EECE 5698 – Special Topics
GNSS Signal Processing
Final Project

Spring 2023

A MATLAB-Based Investigation of GDOP for Satellite Positioning

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Summary

The report discusses Geometric Dilution of Precision (GDOP), which is a measure of the accuracy of a positioning system based on the number and geometry of satellites in view. It begins with an explanation of Dilution of Precision (DOP), followed by a detailed discussion of Geometric Dilution of Precision and its importance in determining positioning accuracy. The relationship between GDOP and the number of satellites is proven, and a MATLAB simulation is presented to observe changes in GDOP values under different geometries and varying numbers of satellites. Then we explore the relationship between GDOP and the Cramer Rao Bound of Positioning Error, as well as the relationship between GDOP and horizontal and vertical errors. The drawbacks of GDOP are discussed, including its reliance on the number and geometry of satellites and its inability to account for atmospheric effects. We conclude the report with a discussion of mitigation strategies for GDOP, such as using differential GPS and multiple frequency receivers, and various applications of GDOP in the fields of surveying and navigation. Overall, the report provides a comprehensive overview of GDOP and its relevance in determining the accuracy of positioning systems.

Introduction

The concept of Geometric Dilution of Precision (GDOP) has its roots in early studies of satellite-based navigation systems, such as the Transit Navigation System developed by the US Navy in the 1960s. The Transit system used a network of low Earth orbiting satellites to provide positioning and navigation information to military and civilian users. In 1974, a team of researchers at the University of Texas at Austin proposed the concept of Geometric Dilution of Precision as a measure of satellite geometry quality. They defined GDOP as the square root of the sum of the squares of the partial derivatives of the position equations with respect to the receiver's coordinates, weighted by the inverse of the variance of the measurement errors. The concept of GDOP was further developed in the following years, and by the early 1980s, it had become a standard metric used in satellite navigation systems, including GPS,

Geometric Dilution of Precision or GDOP is often the preferred metric to determine a good selection of satellite geometry. The accuracy of a GNSS navigation solution depends not only on the accuracy of the ranging measurements, but also on the signal geometry. The position information along a given axis obtainable from a given ranging signal is maximized when the angle between that axis and the signal line of sight is minimized. Therefore, the horizontal GNSS positioning accuracy is optimized where signals from low-elevation satellites are available and the line-of-sight vectors are evenly distributed in azimuth. Vertical accuracy is optimized where signals from higher elevation satellites are available. The effect of signal geometry on the navigation solution is quantified using the dilution of precision (DOP) concept. he uncertainty of each pseudo-range measurement, known as the user-equivalent range error (UERE), is σ_p . The DOP is then used to relate the uncertainty of various parts of the navigation solution to the pseudo-range uncertainty.

Dilution of Precision (DOP)

In contrast to the elevation distribution, which is biased toward lower heights, particularly at high latitudes, the distribution of GNSS signals about azimuth is approximately uniform through time. As a result, GNSS horizontal location accuracy is frequently superior to vertical precision. The exception is seen in mountainous and urban valleys, where a large number of low-elevation signals are blocked. The idea of dilution of precision (DOP) is used to quantify the impact of signal geometry on the navigation solution. The user-equivalent range error (UERE), also known as the uncertainty of each pseudo-range measurement, is σ_p. Then, using the DOP, various components of the navigation solution's uncertainty are related to the pseudo-range uncertainty using the following equations:-

$\sigma_N = D_N\sigma_p,\ \sigma_E = D_E\sigma_p,\ \sigma_D = D_V\sigma_p,\ \sigma_H = D_H\sigma_p,\ \sigma_P = D_P\sigma_p,\ \sigma_{rc} = D_T\sigma_p,\ \sigma_G = D_G\sigma_p$

Following is a table which denotes the uncertainty and the corresponding dilution of precision:-

Uncertainty	Symbol	Dilution of precision	Symbol
North position	σ_N	North dilution of precision	D_N
East postion	σ_E	East dilution of precision	D_E
Vertical position	σ_D	Vertical dilution of precision (VDOP)	D_V
Horizontal position	σ_H	Horizontal dilution of precision (HDOP)	D_H
Overall position	σ_P	Position dilution of precision (PDOP)	D_P
Receiver clock offset	σ_{rc}	Time dilution of precision (TDOP)	D_T
Total position and clock	σ_G	Geometric dilution of precision (GDOP)	D_G

Table 1: Uncertainty and corresponding dilution of precision.

