CHAPTER 1

# Set Theory

*In this chapter, the notation, terminology and concepts of set theory are introduced, which are useful in studying any branch of mathematics. Venn diagram representation of sets will be introduced in order to give a clear picture of set operations.*

## 1.1 SETS AND ITS ELEMENTS

What do the following have in common ?

- A bouquet of flowers,
- A set of all Indians,
- A group of children,
- A herd of animals,
- A collection of books.

In each case, we are dealing with a collection of objects of a certain type. Instead of using different word for each type of collections, we denote them by the one word "set".

Thus, we give the formal definition of set as :

DEF. A set is any well-defined collection of objects, called the *elements* or *members* of the set.

Well-defined just means there exists a rule which helps us in deciding whether a given objects belongs to the collection or not.

Capital letters, $A, B, C, \ldots\ldots, X, Y, Z$, will be used throughout to denote sets, and lower case letters, $a, b, x, y, \ldots\ldots$, will be used to denote elements of sets.

If an element $a$ belongs to a set $A$, we write $a \in A$ read as "$a$ belongs to $A$" or "$a$ is an element of $A$" or "$a$ is in $A$".

The symbol $\notin$ is used to indicate, "does not belong to". Thus, if there exists an object $b$ which does not belong to $A$, we write $b \notin A$.

There are *two* ways to describe a set :

1. Describe a set by listing all its elements.
2. Describe a set by describing the properties of the elements of the set.

**Example 1.** Describe the set which contains all the positive integers less than or equal to 4.

Let us denote this set by $A$. Then

1. $A = \{1, 2, 3, 4\}$
2. $A = \{x \mid x$ is a positive integer less than or equal to $4\}$.

**Example 2.** Describe a set $A$ containing all the even positive integers.

The set $A$ can be written as

1. $A = \{2, 4, 6, 8, \ldots\ldots\}$
2. $A = \{x \mid x$ is an even integer, $x > 0\}$

### Standard Sets and Symbols

We introduce here several sets and their notation that will be used throughout this book.

(*i*) $N$ or $Z^+ = \{x \mid x$ is a positive integer$\}$

Thus $Z^+$ consists of the numbers 1, 2, 3, ......

(*ii*) $Z = \{x \mid x$ is an integer$\}$

$= \{\ldots\ldots -2, -1, 0, 1, 2, \ldots\ldots\}$

(*iii*) $Q = \{x \mid x$ is a rational number$\}$

(*iv*) $R = \{x \mid x$ is a real number$\}$

(*v*) $C = \{x \mid x = a + ib\,;\, a, b \in R, i = \sqrt{-1}\}$, the set of complex numbers.

☞ **DEF.** Let $A$ and $B$ be two sets. If every element of $A$ is also an element of $B$, then $A$ is called a **subset** of $B$ or $A$ is contained in $B$ and we write $A \subseteq B$ or $B \supseteq A$.

If $A$ is not a subset of $B$ *i.e.*, if at least one element of $A$ does not belong to $B$, we write $A \not\subseteq B$ or $B \not\supseteq A$.

**Example 3.** We have $Z^+ \subseteq Z$.

Also $Z \subseteq Q \subseteq R \subseteq C$.

**Example 4.** Let $A = \{1, 2, 3, 4, 5, 6, 7\}$, $B = \{2, 4, 6\}$ and $C = \{1, 3, 5, 7\}$, then $B \subseteq A$ and $C \subseteq A$.

However $A \not\subseteq B$, $A \not\subseteq C$, $B \not\subseteq C$ and $C \not\subseteq B$.

☞ **DEF.** Two sets $A$ and $B$ are said to be **equal sets** if they have the same elements.

We write $A = B$.

Equivalently, $A = B$ iff $A \subseteq B$ and $B \subseteq A$.

