Lesson-5: Applications of Set Theory to Solve Business Problems

After studying this lesson, you should be able to:

- Explain the relationship of sets;
- Apply the principles of set theory in business problems.

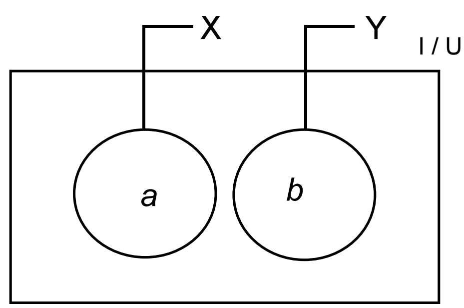
Introduction

The numbers of elements in a set X is denoted by 'n(X)'. Again, the number of elements in a set Y is expressed by n(Y). Here we derive a formula for $n(X \cup Y)$ in terms of n(X), n(Y) and $n(X \cap Y)$. First we observe that if X and Y set are disjoint, i.e., if $(X \cap Y) = Ø$, then $n(X \cup Y) = n(X) + n(Y)$

union of two joint are $n(X \cup Y) =$) $+ n(Y) - (X \cap Y)$.

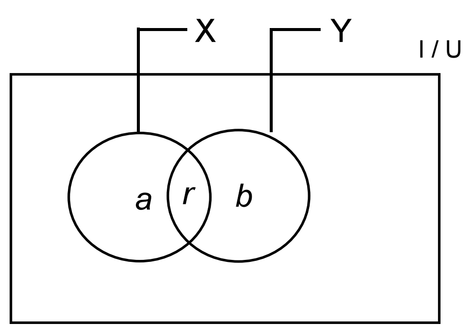
Later we take the case of the union of two finite sets which are not mutually disjoint, that is, there are some common elements between the two sets, i.e., the union of two joint sets are $n(X \cup Y) = n(X) + n(Y) - (X \cap Y)$.

The following Venn diagram presents two disjoint sets:



Here, the numbers of the elements of X set, n(X) = *a,* and Y set, n(Y) = *b*. If the sets are disjoint, then, $n(X \cup Y) = (a + b) = n(X) + n(Y)$

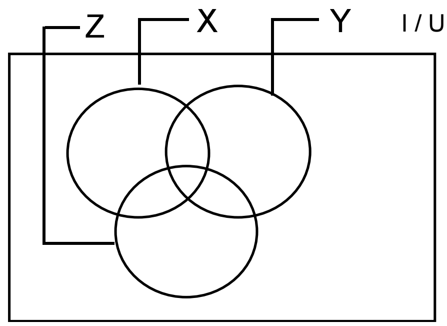
On the other hand, if 'r' is the common element of both the sets X and Y, the total elements of X set n(X) = a + r and Y set, n(Y) = b + r. i.e., $n(X \cup Y) = \{a, b, r\}$



But if $n(X \cup Y) = n(X) + n(Y)$, then the elements of $n(X \cup Y)$ are $\{(a + r) + (b+ r)\} = \{a + b + r + r\}$. Here the element 'r ' will be deleted because it is added more than one time.

i.e., $n(X \cup Y) = [\{ a+ r\} + \{ b+ r\} - r] = n(X) + n (Y) - n (X \cap Y)$

Again for the union of any three sets X, Y and Z, which are mutually disjoint, we have, $n(X \cup Y \cup Z) = n(X) + n(Y) + n(Z)$. But when these three sets are joint, then the Venn diagram would be as under:



In the above diagram, the three sets are mutually joint. For the union of these three sets the elements are, $n(X \cup Y \cup Z)$.

Now, if we consider that X is a set and $(Y \cup Z)$ is another set, then as per union of sets, wet get:

$$n(X \cup Y \cup Z) = n\,[\{X \cup (Y \cup Z)\,].$$
$$= n(X) + n\,(Y \cup Z\,) - n[\,X \cap (Y \cup Z)\,]$$

Again $n(Y \cup Z) = n(Y) + n(Z) - n(Y \cap Z)$

and $n[X \cap (Y \cup Z\,) = n[(X \cap Y) \cup (X \cap Z)]$ (using distributive law)

$$= n(X \cap Y) + n(X \cap Z) - n[(X \cap Y) \cap (X \cap Z)]$$
$$= n(X \cap Y) + n(X \cap Z) - n(X \cap Y \cap Z)$$

Therefore

$$n(X \cup Y \cup Z) = n(X) + n(Y) + n(Z) - n\,(X \cap Y) - n(Y \cap Z) - n(X \cap Z) + n(X \cap Y \cap Z)$$

Here, $n(X) \Rightarrow$ The elements of X set

$n(Y) \Rightarrow$ The elements of Y set

$n(Z) \Rightarrow$ The elements of Z set

$n(X \cap Y) \Rightarrow$ The common elements of X and Y set.

