1.9.6 Venn Diagram: Applications

Example 28: *Use Venn diagram to show that the following argument is valid.*

P_1 : *Babies are illogical*

P_2 : *Nobody is despited who can manage a crocodile*

P_3 : *Illogical people are despised*

P_4 : *Babies cannot manage crocodile.*

Solution: The given premises lead to the Venn diagram in the Fig. 1.22

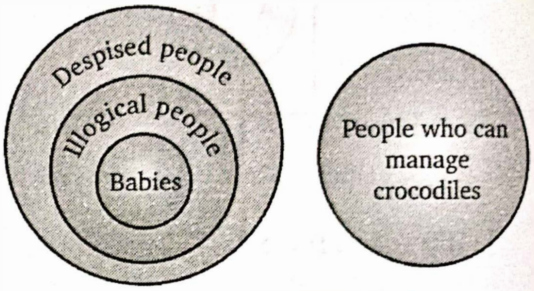

Fig. 1.22

In the Venn diagram, the set of babies and the set of people who can manage crocodile are disjoint. Hence babies can not manage crocodiles and the conclusion P is valid.

Illustration: *Show that the following argument is not valid*

P_1 : All students are Lazy

P_2 : Everybody who is not wealthy is a student

P_3 : Lazy people are not wealthy.

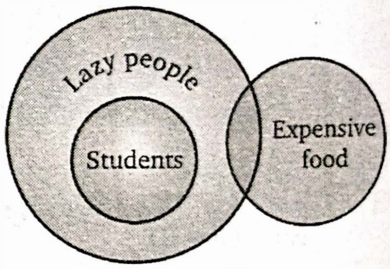

Fig. 1.23

Here both premises hold *i.e.* P_1 and P_2 but conclusion does not hold. Thus the argument is invalid.

Example 29: *If A and B are two sets such that*

$$n(A) = 27,\ n(B) = 35 \quad \text{and} \quad n(A \cup B) = 50 \text{ find } n(A \cap B)$$

Solution: We know that

$$n(A \cup B) = n(A) + n(B) - n(A \cap B)$$

$$n(A \cap B) = 27 + 35 - 50 = 12$$

Hence $n(A \cap B) = 12$

Example 30: *In group of 50 people, 35 speaks Hindi, 25 speaks both English and Hindi and all people speak at least one of the two languages. How many people speak only English and not Hindi? How many speak English?*

Solution: We have

Total number of people = 50

Total number of people speaking Hindi = 35

Let A be the set of people that speak Hindi, B be the set of people that speak English.

Let $n(A)$ = The number of people in the set A

$n(B)$ = The number of people in the set B

$\therefore \quad n(A \cup B) = 50,\ n(A) = 35,\ n(A \cap B) = 25$

We know $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\Rightarrow$ $n(B) = n(A \cup B) - n(A) + n(A \cap B)$

$\Rightarrow$ $n(B) = 50 - 35 + 25 = 40$

And number of people that speak English only

$= n(B) - n(A \cap B)$

$= 40 - 25 = 15$

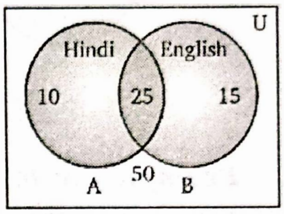

Fig. 1.24

Thus, the number of people speaking English only $= 15$

And, the number of people speaking English $= 40$

Alternatively: This problem can also be solved by Venn diagram in Fig. 1.24.

Example 31: *A computer company must hire 20 programmers to handle system programming jobs and 30 programmers for applications programming. Of these hired, 5 are expected to perform jobs of both types. How many programmers must be hired?*

Solution: We have

20 Programmers = Programming jobs $= n(A)$

30 Programmers = Application programming $= n(B)$

5 Programmers = Both the jobs $= n(A \cap B)$

Then $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$= 20 + 30 - 5 = 50 - 5 = 45$

$\therefore$ The total number of programmers hired are 45.