**Example 5.** If $A = \{1, 2, 3\}$ and $B = \{x \mid x$ is a positive integer and $x^2 < 12\}$, then $A = B$.

**Example 6.** If $A = \{1, 2\}$ and $B = \left\{2, 1, \frac{20}{10}, \frac{6}{3}, 1\right\}$, then $A = B$.

The ordering of the elements and the number of times an element appears doesn't affect a set as seen in Example 6.

If $A \subseteq B$ but $A \neq B$, then we say that $A$ is a **proper subset** of $B$ and we write $A \subset B$.

**Example 7.** Let $A = \{1, 2, 3\}$, $B = \{1, 3, 2\}$, $C = \{1, 3\}$.

Then $C \subset A$, $C \subset B$, $A \subseteq B$, $B \subseteq A$ *i.e.*, $C$ is a proper subset of $A$ and $B$ and $B = A$.

☞ **DEF.** A set is called a **universal set** if it includes every set under discussion. A universal set is generally denoted by $U$.

☞ **DEF.** A set which does not contain any element is called an **empty set or null** set and it is denoted by $\phi$ or $\{\ \}$.

**Example 8.** *Given* $S = \{2, a, \{3\}, 4\}$ *and* $R = \{\{a\}3, 4, 1\}$. *Answer whether the following are true or false* :

(*i*) $\{a\} \in S$ (*ii*) $\{a\} \in R$

(*iii*) $\{a, 4, \{3\}\} \subseteq S$ (*iv*) $\{\{a\}, 1, 3, 4\} \subset R$

(*v*) $R = S$ (*vi*) $\phi \subset R$

**Solution.** (*i*) False. $a \in S$ and $\{a\} \subset S$

(*ii*) True

(*iii*) True

(*iv*) False. $\{\{a\}, 1, 3, 4\} = R$ and is not a proper subset of $R$.

(*v*) False

(*vi*) True.

**Example 9.** *Determine whether each of the following statements is true for arbitrary sets A, B, C. Justify your answer.*

(*a*) *If* $A \in B$ *and* $B \subseteq C$, *then* $A \in C$.

(*b*) *If* $A \in B$ *and* $B \subseteq C$, *then* $A \subseteq C$.

(*c*) *If* $A \subseteq B$ *and* $B \in C$, *then* $A \in C$

(*d*) *If* $A \subseteq B$ *and* $B \in C$, *then* $A \subseteq C$.

(*e*) $\{a, \phi\} \in \{a, \{a, \phi\}\}$

**Solution.** (*a*) **True.**

*e.g.*, $A = \{1, 2\}$, $B = \{\{1, 2\}, 3, 4\}$, $C = \{\{1, 2\}, 3, 4, 5\}$

Here, $A \in B$ and $B \subseteq C$ and $A \in C$ also.

(*b*) **False.**

*Example* : $B = \{A, 1, 2, 3\}$

$C = \{1, 2, 3, 4, A\}$

Here $A \in B$, $B \subseteq C$ but $A \not\subseteq C$.

(*c*) **False.**

*Example* : $A = \{1, 2\}$

$B = \{1, 2, 3\}$

$C = \{\{1, 2, 3\}, 4, 5\}$

Here $A \subseteq B$ and $B \in C$ but $A \notin C$.

(*d*) **False.**

*Example*. Consider the example given in (*c*), then $A \subseteq B$ and $B \in C$ but $A \not\subseteq C$.

(*e*) **True.**

**Example 10.** *Write the sets :*

(*i*) $\phi \cup \{\phi\}$ (*ii*) $\{\phi\} \cap \{\phi\}$ (*iii*) $\{\phi, \{\phi\}\} - \phi$

**Solution.** (*i*) $\phi \cup \{\phi\} = \{\phi\}$

(*ii*) $\{\phi\} \cap \{\phi\} = \{\phi\}$

(*iii*) $\{\phi, \{\phi\}\} - \phi = \{\phi, \{\phi\}\}$.