$n(Y \cap Z) \Rightarrow$ The common elements of Y and Z set.

$n(X \cap Z) \Rightarrow$ The common elements of X and Z set.

$n(X \cap Y \cap Z) \Rightarrow$ The common elements of X, Y and Z set.

Now we have got an idea regarding the operation of set theory, which can be applied in the field of business.

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The following section of this lesson contains some model applications of set theory.

Example-1:

There are 1,500 students who appeared at the CMA examination under the ICMAB. Out of these students, 450 failed in Accounting, 500 failed in Business Mathematics and 475 failed in Costing. Those who failed in both Accounting and Business Mathematics were 300, those who failed in both Business Mathematics and Costing were 320 and those who failed in both Accounting and Costing were 350. The students who failed in all the three subjects were 250.

Find (i) How many students failed in at least any one of the subjects?

(ii) How many students failed in no subjects?

(iii) How many students failed in only one subjects?

(iv) How many students failed in both Accounting and Business Mathematics only?

Solution:

Let U is the set of the students who appeared at the CMA examination and A, B and C denote the set of students who failed in Accounting, Business Mathematics and Costing respectively. Now we are given,

$n(U) = 1500 \qquad n(A \cap B) = 300$

$n(A) = 450 \qquad n(B \cap C) = 320$

$n(B) = 500 \qquad n(A \cap C) = 350$

$n(C) = 475 \qquad n(A \cap B \cap C) = 250.$

(i) Number of students who failed in at least any one of the subjects.

Now, $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A\cap B) - n(B\cap C) - n(A\cap C) + n(A\cap B\cap C)$

$= 450 + 500 + 475 - 300 - 320 - 350 + 250$

$= (1675 - 970) = 705$

Therefore, the number of students who failed at least in any one of the subjects is 705.

(ii) Number of students who failed in no subjects.

$n(A \cap B \cap C)' = n(U) - n(A \cup B \cup C)$

$= (1500 - 705) = 795$

Hence the numbers of students who failed in no subjects is 795

(iii) Number of students who failed in only one subject:

Now, $n(A \cap B' \cap C') = n(A) - n(A \cap B) - n(A\cap C) + n(A \cap B \cap C)$

$= (450 - 300 - 350 + 250) = 50$

$n(A' \cap B \cap C') = n(B) - n(A \cap B) - n(B \cap C) + n(A \cap B \cap C)$

$= (500 - 300 - 320 + 250) = 130$

$n(A' \cap B' \cap C) = n(C) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$

$= (475 - 350 - 320 + 250) = (725 - 670) = 55$

Hence the number of students who failed in only one subject is,

$n(A \cap B' \cap C') + n(A' \cap B \cap C)' + (A' \cap B' \cap C)$

$= (50 + 130 + 55) = 235.$

(iv) Number of students who failed in both Accounting and Business mathematics only:

Now, $n(A \cap B \cap C') = n(A \cap B) - n(A \cap B \cap C)$

$= (300 - 250) = 50$

So the number of students who failed in both Accounting and Business Mathematics only is 50.

Example-2:

A Survey of 600 workers in a plant indicated that 410 owned their own houses, 500 owned cars, 550 owned televisions, 410 owned cars and televisions, 340 owned cars and houses, 370 owned houses and television and 300 owned all three. Illustrate by a Venn diagram and prove that the above data is not correct. What set is empty?

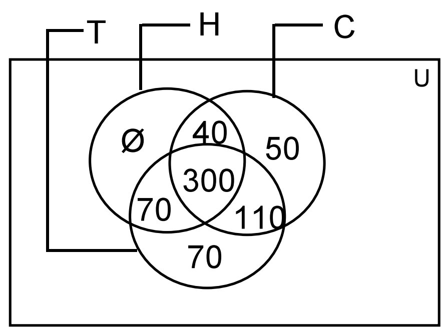
Solution:

Let U is the set of the workers who were surveyed and H, C and T are the sets of workers who owned their houses, cars and televisions respectively. Now we are given,

$n(U) = 600$; $n(H) = 410$; $n(C) = 500$; $n(T) = 550$; $n(C \cap T) = 410$;

$n(H \cap T) = 370$; $n(C \cap H) = 340$; $n(C \cap H \cap T) = 300$.

The following Venn diagram shows the result of the survey of ownership:



Hence, the total number of workers in the survey is,

$n(H \cup C \cup T) = n(H) + n(C) + n(T) - n(H \cap C) - n(C \cap T) - n(H \cap T) + n(H \cap C \cap T)$

$= (410 + 500 + 550 - 340 - 410 - 370 + 300) = 640$

This figure exceeds the total number of workers who were surveyed. Hence the given data is not correct or consistent.

In Venn diagram, $(H \cap C' \cap T')$ set is empty.