Example 32: *In a group of 60 people, 40 speak Hindi, 20 speak both English and Hindi and all people speak at least one of the two languages. How many people speak only English and not Hindi? How many speak English?*

Solution: Let total people = 60

Hindi Speaking = 40

English, Hindi Speaking = 20

Let $n(A)$ = The set of Hindi Speaking

$n(B)$ = The set of English speaking

$\therefore$ $n(A) = 40$, $n(A \cap B) = 20$, $n(A \cup B) = 60$

Number of people that speak only English and not Hindi is

$n(B - A) = n(A \cup B) - n(A) = 60 - 40 = 20$

By venn diagram the shaded (stripes) portion (see Fig. 1.25) is the required number of people. Now the number of people, speaking English $n(B)$.

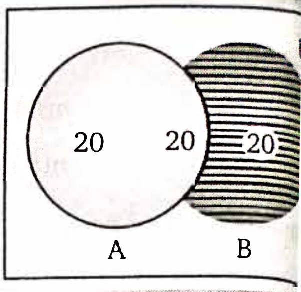

Fig. 1.25

$\therefore$ We have $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\Rightarrow \quad n(B) = n(A \cup B) + n(A \cap B) - n(A)$

$\Rightarrow \quad n(B) = 60 + 20 - 40 = 40$

Example 33: *In a class of 25 students, 12 have taken Mathematics, 8 have taken Mathematics b Biology. Find the number of students who have taken Mathematics and Biology and those who taken B but not Mathematics?*

Solution: By Venn Diagram.

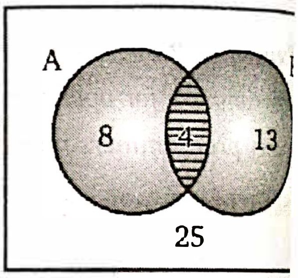

Fig. 1.26

Let A denotes students taken Mathematics and set B denotes students taken Biology. Shaded (stripes) portion define those students taken both Mathematics and Biology.

Total number of students = 25

The students taken Mathematics only = 8

The students taken Mathematics and Biology = 12 – 8 = 4

The Students taking Biology only = 25 – (8 + 4) = 13

Example 34: *A school has 21 boys in basket ball team, 26 in hockey and 29 in football team. Now if 1 play hockey and basketball, 15 boys play hockey and football, 12 boys play football and basketball and e play hockey, football and basketball all three games, then what is total number of boys playing these gam*

Solution: Let B, H, F denote the total number of boys playing basketball, hockey and football.

Then $\quad n(B) = 21, \quad n(H) = 26, \quad n(F) = 29$

$n(H \cap B) = 14; \quad n(H \cap F) = 15; \quad n(F \cap B) = 12$

and $\quad n(B \cap H \cap F) = 8$

Then $\quad n(B \cup H \cup F) = n(B) + n(H) + n(F) - n(H \cap B) - n(H \cap F) - n(F \cap B) + n(B \cap H \cap F)$

$= 21 + 26 + 29 - 14 - 15 - 12 + 8 = 84 - 41 = 43$

Example 35: *A class has 175 students. The following description gives the number of students studyi more of the subjects in this class.*

Mathematics 100; *Physics* 70; *Physics and Chemistry* 46; *Mathematics and physics* 30; *Mathemati Chemistry* 28; *Physics and Chemistry* 23; *Mathematics, Physics and Chemistry* 18.

Find:

(i) *How many students are enrolled in Mathematics alone; Physics alone and Chemistry alone?*

(ii) *The number of students who have not offered any of these subjects.*

lution: Let M,P,C denote the sets of students enrolled in Mathematics, ysics and Chemistry.

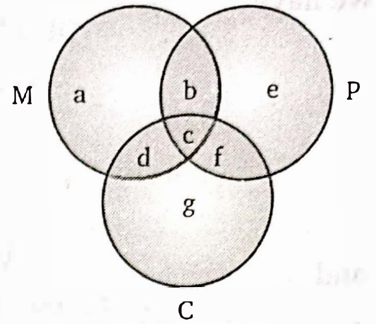

Fig. 1.27

a, b, c, d, e, f, g be number of elements contained in bounded regions.