**Example 11.** Consider $U$ as the set of all positive numbers. Then $A = \{x \in U \mid x + 1 = 0\}$ is an empty set since there is no positive number which satisfies the equation $x + 1 = 0$.

The empty set is a subset of every set.

$\phi$ and $\{\phi\}$ are different sets. $\phi$ has no element whereas $\{\phi\}$ has a unique element $\phi$.

☞ **DEF.** A set is **finite** if it has a finite number of distinct elements. Any set which is not finite is said to be an **infinite set.**

☞ **DEF.** Let $A$ be a finite set. The number of distinct elements in $A$ is called the **cardinality** of $A$ and is denoted by $n(A)$ or $|A|$.

**Example 12.** The set $A = \{1, 2, 3, 4\}$ is a finite set whereas the set $Z$ of integers is an infinite set. Also $|A| = 4$.

Note that $|\phi| = 0$ and $|\{\phi\}| = 1$.

☞ **DEF.** If $A$ is a set, then the set of all subsets of $A$ is called the **power set** and is denoted by $P(A)$ *i.e.*, $P(A) = \{X \mid X \subseteq A\}$

**Example 13.** Let $A = \{a, b, c\}$. Then

$$P(A) = \{\phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{c, a\}, \{a, b, c\}\}.$$

We can see that the $|P(A)| = 2^{n(A)}$.

☞ **DEF.** A diagram of a set is called a **Venn diagram** (named after John Venn, 1834-1883). The universal set $U$ is represented by a large rectangular area. Subsets within this universe are represented by circles.

**Example 14.** Let $A = \{1, 2, 3, 4, ......, 10\}$ and $U = Z^+$. Then $A \subset U$ and its Venn diagram is given by Fig. 1.1.

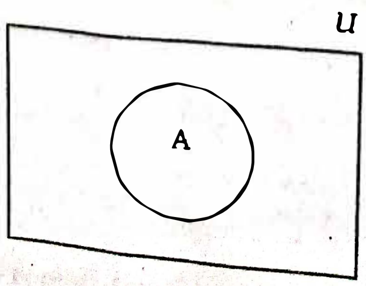



**Fig. 1.1.**

## 1.2 OPERATIONS OF SETS

In this section we introduce some basic operations on sets. Using these operations, we can construct new sets from given sets.

☞ **DEF.** Let $A$ and $B$ be two sets. The **intersection** of $A$ and $B$, written as $A \cap B$, is the set of elements which belong to both $A$ and $B$, *i.e.*, $A \cap B = \{x \mid x \in A \text{ and } x \in B\}$.

☞ **DEF.** For any two sets $A$ and $B$, the **union of $A$ and $B$**, written as $A \cup B$, is the set of all elements which belong to $A$ or to $B$ or both. Symbolically, it is written as

$$A \cup B = \{x \mid x \in A \quad \text{or} \quad x \in B\}$$

☞ **DEF.** Two sets $A$ and $B$ are said to be **disjoint** if they have no element in common, that is, if $A \cap B = \phi$.

**Example 15.** Let $A = \{1, 3, 9\}$, $B = \{-2, 0, 3, 8, 9\}$.
Then $A \cap B = \{3, 9\}$ and $A \cup B = \{-2, 0, 1, 3, 8, 9\}$.

**Example 16.** Let $B$ denote the set of boys in a school $S$ and $G$ denote the set of girls in a school $S$. Then

$$B \cup G = S \qquad \text{and} \qquad B \cap G = \phi.$$

**Example 17.** For any set $A$, $A \cup \phi = A$ and $A \cap \phi = \phi$.

☞ **DEF.** Let $U$ be the universal set and let $A$ be any subset of $U$. Then the ***absolute complement*** **of $A$** or simply, ***complement*** **of $A$**, denoted by $\overline{A}$ or $A^c$, is the set of elements that are in $U$ and not in $A$, *i.e.*, $\overline{A} = \{x \mid x \in U \text{ and } x \notin A\}$.