The DOPs are defined in terms of the geometric matrix by:

$$\begin{pmatrix} D_N^2 & . & . & . \\ . & D_E^2 & . & . \\ . & . & D_V^2 & . \\ . & . & . & D_T^2 \end{pmatrix} = (G_n^T G_n)^{-1}$$

Where, G_n is the local navigation frame matrix.

And

$D_H = \sqrt{D_N^2 + D_E^2}$

$D_P = \sqrt{D_N^2 + D_E^2 + D_V^2}$

$$D_G = \sqrt{D_N^2 + D_E^2 + D_V^2 + D_{rc}^2} = \sqrt{tr[(G_n^T G_n)^{-1}]}$$

As $G_n^T G_n$ is symmetric about the diagonal, the matrix inversion is simplified.

Even though the HDOP takes into consideration two axes, VDOP is significantly greater than the HDOP, particularly in polar locations. Despite the relatively little difference, equatorial locations exhibit the best overall performance. Early GPS receivers could only track signals from four or five satellites at once, and many employed GDOP or PDOP optimization to select the satellites to track.

Geometric Dilution of Precision (GDOP)

Geometric Dilution of Precision (GDOP) is a measure of the geometric quality of a satellite constellation used for Global Navigation Satellite Systems (GNSS) such as GPS, GLONASS, Galileo, and BeiDou. The geometry of the satellites in relation to the receiver affects the GDOP. Particularly, the satellites' position, elevation, and azimuth have an impact on the GDOP. While the elevation and azimuth angles form the geometry of the satellite constellation, the positions of the satellites determine the distances between them and the receiver.

The concept of GDOP can be understood by visualizing the three-dimensional space around the receiver as a sphere, with the receiver at the center. In GNSS positioning, a minimum of four satellites is required to calculate the position of the receiver. Ideally, these four satellites should be distributed uniformly around the sphere, providing the best geometric configuration for the position calculation. However, this is not always the case in practice, as the relative position and orientation of the satellites can vary depending on the time of day, the receiver's location, and the movement of the satellites.

A good satellite geometry is one that has a lot of satellites visible, are at high altitudes, and are spaced out in azimuth. With this geometry, there are more independent observations available, and the correlation between mistakes in location estimates caused by various satellites is decreased. In contrast, a satellite geometry that is unfavorable has few satellites visible, are at low altitudes, and are crowded in azimuth. Due to this shape, there are fewer independent measurements made, and the mistakes in location estimates caused by the many satellites are more correlated.

The GDOP value is calculated based on the position and velocity of the receiver, and the position and velocity of the satellites. It is an indicator of the degree of uncertainty in the position solution due to the satellite geometry. A lower GDOP value indicates a better geometric configuration, and thus a more accurate position solution, while a higher GDOP value indicates a poorer geometric configuration, and thus a less accurate position solution.

The value of GDOP ranges from 1 to infinity, with a lower value indicating a better geometric quality. A GDOP value of 1 represents an ideal geometric configuration where the satellites are uniformly distributed around the sphere, providing the best possible position solution. On the other hand, a GDOP value of infinity indicates that the position solution is indeterminate due to a poor geometric configuration.

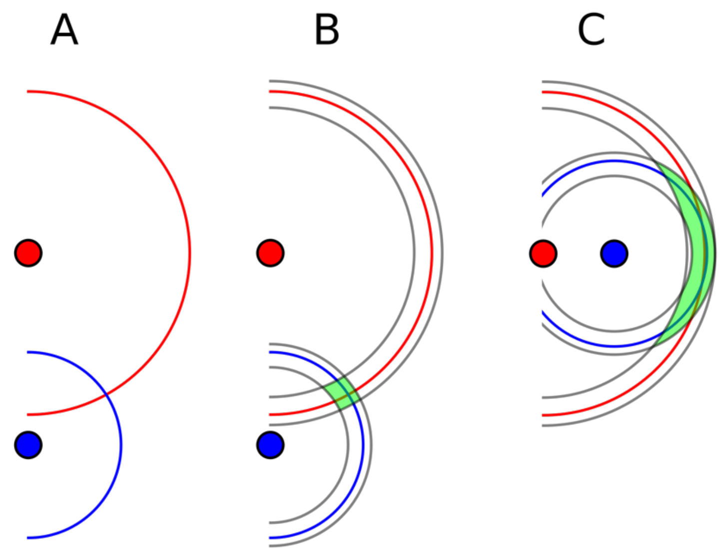

Figure 1: Understanding geometric dilution of precision [3]

A straightforward example for comprehending geometric dilution of precision (GDOP). In A, the intersection of two circles with the measured radii serves as the spot where two landmarks' distances were measured. In B, there are some error boundaries for the measurement, and their actual location might be anywhere in the area of green. The measurement error is the same in C, but the arrangement of the landmarks has significantly increased the inaccuracy in their position.