ing the given data, we have

$$a+b+c+d=100, \quad b+c+e+f=70, \quad c+d+f+g=46$$

$$b+c=30, \quad c+d=28, \quad c+f=23, \quad c=18$$

lve these equations, we obtain

$c=18,\ f=5,\ d=10,\ b=12,\ g=13,\ e=35$ and $a=60$

) Number of students enrolled in

Mathematics alone $= a = 60$

Physics alone $= e = 35$

Chemistry alone $= g = 13$

i) Number of students not offering any of these subjects

$$=175-(a+b+c+d+e+f+g)$$

$$=175-(60+12+18+10+35+5+13)=175-153=32$$

xample 36: *A survey shows that 73% of the Indians like apples, whereas 65% like oranges. What ercentage of Indians like both apples and oranges?*

olution: Let $A=$ Set of Indians who like apples

$B=$ Set of Indians who like oranges

Then $n(A)=73,\ n(B)=65,\ n(A\cup B)=100$

$\therefore\ n(A\cap B)=n(A)+n(B)-n(A\cup B)=73+65-100=38$

Hence, 38 of the Indians like both apples and oranges

xample 37: *In a survey it is found that 21 people like product A, 26 people like product B and 29 like roduct C. If 14 people like products A and B; 15 people like products B and C; 12 people like products C and A; nd 8 people like all the three products.*

Find:

(i) *How many people are surveyed in all?*

(ii) *How many like product C only?*

Solution:

Let A = the set of people who like product A

B = the set of people who like product B

C = the set of people who like product C

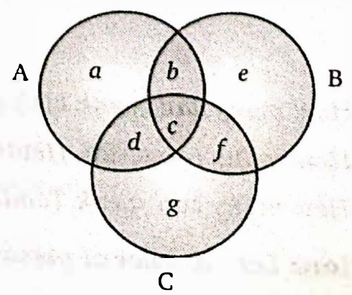

Fig. 1.28

Let, a, b, c, d, e, f, g denote the number of elements containing in bounded regions.

Then, we have

$$a + b + c + d = 21$$
$$b + c + e + f = 26$$
$$c + d + f + g = 29$$
$$b + c = 14, \quad c + f = 15, \quad c + d = 12$$

and $$c = 8$$

$\therefore$ $$c = 8,\ d = 4,\ f = 7,\ b = 6,\ g = 10,\ e = 5,\ a = 3$$

(i) Total number of surveyed people $= a + b + c + d + e + f + g = 43$

(ii) Number of people who like product C only $= g = 10$

Example 38: *In a group of 52 persons, 16 drink tea but not coffee and 33 drink tea.*

(i) How many drink tea and coffee both?

(ii) How many drinks tea and coffee both?

(iii) How many drink coffee but not tea?

Solution:

Let A = Set of persons who drink tea

and B = Set of persons who drink coffee

Then, $A - B$ = Set of persons who drink tea but not coffee

And $B - A$ = Set of persons who drink coffee but not tea

But $n(A \cup B) = 52$, $n(A - B) = 16$ and $n(A) = 33$

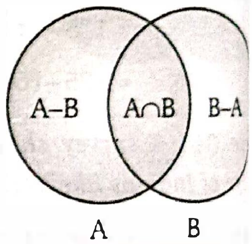

Fig. 1.29

(i) Set of persons who drink tea and coffee both $= (A \cap B)$

Now, $n(A - B) + n(A \cap B) = n(A)$

$\Rightarrow$ $n(A \cap B) = n(A) - n(A - B) = (33 - 16) = 17$

Thus, 17 drinks tea and Coffee both

(ii) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\Rightarrow$ $n(B) = n(A \cup B) + n(A \cap B) - n(A) = 52 + 17 - 33 = 36$

Now $n(B - A) + n(A \cap B) = n(B)$

$\Rightarrow$ $n(B - A) = n(B) - n(A \cap B) = 36 - 17 = 19$

Thus, 19 drinks coffee but not tea.

Example 39: *In a group of 850 persons, 600 can speak Hindi and 340 can speak Tamil.*

Find:

(i) How many can speak both Hindi and Tamil?

(ii) How many can speak Hindi only?

(iii) How many can speak Tamil only?

Solution: Let A = Set of persons who can speak Hindi.