If $A$ and $B$ are sets, then the **relative complement of $A$ with respect to $B$** denoted as $B - A$, is defined as

$$B - A = \{x \mid x \in B \text{ and } x \notin A\}$$

that is, the set of elements that are in $B$ but not in $A$.

☞ **DEF.** The **symmetric difference** of two sets $A$ and $B$, denoted by $A \Delta B$ or $A \oplus B$ is

$$A \Delta B = \{x \mid x \in A \text{ or } x \in B \text{ but not both}\}$$

$$= (A \cup B) - (A \cap B).$$

**Example 18.** Let $A = \{x \mid x \text{ is an integer and } x \leq 2\}$ and $U = Z$. Then
$\overline{A} = \{x \mid x \text{ is an integer and } x \geq 3\}$.

**Example 19.** Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{4, 5, 6, 7, 8\}$ then $A - B = \{1, 2, 3\}$ and $B - A = \{6, 7, 8\}$ and $A \Delta B = \{1, 2, 3, 6, 7, 8\}$.

**Example 20.** *What can you say about the sets A and B if $A \oplus B = A$ ?*

**Solution.**

$$A \oplus B = (A \cup B) - (A \cap B)$$

and for

$$A \oplus B = A$$

$$B = \phi.$$

**Example 21.** *What can you say about the sets A and B if the following are true ?*

(*a*) $A \cup B = A$

(*b*) $A \cap B = A$

(*c*) $A - B = A$

(*d*) $A \cap B = B \cap A$

(*e*) $A - B = B - A$.

**Solution.**

(*a*) If $A \cup B = A$, then $B \subseteq A$

(*b*) $A \cap B = A \quad \Rightarrow \quad A \subseteq B$

(*c*) $A - B = A \quad \Rightarrow \quad A \cap B = \phi$

(*d*) Since this is always true we can say nothing.

(*e*) $A - B = B - A$

$\Rightarrow \quad A = B$.

**Example 22.** *Find the sets A and B if*

$$A - B = \{1, 5, 7, 8\},\ B - A = \{2, 10\} \text{ and } A \cap B = \{3, 6, 9\}$$

**Solution.**

$$A = \{1, 5, 7, 8, 3, 6, 9\}$$

$$B = \{2, 10, 3, 6, 9\}$$

**Example 23.** *If $A \subset B$, show that :*

$$A \cup (B - A) = B.$$

**Solution.** Let $U$ be the universal set of which $A$ and $B$ are subsets.

$$\text{L.H.S.} = A \cup (B - A)$$

$$= A \cup (B \cap \bar{A})$$

$$= (A \cup B) \cap (A \cup \bar{A}) \qquad \text{[Using distributive law]}$$

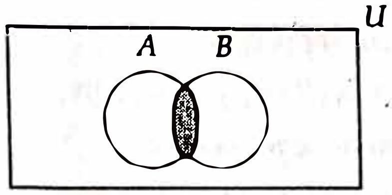
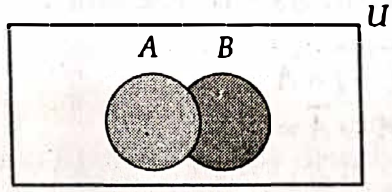
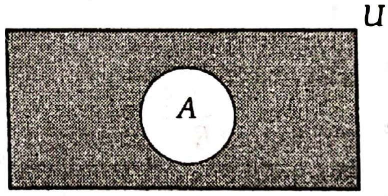
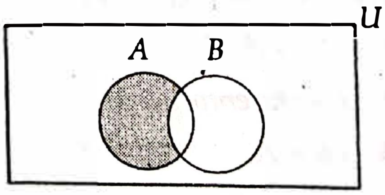
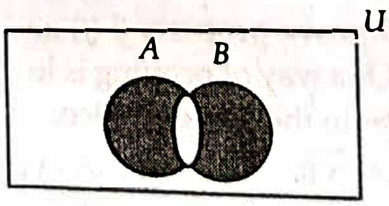
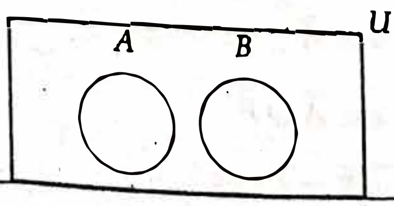
$$= (A \cup B) \cap E$$