Relation between GDOP and Number of satellites

The authors in [2] have stated that increasing the number of satellites will only reduce the GDOP. The proof is as given below:-

Let,

s – Vector from the Earth's centre to a satellite

u – Vector from the Earth's centre to a user's position

r – Vector offset from the user to the satellite

then the relation between them is ,

r = s-u,

The pseudorange p_j for the jth satellite is given by ,

$$p_j = \left|\left|s_j - u\right|\right| + ct_b$$

Where , c is the speed of light and t_b is the receiver clock offset or bias from system time

The user position in three dimensions is denoted by (x_u, y_u, z_u)

Considering four satellites, the pseudorange measurements are given as,

$p_{j=}\sqrt{(x_{j-}x_u)^2 + (y_{j-}y_u)^2 + (z_{j-}z_u)^2} + ct_b$, j=1,2,3,4

Where (x_j, y_j, z_j) denotes the j[th] satellite's position in three dimensions

Defining $\widehat{p_j}$ as p_j at $(\widehat{x_u}, \widehat{y_u}, \widehat{z_u})$ we can write,

$$\Delta p_j = p_j - \widehat{p_j}$$

Writing the equation in compact matrix formulation, the value of Δp is as follows,

$$\Delta p \approx H\Delta x$$

Where, H is the visibility matrix and in general is a nx4 matrix with n≥ 4

The pseudorange errors are generally considered random as $dx \ and \ dp$ and are related as. $dp = Hdx + e$ where e is an error vector.

For i.i.d range error of variance σ_{uere}^2, the covariance is expressed in a matrix form as follows,

$$cov(dp) = K_n\sigma_{uere}^2$$

Where, K_n is a symmetric positive definite matrix and σ_{uere}^2 is the user equivalent range error variance

Let $cov(dx) = \begin{pmatrix} \sigma_{x_u}^2 & . & . & . \\ . & \sigma_{y_u}^2 & . & . \\ . & . & \sigma_{z_u}^2 & . \\ . & . & . & \sigma_{ct_b}^2 \end{pmatrix}$ where the off-diagonal entries are not important for the discussion,

The GDOP is given by,

$$GDOP = \sqrt{\sigma_{x_u}^2 + \sigma_{y_u}^2 + \sigma_{z_u}^2 + \sigma_{ct_b}^2}/\sigma_{uere}$$

We will now consider the ideal case where

$$cov(dp) = I_n\sigma_{uere}^2$$

Where I_n is the nxn identity matrix. For $n \geq 4$ we have

$$cov(dx) = (H^T H)^{-1} H^T H (H^T H)^{-1} I_n \sigma_{uere}^2$$

$$= (H^T H)^{-1} I_n \sigma_{uere}^2$$

Let $B = Diag(H^T H)^{-1} = diag(b_{11}, b_{22}, b_{33}, b_{44})$

Then we can write

$$GDOP = \sqrt{b_{11} + b_{22} + b_{33} + b_{44}}$$

$$= \sqrt{Tr(H^T H)^{-1}}$$

If we consider the number of satellites as four,

$$H_4 = (a_x \; a_y \; a_z \; I)$$

Where a_x, a_y and a_z are the unit vectors that point from the linearization point to the location of the ith satellite

If the number of satellites are more than four then the H matrix can be modified by adding row vectors Eg: If n=5,

$$H_5 = \binom{H_4}{h}$$

Here there is an assumption that $H_4$is nonsingular and h is a nonzero vector

Now clealy,

$$H_5^T H_5 = (H_4^T H_4) + h^t h$$

Let, $(H_4^T H_4) = P^T \Lambda P$

Where P is an orthogonal matrix and Λ is a diagonal matrix. Then we can write

$$P^T (H_5^T H_5) P = P^T (H_4^T H_4) P + (P^T h^t)(hP)$$

Substituting $\alpha = hP$,

$$= \Lambda + \alpha^T \alpha$$

$$= diag(\lambda_{11}, \lambda_{22}, \lambda_{33}, \lambda_{44}) + \alpha^T \alpha$$