$n(H \cap C' \cap T') = n(H) - n(H \cap C) - n(H \cap T) + n(H \cap C \cap T)$

$$= (410 - 340 - 370 + 300) = (710 - 710) = Ø.$$

Example-3:

Mr. Arefin has 165 workers in process-X, 110 workers in process-Y and 97 workers in process-Z in his firm. Out of these workers, 281 workers are skilled in the activities of X and/or Y, 269 workers are skilled in the activities of Y and/or Z, 241 workers are skilled in the activities of X and/or Z. However, 44 workers are unskilled.

Accounts department of his firm has informed that the average monthly earnings of different types of workers are as follows:

Workers who are skilled in the activities of at least two processes: Tk.3,500

Workers who are skilled in the activities of any one process: Tk.2,500

Workers who are not skilled: Tk.1,500

Find the monthly amount of total earnings of all workers of the firm.

Solution:

Let U is the set of the workers who are skilled in all the processes. X, Y and Z are the sets of workers who are skilled in Process-X, Process-Y and Process–Z respectively.

We are given,

$n(X) = 165$; $n(Y) = 110$; $n(Z) = 97$; $n(X \cup Y) = 281$; $n(Y \cup Z) = 269$; $n(X \cup Z) = 241$.

The numbers of workers who are not skilled is 44.

Now, the total number of workers, $n(U) = (Px + Py + Pz) = (165+110+97) = 372$

The total number of the workers who are skilled in at least one of the processes;

$$n(X \cup Y \cup Z) = (372 - 44) = 328$$

The number of workers who are skilled in only X,

$$n(X \cup Y \cup Z) - n(Y \cup Z) = (328 - 269) = 59$$

The number of workers who are skilled in only Y,

$$n(X \cup Y \cup Z) - n(X \cup Z) = (328 - 241) = 87$$

The number of workers who are skilled in only Z,

$$n(X \cup Y \cup Z) - n(X \cup Y) = (328 - 281) = 47$$

So, the total number of workers who are skilled in only one process,

$$= (59 + 87 + 47) = 193.$$

Therefore the number of workers who are skilled in at least two processes is,

$$= (328 - 193) = 135.$$

Hence the monthly amount of total earnings of all workers of the firm is,

$$=135.(3,500) + 193.(2,500) + 44.(1,500)$$

$$= (4,72,500 + 482,500 + 66,000) = \text{Tk.}10,21,000.$$

Questions for Review

These questions are designed to help you assess how far you have understood and can apply the learning you have accomplished by answering (in written form) the following questions:

1. Prove that $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$.

2. In a survey, only 60% of 1000 questionnaires are found correct. Survey result indicates that only 42% prefer their present job responsibilities and 55% prefer their job environment. If 30% prefer both the job responsibility and job environment, how many do not prefer any one of these two?

3. In a survey of 100 families, the numbers that read the recent issues of various magazines were found to be as follows:

   Dhaka Courier 28; Readers Digest 30; Bangladesh Time 5; Courier and Readers Digest 8; Courier and Bangladesh Time 10. Readers Digest and Bangladesh Time 42; All the three magazines 3. With the help of set theory, find

   (i) How many read none of the three magazines?

   (ii) How many read Bangladesh Time as their only magazine?

   (iii) How many read Reader's Digest if and only if they read Bangladesh Time?

4. The production manager of MIC House, M. Nuruddin has 95 workers in Division-P, 80 workers in Division-Q and 120 workers in Division-R in his firm. Three different types of products are produced in these three divisions and workers in each division can easily perform the activities of that division. However, out of these workers, 25 workers can perform the activities of P and/or Q, 32 workers can perform the activities of Q and/or R, 39 workers can perform the activities of P and/or R. There are only 12 workers who can perform any activity of the three divisions. Due to change in the demand of the product of three divisions, M. Nuruddin has to shift workers from one division to another. In a 40-hour week, what would be the maximum labor hours in each division that can be worked by this work force?

Multiple Choice Questions (√ the appropriate answer)

1. In a group of persons, each one knows either Bengali or English. If 100 know Bengali, 50 know English and 30 know both, how many persons are there in the group?

   (a) 130, (b) 120, (c) 150

2. If 63% of Bangladeshi like milk and 76% like tea, how many like both?

   (a) 39%, (b) 26%, (c) 13%

3. In a group of 52 persons, 16 drink tea but not coffee and 33 drink tea. How many drink coffee but not tea?

   (a) 17, (b) 3, (c) 19

4. A dinner party is to be fixed for a group consisting of 100 persons. In this party, 50 persons do not prefer fish, 60 prefer chicken and 10 do not prefer either chicken or fish. The number of persons who prefer both fish and chicken is:

   (a) 30, (b) 10, (c) 20

5. In a class consisting of 100 students, 20 know English, 20 do not know Hindi and 10 know neither English nor Hindi. The number of students knowing both Hindi and English is:

   (a) 15, (b) 20, (c) 10