$$= A \cup B = B \qquad \text{(Since } A \subset B\text{)}$$

$$= \text{R.H.S.}$$

Venn diagrams of set operations are given in Table 1.1. The shaded area represents the sets indicated in symbol column.

**Table 1.1.**

| *Set operation* | *Symbol* | *Venn diagram* |
|---|---|---|
| Intersection of two sets $A$ and $B$ | $A \cap B$<br>Common in both |  |
| Union of two sets $A$ and $B$ | $A \cup B$<br>Union |  |
| Absolute complement of a set $A$ | $\overline{A}$ |  |
| Relative complement of $B$ with respect to $A$. | $A - B$<br>$A \cap \overline{B}$ |  |
| The symmetrical difference of sets $A$ and $B$ | $A \Delta B$ or $A \oplus B$, |  |
| Disjoint sets $A$ and $B$ | $A \cap B = \phi$ |  |

## 1.3 ALGEBRAIC PROPERTIES OF SET OPERATIONS

Set operations defined in section 1.2 satisfy many algebraic properties which in many instances are similar to the algebraic properties of the real number where "$\cup$" plays the role of "+" and "$\cap$" plays the role of "$\times$". However, there are several differences.

For any sets $A$, $B$, $C$ contained in a universal set $U$, the following properties are satisfied :

1. *Idempotent properties*
   (*a*) $A \cup A = A$, (*b*) $A \cap A = A$
2. *Commutative properties*
   (*a*) $A \cap B = B \cap A$ (*b*) $A \cup B = B \cup A$
3. *Associative properties*
   (*a*) $A \cap (B \cap C) = (A \cap B) \cap C$ (*b*) $A \cup (B \cup C) = (A \cup B) \cup C$
4. *Distributive properties*
   (*a*) $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ (*b*) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
5. *Properties of the complement*
   (*a*) $\overline{(\overline{A})} = A$ (Double complement law)
   (*b*) $A \cup \overline{A} = U$
   (*c*) $A \cap \overline{A} = \phi$
   (*d*) $\overline{\phi} = U$
   (*e*) $\overline{U} = \phi$
   (*f*) $\overline{A \cap B} = \overline{A} \cup \overline{B}$ (De Morgan's laws)
   (*g*) $\overline{A \cup B} = \overline{A} \cap \overline{B}$
6. *Properties of a universal set*
   (*a*) $A \cap U = A$ (*b*) $A \cup U = U$
7. *Property of the empty set*
   (*a*) $A \cup \phi = A$ (*b*) $A \cap \phi = \phi$
8. *Absorption property*
   (*a*) $A \cup (A \cap B) = A$ (*b*) $A \cap (A \cup B) = A$.

We will prove property 5 (*f*) and leave the proofs of remaining properties as an exercise. One way of proving is to choose an element in one of the sets and show that it belongs to the other set also.

Let $x \in \overline{A \cap B} \Rightarrow x \notin A \cap B$

$\Rightarrow x \notin A$ or $x \notin B$

$\Rightarrow x \in \overline{A}$ or $x \in \overline{B}$

$\Rightarrow x \in \overline{A} \cup \overline{B}$

Thus $\overline{A \cap B} \subseteq \overline{A} \cup \overline{B}$ ...(1)

Conversely, suppose that $x \in \overline{A} \cup \overline{B}$

$\Rightarrow x \in \overline{A}$ or $x \in \overline{B}$

$\Rightarrow x \notin A$ or $x \notin B$

$\Rightarrow x \notin A \cap B$

$\Rightarrow \qquad x \in \overline{A \cap B}$

and thus $\qquad \bar{A} \cup \bar{B} \subseteq \overline{A \cap B}$ ...(2)

and hence from (1) and (2)

$$\overline{A \cap B} = \bar{A} \cup \bar{B}.$$

Second way of proving is by use of Venn diagrams.