Considering the form,

$$(D + \alpha^T \alpha)^{-1} = D^{-1} - v(\alpha^*)^T (\alpha^*)$$

Here, $\alpha = (\alpha_1\ \alpha_2\ \alpha_3\ \alpha_4)$ and $(\alpha^*) = (\frac{\alpha_1}{\lambda_{11}}\ \frac{\alpha_2}{\lambda_{22}}\ \ \frac{\alpha_1}{\lambda_{11}}\ \frac{\alpha_1}{\lambda_{11}})$ and $v = \frac{1}{1+\sum_{i=1}^{4}\frac{\alpha_i^2}{\lambda_{ii}}}$

How the above equations we can deduce that v is positive (and assumed to be finite), D is a positive definite diagonal matrix, $(\alpha^*)^T(\alpha^*)$ is a positive semidefinite matrix, $H_5^T H_5$is a positive definite matrix. From this we can infer that $D^{-1} - v(\alpha^*)^T(\alpha^*)$ is positive definite and

$Tr(D + \alpha\alpha^T)^{-1} = Tr(D^{-1})[v(\alpha^*)^T(\alpha^*)]$

Now we can write,

$$Tr(H_5^T H_5)^{-1} = Tr(D + \alpha\alpha^T)^{-1}$$

$$= Tr(D^{-1}) - Tr\ [v(\alpha^*)^T(\alpha^*)]$$

We can see here that the trace decreases

Where, $Tr(D^{-1}) = Tr[(H_4^T H_4)^{-1}]$

And

$$Tr\ [v(\alpha^*)^T(\alpha^*)] = v\sum_{i=1}^{4}(\frac{\alpha_i}{\lambda_{ii}})^2$$

$$= \frac{\sum_{i=1}^{4}(\frac{\alpha_i}{\lambda_{ii}})^2}{1 + \sum_{i=1}^{4}(\frac{\alpha_i}{\lambda_{ii}})^2\ \lambda_{ii}}$$

This proves the assertion that if we increase the number of satellites then the value of GDOP will decrease.

Simulation and Results

To observe how the satellite positions and number of satellite affects the value of GDOP, I implemented a code in MATLAB that computes GDOP for varying number of satellites. Following are the results from the simulations:

A) GDOP values for varying satellite geometries

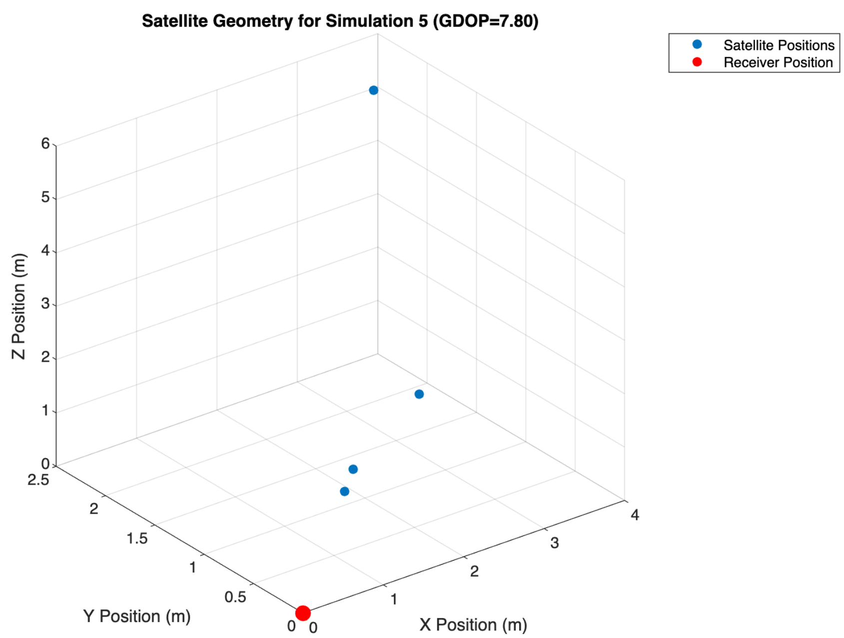

Figure 1: Bad satellite geometry

Above is a case of bad satellite geometry. Here the satellites are too close to each other and are clustered in a particular part. Therefore, this configuration results in a high GDOP value.