B = Set of persons who can speak Tamil

$\therefore$ $n(A) = 600$, $n(B) = 340$ and $n(A \cup B) = 850$

(i) Set of persons who can speak both Hindi and Tamil = $(A \cap B)$

$\therefore$ $$n(A \cap B) = n(A) + n(B) - n(A \cup B)$$

$$= 600 + 340 - 850 = 90$$

Thus, 90 persons can speak both Hindi and Tamil

(ii) Set of persons who can speak Hindi only = $(A - B)$

Now $$n(A - B) + n(A \cap B) = n(A)$$

$\Rightarrow$ $$n(A - B) = n(A) - n(A \cap B) = 600 - 90 = 510$$

Thus, 510 persons can speak Hindi only

(iii) Set of persons who can speak Tamil only = $(B - A)$

Now $$n(B - A) + n(A \cap B) = n(B)$$

$\Rightarrow$ $$n(B - A) = n(B) - n(A \cap B) = 340 - 90 = 250$$

Hence, 250 persons can speak Tamil only

Example 40: *In an examination, 56% of the candidates fail in English and 48% failed in Science. If 18% failed in both English and Science, find the percentage of those who passed in both the subjects.*

Solution:

Failed in English only = $56 - 18 = 38$

Failed in Science only = $48 - 18 = 30$

Failed in both English and Science = 18

Failed in one or both of the subjects = $38 + 30 + 18 = 86$

Passed in both the subjects = $(100 - 86) = 14$

Example 41: *In a group of 65 people, 40 like cricket, 10 like both cricket and tennis. How many like tennis only and not cricket? How many like tennis?*

Solution:

Let A = Set of people who like cricket

B = Set of people who like tennis

i.e., $n(A) = 40$, $n(A \cap B) = 10$

$\therefore$ $n(A - B) = (40 - 10) = 30$

$n(B - A) = 65 - (30 + 10) = 25$

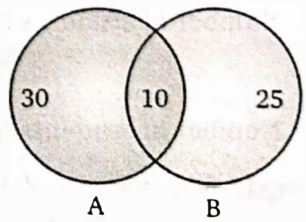

Fig. 1.30

Number of people who like tennis only = 25

Number of people who like tennis = $25 + 10 = 35$

Example 42: *In a canteen, out of 123 students, 42 students buy ice-cream, 36 buy buns and 10 buy cakes, 15 students buy ice-cream and buns, 10 buy ice- cream and cakes, 4 buy cakes and buns but not ice-cream and 11 buy ice-cream and buns but not cakes. Draw Venn diagram to illustrate the above information and find:*

(i) How many students buy nothing at all?

(ii) How many students buy at least two items?

(iii) How many students buy all the three items?

Solution:

Define the sets A, B and C such that

A = Set of students who buy cakes,

B = Set of students who buy ice-cream,

C = Set of students who buy buns.

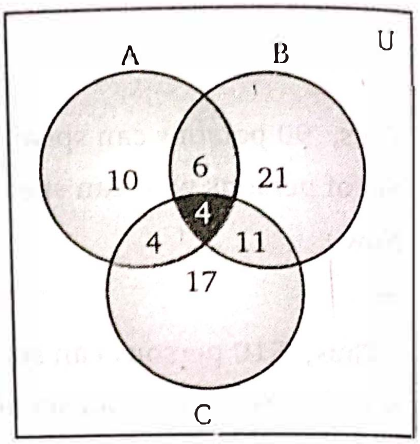

Fig. 1.31

According to Question, we have

$$n(A) = 10, \quad n(B) = 42, \quad n(C) = 36$$

$$n(B \cap C) = 15$$

$$n(A \cap B) = 10$$

$$n\{(A \cap C) - B\} = 4$$

$$n\{(B \cap C) - A\} = 11$$

and $\quad n\{(A - B \cup C) - B\} = 10$

Now, we have $\quad n(B \cup C) = n(B) + n(C) - n(B \cap C)$

$$= 142 + 36 - 15 = 63$$

$$n(B \cup C) - n(B) = 63 - 42 = 21$$

and $\quad n(B \cup C) - n(C) = 63 - 36 = 27$

The above distribution of the student can be illustrated by Venn diagram (Fig. 1.31).