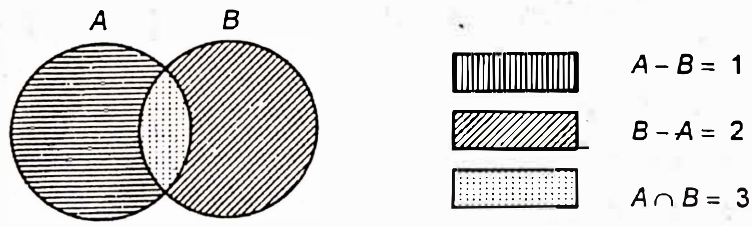
**Example 24.** *Prove the identity :*

$$(A \cup B) - (A \cap B) = (A - B) \cup (B - A).$$

**Solution. 1st method :**

$$\begin{aligned}
\text{L.H.S.} &= (A \cup B) - (A \cap B) \\
&= (A \cup B) \cap (\overline{A \cap B}) \\
&= (A \cup B) \cap (\bar{A} \cup \bar{B}) \\
&= [(A \cup B) \cap \bar{A}] \cup [(A \cup B) \cap \bar{B}] \qquad \text{[By distributive property]} \\
&= [(A \cap \bar{A}) \cup (B \cap \bar{A})] \cup [(A \cap \bar{B}) \cup (B \cap \bar{B})] \\
&= \phi \cup (B \cap \bar{A}) \cup (A \cap \bar{B}) \cup \phi \\
&= (B - A) \cup (A - B) = (A - B) \cup (B - A) \\
&= \text{R.H.S.}
\end{aligned}$$

**2nd method :** By using Venn diagrams, we can also prove this identity.




$$\begin{aligned}
\text{L.H.S.} &= (A \cup B) - (A \cap B) \\
&= (A \cup B) - 3 \\
&= 1 + 2 \\
&= (A - B) \cup (B - A)
\end{aligned}$$

$\Rightarrow A \cap (\overline{A \cap B})$
$A \cap (\bar{A} \cup \bar{B})$
$(A \cap \bar{A}) \cup (A \cap \bar{B})$
$\phi \cup A \cap \bar{B}$
$A - B$

**Example 25.** *Prove that* $A - (A \cap B) = A - B.$

**Solution. 1st method.** Since $B$ and $\bar{B}$ are mutually exclusive sets, $B \cup \bar{B} = U$, where $U$ is the universal set of which $A$ and $B$ are subsets.

$$\begin{aligned}
\text{Hence,} \quad A \cap (B \cup \bar{B}) &= (A \cap B) \cup (A \cap \bar{B}) \\
&= (A \cap B) + (A \cap \bar{B})
\end{aligned}$$

[Since $(A \cap B)$ and $(A \cap \bar{B})$ are disjoint]

$$= (A \cap B) + (A - B)$$

$$A \cap (B \cup \bar{B}) = A \cap U = A$$

Hence, $\qquad A = (A \cap B) + (A - B)$

$\Rightarrow \qquad A - (A \cap B) = A - B.$

**2nd method.**

$$\begin{aligned} \text{L.H.S.} &= A - (A \cap B) \\ &= A \cap (\overline{A \cap B}) = A \cap (\overline{A} \cup \overline{B}) \\ &= (A \cap \overline{A}) \cup (A \cap \overline{B}) \\ &= \phi \cup (A \cap \overline{B}) = (A \cap \overline{B}) \\ &= A - B = \text{R.H.S.} \end{aligned}$$

