Q:- $T(n) = T\left(\frac{3n}{8}\right) + T\left(\frac{5n}{8}\right) + n^2 + C$

$\rightarrow T(n) = 3T\left(\frac{n}{8}\right) + 5T\left(\frac{n}{8}\right) + n^2 + C$

$= 8T\left(\frac{n}{8}\right) + n^2 C$

$= 8\left(8\left(T\left(\frac{n}{8^2}\right)\right) + n^2\right) + C + n^2$

$= 8^2 T\left(\frac{n}{8^2}\right) + 8n^2 + C + n^2$

after k iterations

$= 8^k T\left(\frac{n}{8^k}\right) + 8k n^2 + C + n^2$

Let $\frac{n}{8^k} = 1$

$n = 8^k$

$k = \log_8 n$

$T(n) = 8^{\log_8 n} T(1) + 8 \log_8 n \; n^2 + C + n^2$

$O(n^2 \log_8 n)$

Q:- $T(n) = T\left(\frac{n}{9}\right) + T\left(\frac{8n}{9}\right) + \log_2 n + C$

→ For simplicity we can assum

$T\left(\frac{n}{9}\right) = T\left(\frac{8n}{9}\right) = T\left(\frac{n}{2}\right)$

as both equal to n i.e

$\frac{n}{9} + \frac{8n}{9} = n$

and $\frac{n}{2} + \frac{n}{2} = n$

and in complexity we need only have an approximate

$T(n) = 2^{*}\, T\left(\frac{n}{2}\right) + \log_2 n + C$

$= 2^{*}\left(2^{*}\, T\left(\frac{n}{2^2}\right) + \log_2 \frac{n}{2}\right) + \log_2 n + C$

$= 2^2\, T\left(\frac{n}{2^2}\right) + 2 \log_2 \frac{n}{2} + \log_2 n + C$

After h iteration

$= 2^h\, T\left(\frac{n}{2^h}\right) + 2^{h-1} \log_2 \frac{n}{2^{h-1}} + 2^{h-2} \log_2 \frac{n}{2^{h-2}}$

$+ \text{---}\ 2 \log_2 \frac{n}{2} + \log_2 n + C$

$= 2^h\, T\left(\frac{n}{2^h}\right) + 2^{h-1} \log_2 n + 2^{h-2} \log_2 n + \text{--}\ 2 \log_2 n$

$- 2^{h-1} \log_2 2^{h-1} - 2^{h-2} \log_2 2^{h-2} \text{ -- } 2 \log 2$

$+ \log_2 n + C$

Put $\frac{n}{2^h} = 1$

$\log_2 n = h$

$$T(n) = 2^h T\left(\frac{n}{2^h}\right) + (2^{h-1} + 2^{h-2} + 2^{h-3} + \dots 2)\log_2 n$$
$$- (2^{h-1}(h-1) + 2^{h-2}(h-2) + \dots 2\log 2))$$
$$+ \log_2 n + C$$

$$= 2^{\log_2 n}(1) + \frac{2^{h-1} - 2}{2-1}\log_2 n - (2^{h-1}(h-1)$$
$$- (2^{h-2}(h-2) \dots + 2\log 2) + \log_2 n + C$$

$$= n + n\log n + \log n + C$$
$$- (2^{h-2}(h-2) + \dots + 2(\log 2))$$

We assume
$$2^{h-2}(h-2) + 2^{h-1}(h-1) \dots 2(\log 2)$$
is approximately
$$(\log n)^2$$
log sum is approximately log n & we know the multiplication is b/w n & log n (Somewhere)

So;
$$n(\log n)^2$$