Now, total number of students buying some thing

$$= 10 + 6 + 21 + 4 + 4 + 11 + 17 = 73$$

(i) Number of students who did not buy anything.

$$= 123 - 73 = 50$$

(ii) Number of students buying at least two items

$$= 6 + 4 + 4 + 11 = 25 \quad \text{and}$$

(iii) Number of students buying all three items = 4.

Example 43: *In a city of 1000 families, it was found that 40% families buy newspaper A, 20 families buy newspaper B, and 10% buy C. Only 5% families buy A and B, 3% buy B and C and 4% buy A and C and 2% families buy all the three newspapers. Find the number of families which buy:*

(i) A only *(ii) B only* *(iii) None of A, B and C.*

Solution: Let P, Q and R denote the set of families buying newspaper A, B and C respectively.

Then according to question, we have

$$n(P) = 40\% \text{ of } 1000 = 400$$

$$n(R) = 100, \quad n(Q) = 200$$

$$n(P \cap Q) = 50, \quad n(Q \cap R) = 30$$

$$n(R \cap P) = 40$$

and $n(P \cap Q \cap R) = 20$ and $n(U) = 1000$,

Where U is the universal set.

(i) Number of families which buy newspaper A only.

$$= n(P \cap Q' \cap R')$$
$$= n[P \cap (Q \cup R)'] \qquad \{\because (Q \cup R)' = Q' \cap R'\}$$
$$= n(P) - n[(P \cap Q) \cup (P \cap R)] \{\because n(A \cap B') = n(A) - n(A \cap B)\}$$
$$= n(P) - [n(P \cap Q) + n(P \cap R) - n\{(P \cap Q) \cap (P \cup R)\}]$$
$$= n(P) - [n(P \cap Q) + n(P \cap R) - n(P \cap Q \cap R)]$$
$$= 400 - (50 + 40 - 20) = 330$$

(ii) Number of families which buy newspaper B

$$= n(P' \cap Q' \cup R') = n(Q \cap P' \cap R')$$
$$= n[Q \cap (P \cup R)']$$
$$= n(Q) - n[Q \cap (P \cup R)]$$
$$= n(Q) - n[(Q \cap P) \cup (Q \cap R)]$$
$$= n(Q) - \{n(Q \cap P) + n(Q \cap R) - n(Q \cap P) \cap (Q \cap R)\}$$
$$= n(Q) - \{n(P \cap Q) + n(Q \cap R) - n(P \cap Q \cap R)\}$$
$$= 200 - [50 + 30 - 20] = 140.$$

(iii) The Number of families which buy none of A, B and C.

$$= n(P' \cap Q' \cap R')$$
$$= n[P \cup Q \cup R)'] \text{ (By de-morgan's law)}$$
$$= n(U) - [n(P) + n(Q) + n(R) - n(P \cap Q) - n(Q \cap R) - n(R \cap P) + n(P \cap Q \cap R)]$$
$$= 1000 - (400 + 200 + 100 - 50 - 30 - 40 + 20) = 400.$$

1.10 Countable and Uncountable Gets

A set A is said to be ***Finite*** if either it is empty or there exists a one-to-one mapping from the set $\{1, 2, 3 \dots n\}$ onto the set A for some natural number N. A set which is not finite is called infinite.

Illustration : *The set ϕ is a finite set.*

Illustration: *The set N of all natural numbers is infinite.*

Illustration: *The set Q all rational numbers is infinite.*

1.10.1 Equivalent Sets

Two sets A and B are said to be equivalent if there exists a bijective map $f : A \to B$. If A and B are equivalent than we can write $A \sim B$

Illustration: *Let* $A = \{1, 2, 3, 4 \dots n\}$ *and* $B = \{m, 2m, 3m, \dots mn\}$

Then A and B are equivalent because map $f : A \to B$ defined by $f(i) = mi,\ i = 1, 2, 3 \dots n$ is bijective.

Illustration: *Any two closed intervals* $[a, b]$ *and* $[c, d]$ *are equipotent. We defined a mapping*

$$f : [a, b] \to [c, d] \text{ such that}$$
$$f(x) = c + \left(\frac{d-c}{b-a}\right)(x - a)$$

Then, f is a bijective mapping.