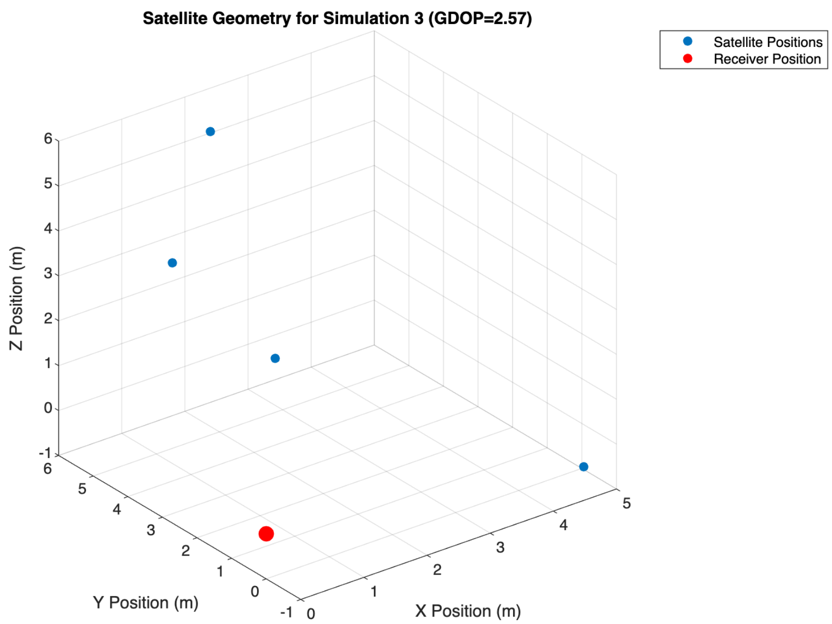

Figure 2: Good Satellite Geometry

In Case 2, the satellites are spread out moderately and are spaced out in azimuth. This results in a low GDOP value as the location estimates from the satellites have a higher correlation.