**Example 26.** *Let A, B, C be subset of u. Given that $A \cap B = A \cap C$, is it necessary that $B = C$? Justify your answer.*

**Solution.** It is not necessary that $B = C$.

For example, $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$

$A = \{1, 2, 3, 4, 5\}$

$B = \{3, 4, 5, 8, 9, 10\}$

$C = \{3, 4, 5, 6, 7, 11, 12\}$

Then, $A \cap B = \{3, 4, 5\}$, $A \cap C = \{3, 4, 5\}$

$A \cap B = A \cap C$

But $B \neq C$.

**Example 27.** *Prove the following or provide a counter example :*

$$A \cup B \subseteq A \cap B \Rightarrow A = B.$$

**Solution.** Let $x \in A \Rightarrow x \in A \cup B$

$\Rightarrow x \in A \cap B$ $\quad [\because A \cup B \subseteq A \cap B, \text{ given}]$

$\Rightarrow x \in B$

$\Rightarrow A \subseteq B.$ ...(1)

Let $x \in B \Rightarrow x \in A \cup B$

$\Rightarrow x \in A \cap B$

$\Rightarrow x \in A$

$\Rightarrow B \subseteq A$ ...(2)

From (1) and (2), we get

$A = B.$

☞ **DEF.** Let $A$ and $B$ be sets. The **cartesian product** of $A$ and $B$, denoted by $A \times B$, is the set of all possible ordered pairs whose first component is an element of $A$ and the second component is an element of $B$ *i.e.*, $A \times B = \{(a, b) \mid a \in A \text{ and } b \in B\}$.

**Example 28.** If $A = \{1, 2\}$ and $B = \{a, b\}$, then

$A \times B = \{(1, a), (1, b), (2, a), (2, b)\}$

$B \times A = \{(a, 1), (a, 2), (b, 1), (b, 2)\}$

$A \times A = \{(1, 1), (1, 2), (2, 1), (2, 2)\}$

$B \times B = \{(a, a), (a, b), (b, a), (b, b)\}$

Note that $n(A \times B) = 4 = n(A) \cdot n(B)$.

## Exercises

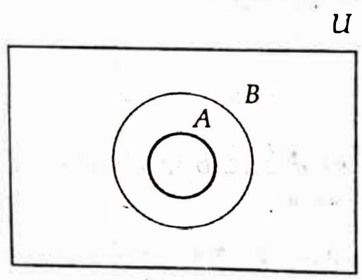
1. List the elements in the following sets.
   (a) The set of letters in the word SUBSETS.
   (b) $\{x \mid x$ is an integer and $x^2 \leq 25\}$
   (c) The set of positive integers that are less than 10.
2. Let $U = \{a, b, c, d, e, f, g, h, k\}$, $A = \{a, b, c, g\}$ $B = \{d, e, f, g\}$, $C = \{a, c, f\}$ :
   Determine the following sets :

