	12
2	2	5
3	0	15
3	1	12

Counter =

Row	Col	Value
4	4	5
0	3	20
1	3	25
2	0	8
2	2	9
3	1	23

Row	Col	Value
4	4	
0	0	240
0	1	300
0	3	230
2	2	45

Multiplication

Row	Col	Value
4	4	5
0	1	10
0	3	12
2	2	5
3	0	15
3	1	12

Counter =

Row	Col	Value
4	4	5
0	3	20
1	3	25
2	0	8
2	2	9
3	1	23

Row	Col	Value
4	4	
0	0	240
0	1	300
0	3	230
2	2	45
3	2	120

Multiplication

Counter = 6

Row	Col	Value
4	4	5
0	1	10
0	3	12
2	2	5
3	0	15
3	1	12

Row	Col	Value
4	4	5
0	3	20
1	3	25
2	0	8
2	2	9
3	1	23

Row	Col	Value
4	4	6
0	0	240
0	1	300
0	3	230
2	2	45
3	2	120
3	3	276

	[0]	[1]	[2]	[3]
[0]	240	300	0	230
[1]	0	0	0	0
[2]	0	0	45	0
[3]	0	0	120	276