B) GDOP values for varying number of satellites

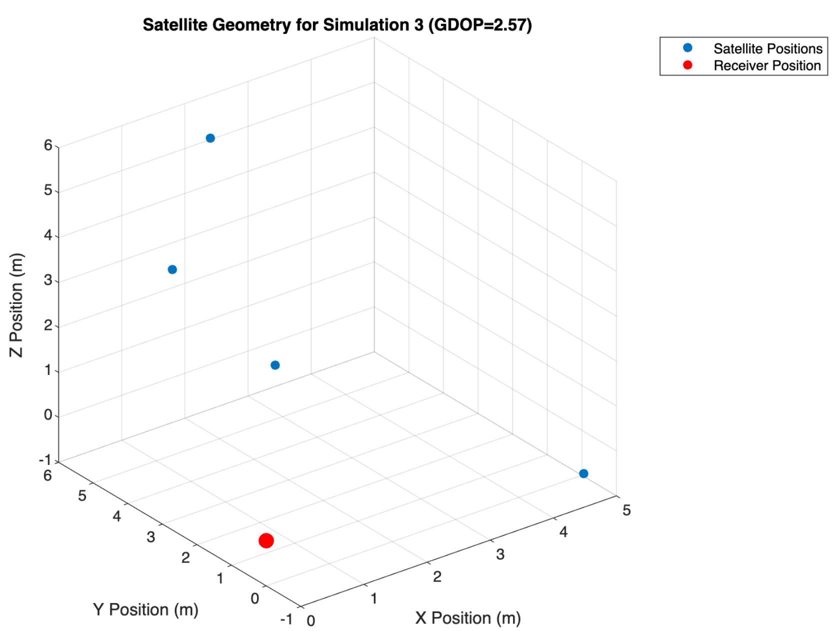

Figure 3: Number of satellites - 3

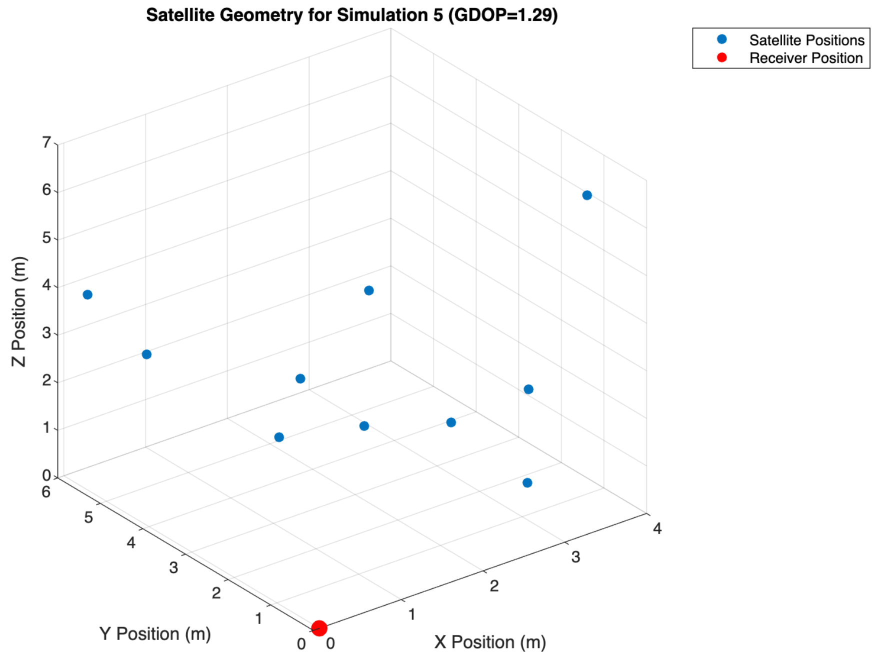

Figure 4: Number of satellites - 10

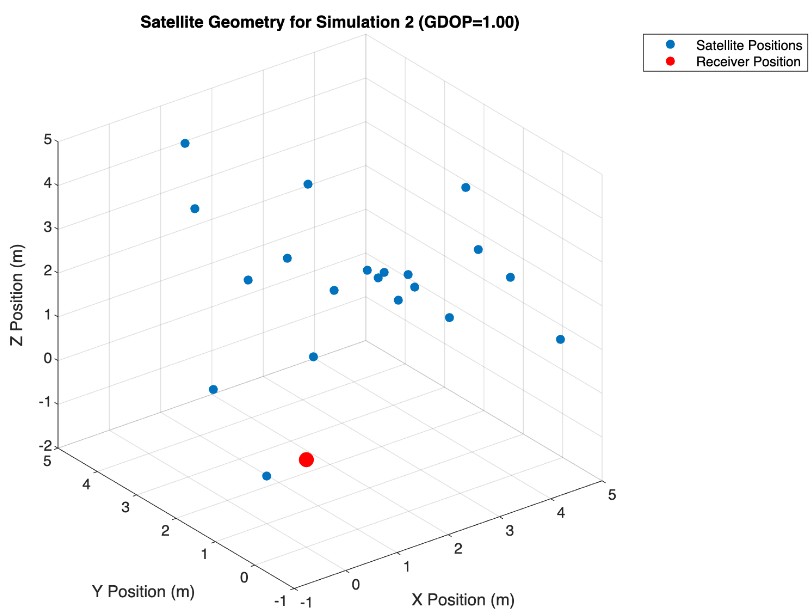

Figure 5: Number of satellites – 20

As observed from the simulation results in Case 1,2 and 3 we can infer that as we increase the number of satellites, the geometric quality increases, which results in a higher positioning accuracy. Therefore a higher number of satellite leads to a lower GDOP value

GDOP and Cramer-Rao Bound of Positioning Error

The GDOP is a measure of the geometric quality of a set of satellite measurements used in the positioning calculation. It is a factor that indicates how well the satellites are positioned relative to the receiver, and how well the receiver's position can be determined based on the available measurements. The CRB, on the other hand, is a fundamental limit on the achievable accuracy of any unbiased estimator of an unknown parameter. In the context of positioning, the CRB represents the minimum variance that can be achieved for the estimation of the receiver's position, given the available measurements.

There is a relationship between the GDOP and the CRB, as the GDOP can be used to derive a lower bound on the positioning error, which is related to the CRB. Specifically, the Cramer-Rao Bound (CRB) of positioning error is inversely proportional to the GDOP. The CRB is given by the inverse of the Fisher information matrix, which is a function of the satellite geometry, the receiver's location, and the measurement noise. The GDOP can be used to estimate the CRB by computing the diagonal elements of the inverse Fisher information matrix, which correspond to the variances of the estimated position components. The diagonal elements of the inverse Fisher information matrix are proportional to the square of the standard deviation of the estimated position error in each direction.

Therefore, a lower GDOP value indicates better positioning accuracy, as it leads to a lower CRB and thus a lower bound on the achievable positioning error. However, the actual positioning error can still be affected by other factors such as measurement noise, receiver hardware, and environmental conditions.