   (a) $A \cup B$ (b) $A \cap B$ (c) $\overline{A}$
   (d) $A \cap \overline{B}$ (e) $A - B$ (f) $A \Delta B$
   (g) $\overline{A \cup B}$ (h) $\overline{A} \cap \overline{B}$ (i) $B - C$
3. Let $A = \{1, 2, 3\}$, $B = \{2, 3\}$ and $C = \{1, 5, 9\}$. Determine which of the following statements are true. Give reasons :
   (a) $2 \in A$ (b) $\{3\} \in A$ (c) $\phi \subseteq C$
   (d) $\phi \in B$ (e) $\{3\} \subseteq B$ (f) $B \subseteq A$
   (g) $B \cap A \subseteq C$ (d) $5 \in C$ (i) $A \cap C \subseteq B$
4. If $A = \{5, 2, 3\}$, find $P(A)$. What is $|A|$ and $|P(A)|$?
5. (a) If $A = \{1, 2\}$, find $P(A)$.
   (b) If $P(A)$ has 256 elements, how many elements are there in $A$ ?
6. Prove that $B - \overline{A} = B \cap A$.
7. Prove that $A - B = A \cap \overline{B}$.
8. Prove that $A - A = \phi$.
9. If $A \cup B = A \cup C$, is it necessary that $B = C$ ? Explain.
10. If $A = \{a, b\}$ and $B = \{b, 1\}$, determine the sets
    (a) $A \times B$ (b) $A \times \{1\} \times B$.
11. Determine the power sets of the following sets :
    (a) $\{a\}$ (b) $\{\{a\}\}$
    (c) $\{a, \{a\}\}$ (d) $\{\phi, \{\phi\}\}$
12. Let $A$ and $B$ be sets such that $(A \cup B) \subseteq B$ and $B \not\subseteq A$. Draw the corresponding Venn diagram.
13. Let $A, B$ and $C$ be sets such that $A \subseteq B$, $A \subseteq C$, $B \not\subseteq C$ and $C \not\subseteq B$. Draw the corresponding Venn diagram.
14. Let $A = \{1, 2, 3, 4\}$. Which of the following sets are equal to $A$ ?
    (a) $\{x \mid x$ is an integer and $x^2 < 25\}$
    (b) $\{2, 3, 4\}$
    (c) $\{4, 1, 2, 3\}$
    (d) $\{x \mid x$ is a positive integer and $x \leq 4\}$
    (e) $\{x \mid x$ is a positive rational number and $x \leq 4\}$

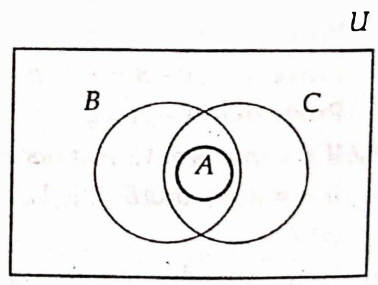
## Answers

1. (a) $\{S, U, B, E, T\}$. (b) $\{-5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5\}$.
   (c) $\{1, 2, 3, \ldots\ldots, 9\}$.

2. (a) $A \cup B = \{a, b, c, d, e, f, g\}$.
   (b) $A \cap B = \{g\}$.
   (c) $\bar{A} = \{d, e, f, h, k\}$.
   (d) $A \cap \bar{B} = \{a, b, c\}$.
   (e) $A - B = \{a, b, c\}$.
   (f) $A \Delta B = \{a, b, c, d, e, f\}$.
   (g) $\overline{A \cup B} = \{h, k\}$.
   (h) $\bar{A} \cap \bar{B} = \{h, k\}$.
   (i) $B - C = \{d, e, g\}$.
3. (a) True.
   (b) False.
   (c) True.
   (d) False.
   (e) True.
   (f) True.
   (g) False.
   (h) True.
   (i) False.
4. $P(A) = \{\phi, \{5\}, \{2\}, \{3\}, \{5, 2\}, \{2, 3\}, \{5, 3\}, A\}$
   $|A| = 3, \quad |P(A)| = 8.$
5. (a) $P(A) = \{\phi, \{1\}, \{2\}, A\}$
   (b) $|A| = 8.$
10. (a) $A \times B = \{(a, b), (a, 1), (b, b), (b, 1)\}$
    (b) $A \times \{1\} \times B = \{(a, 1, b), (a, 1, 1), (b, 1, b), (b, 1, 1)\}$.
11. (a) $\{\phi, \{a\}\}$
    (b) $\{\phi, \{\{a\}\}\}$
    (c) $\{\phi, \{a\}, \{\{a\}\}, \{a, \{a\}\}\}$
    (d) $\{\phi, \{\phi\}, \{\{\phi\}\}, \{\phi, \{\phi\}\}\}$
12.




13.




14. (c) and (d).