Relation with horizontal and vertical errors

Horizontal Errors:

Horizontal errors are the errors in the position estimate of the receiver in the horizontal plane, which includes the East-West and North-South directions. These errors are primarily caused by the following factors:

- Satellite Geometry: The geometry of the satellites in view with respect to the receiver affects the accuracy of the horizontal position estimate. When the satellites are arranged in a way that provides good geometry (i.e., low GDOP), the horizontal errors are minimized.
- Signal Quality: The quality of the received signals from the satellites also affects the accuracy of the horizontal position estimate. If the signals are weak or there is interference, the horizontal errors can be significant.
- Multipath: Multipath occurs when the GPS signals reflect off nearby objects, such as buildings or trees, before reaching the receiver. This can cause the receiver to calculate incorrect distances and lead to horizontal errors.

Vertical Errors:

Vertical errors are the errors in the position estimate of the receiver in the vertical direction, which includes the up-down direction. These errors are primarily caused by the following factors:

- Satellite Elevation Angle: The elevation angle of the satellites in view affects the accuracy of the vertical position estimate. Satellites with higher elevation angles provide better vertical accuracy, while low elevation angles can result in significant errors.
- Signal Quality: As with horizontal errors, the quality of the received signals from the satellites also affects the accuracy of the vertical position estimate.
- Atmospheric Conditions: The GPS signals are affected by the Earth's atmosphere, and this can cause errors in the vertical position estimate. For example, changes in atmospheric pressure and temperature can cause the signal to refract or bend, leading to vertical errors.

GDOP affects both horizontal and vertical errors, but it primarily affects the horizontal errors by determining the number and arrangement of the satellites in view. If the satellites are arranged in a way that provides good geometry (i.e., low GDOP), the horizontal errors are minimized. However, GDOP also indirectly affects the vertical errors, as the same satellites that provide good geometry for the horizontal position estimate also provide better vertical accuracy when they are at higher elevation angles.

In summary, the primary factors affecting horizontal errors are satellite geometry, signal quality, and multipath, while the primary factors affecting vertical errors are satellite elevation angle, signal quality, and atmospheric conditions. GDOP is a measure of the satellite's geometry that affects both horizontal and vertical errors, but it primarily affects the horizontal errors by determining the number and arrangement of the satellites in view.

Drawbacks of GDOP

While GDOP can provide valuable information about the quality of GPS signals, it also has some drawbacks:

- Limited accuracy: GDOP provides an estimate of the potential accuracy of the GPS position solution. However, it does not guarantee that the actual position solution will be accurate. Other factors, such as atmospheric conditions, multipath interference, and receiver errors, can affect the accuracy of GPS position solutions.
- Limited usefulness: GDOP is only one factor that affects GPS accuracy. Other factors, such as the number of visible satellites, the distance to the satellites, and the strength of the signals, also play a role in determining GPS accuracy.
- Limited applicability: GDOP is only applicable to GPS systems that use multiple satellites to calculate position solutions. Other positioning systems, such as those that use ground-based beacons or cellular networks, may not be affected by GDOP.
- Dynamic nature: GDOP is a dynamic factor that can change rapidly as the user's position changes or as new satellites come into view. As a result, it may be difficult to use GDOP to make accurate predictions about the quality of the GPS signal over time.

Mitigation of GDOP

A high GDOP value indicates a poor satellite geometry configuration, which can lead to reduced accuracy and precision in GPS/GNSS positioning. To mitigate this, there are several strategies that can be used:

- Wait for better satellite geometry: The position of satellites in the sky changes over time. Waiting for some time may lead to a better satellite geometry configuration, resulting in a lower GDOP.
- Move to a location with better satellite visibility: If possible, moving to a location where the satellites are distributed more evenly in the sky can lead to a better satellite geometry configuration and a lower GDOP.
- Use additional satellite constellations: Most GNSS receivers are capable of receiving signals from multiple satellite constellations, such as GPS, GLONASS, Galileo, and BeiDou. Using additional constellations can increase the number of visible satellites and improve the satellite geometry configuration, resulting in a lower GDOP.
- Use a GPS/GNSS receiver with advanced signal processing techniques: Modern GPS/GNSS receivers use advanced signal processing techniques such as carrier-phase ambiguity resolution, multipath mitigation, and signal filtering to improve positioning accuracy and reduce the effects of poor satellite geometry.
- Use differential GPS/GNSS: Differential GPS/GNSS techniques involve using a reference receiver at a known location to estimate and correct for errors in the satellite signals received by the user receiver. This technique can improve positioning accuracy and reduce the effects of poor satellite geometry.

Applications

GDOP is an important metric in various applications where satellite positioning is used to determine the location of objects on the Earth's surface. Some of the common applications of GDOP include:

- Navigation: GDOP plays a crucial role in GPS navigation systems, which are widely used in various industries such as aviation, maritime, and land transportation. In these applications, accurate position estimates are critical for ensuring safety and efficiency, and GDOP can help to assess the quality of the position estimate and improve its accuracy.

- Surveying: GDOP is also used in surveying applications to determine the precise location of land boundaries, construction sites, and other features on the Earth's surface. Surveyors use GDOP to evaluate the quality of the satellite geometry and select the best combination of satellites to achieve the desired accuracy.

- Remote sensing: In remote sensing applications such as satellite imaging and weather forecasting, GDOP is used to determine the accuracy of the position estimate and ensure that the image or data is correctly georeferenced.

- Precision agriculture: GDOP is increasingly used in precision agriculture applications, where satellite positioning is used to optimize crop production by providing accurate location information for planting, fertilizing, and harvesting.

- Disaster management: GDOP is also used in disaster management applications, such as search and rescue operations, where accurate position estimates are crucial for locating people or objects in a timely manner.

Understanding the role of GDOP in these applications is crucial for ensuring that satellite positioning is used effectively and efficiently. By assessing the quality of the position estimate and selecting the best combination of satellites, GDOP can help to improve the accuracy and reliability of satellite positioning in a wide range of applications.

Conclusion

In conclusion, Geometric Dilution of Precision (GDOP) is a key idea in assessing positioning system accuracy. The Cramer Rao Bound of Positioning Error, horizontal and vertical errors, and the link between GDOP and the number and geometry of satellites have all been covered in the study. Although there are drawbacks to utilizing GDOP as the only metric for precision, it is nevertheless a widely used metric in geodesy, navigation, and surveying. With the use of the MATLAB simulation and findings reported in this paper, it is possible to see how alternative satellite configurations and geometries affect GDOP values, which may be used to improve the precision of positioning systems. The research emphasizes the value of differential GPS and multiple frequency receivers in reducing the effects of GDOP. Overall, this report has provided a comprehensive overview of GDOP, its relationship with other accuracy measures, and its applications and limitations.

References

[1] Paul D. Groves, Principles of GNSS, Inertial, and Multisensor Integrated Navigation Systems, Artech House, 2008.

[2] R. Yarlagadda, I. Ali, N. Al-Dhahir, and J. Hershey, “GPS GDOP metric,” IEE Proceedings-radar, sonar and navigation, vol. 147, no. 5, pp. 259–264, 2000.

[3] https://en.wikipedia.org/wiki/Dilution_of_precision_(navigation)

Appendix

a) MATLAB code to compute the GDOP for varying satellite geometry and number of satellites

Code:

```
%EECE5698-ST: GNSS signal processing
% MATLAB code to compute the GDOP for varying satellite geometry and number of satellites
% Prathamesh Rege, Spring 2023

clc;
clear all;
close all;

% Specify the position of the receiver
receiver_position = [0; 0; 0];

% Specify the positions of the satellites
num_satellites = 20;
satellite_positions = 2.02E7*rand(num_satellites, 3); % random positions

% Specify the range measurement noise standard deviation
range_noise_std_dev = 5; % meters

% Specify the number of simulations to run
num_simulations = 6;

% Simulate different satellite geometries
for i = 1:num_simulations
    % Add noise to the satellite positions
    satellite_positions_noisy = satellite_positions + range_noise_std_dev *
rand(size(satellite_positions));

    % Calculate the range from the receiver to each satellite
    ranges = sqrt(sum((satellite_positions_noisy - receiver_position').^2, 2));

    % Calculate the unit vectors from the receiver to each satellite
    unit_vectors = (satellite_positions_noisy - receiver_position')./ranges;

    % Calculate the GDOP for the current geometry
    gdop = sqrt(trace(inv(unit_vectors'*unit_vectors)));

    % Plot the satellite positions
    figure;
    scatter3(satellite_positions_noisy(:,1), satellite_positions_noisy(:,2),
satellite_positions_noisy(:,3), 'filled');
    hold on;
    scatter3(receiver_position(1), receiver_position(2), receiver_position(3), 100, 'r', 'filled');
    title(sprintf('Satellite Geometry for Simulation %d (GDOP=%.2f)', i, gdop));
    xlabel('X Position (m)');
    ylabel('Y Position (m)');
    zlabel('Z Position (m)');
    legend('Satellite Positions', 'Receiver Position');
    grid on;

    % Update the positions of the satellites for the next simulation
    satellite_positions = randn(num_satellites, 3); % random positions
end
```