

## Assignment No. 4

Q.1) Explain Instantaneous Description (ID) of PDA.

current so-status of push down automata at particular part of time is known as instantaneous description (ID) of PDA.

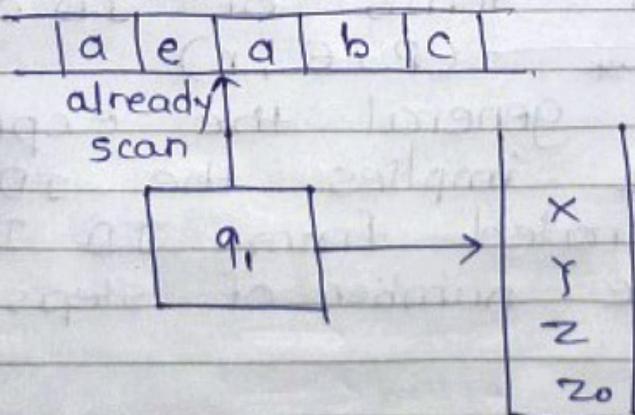
ID informs about current situation of push down automata.

The term ID is basically a tuple consisting of 3 parts.

① current state OR present state of machine.

② The IIP symbol which is being processed currently it is also known as lookahead.

③ current contents of the stack.  
consider the following sinario



By observing above diagram we came to know that machine is currently in a state  $q_1$ .

The string  $\alpha e$  is already scanned the remaining subsequent string  $abc$  is to be scanned yet.

current the stack is containing the string  $x\tau z$  based on this description, we can easily formulate the ID as  $(q_1, "abc", "x\tau z")$ .

so, the ID is set of parameters required to execute the  $\text{transit}^n$  fun.

If we consider the  $\text{transit}^n$  function as  $\delta(q, \alpha, z_0) = (p, r)$

It means that when current character  $\alpha$  (lookahead) is processed from state  $q$  when  $z_0$  is on the top of stack.

Machine changes in response, machine changes  $q$  to  $p$ , machine will load another string  $r$  on another string.

This execution of  $\text{transit}^n$  fun visualized in terms of ID as  $(q, \alpha, z_0) \xrightarrow{*} (p, \epsilon, r)$

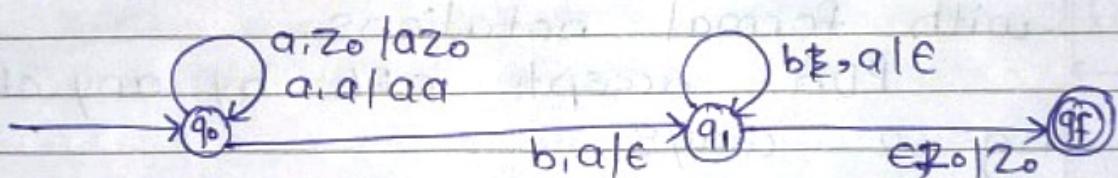
so, in general the expression  $I_1 \xrightarrow{*} I_2$  implies the ID  $I_2$  has been generated from ID  $I_1$  in some finite number of steps.

Q.2) Discuss union of two PDAs with examples?

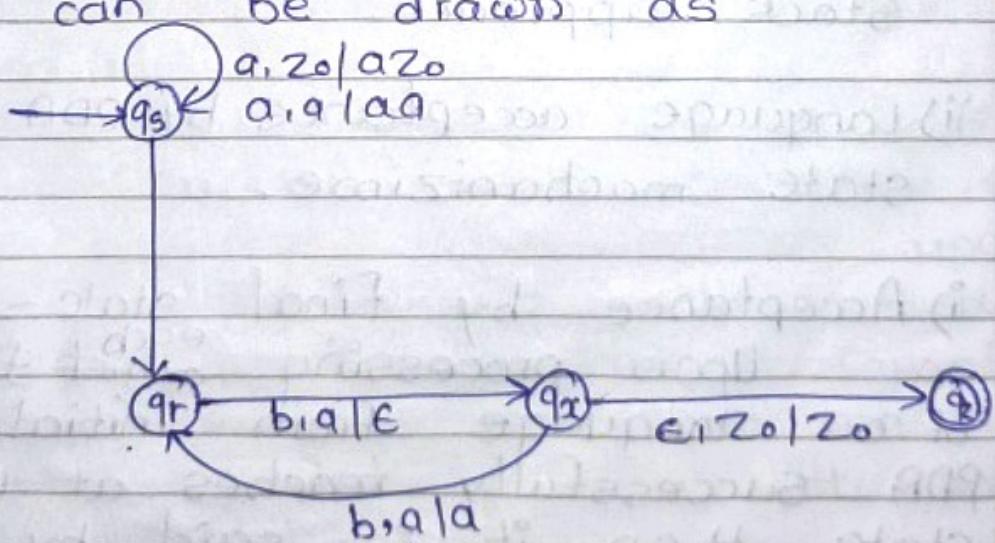
### Union of two PDAs

Just like we can perform union operation on CFG. It is quite possible to perform union operat<sup>n</sup> of PDA. consider, the language  $L_1 = \{a^n b^n \mid n \geq 1\}$ . consider another language  $L_2 = \{a^n b^{2n} \mid n \geq 1\}$

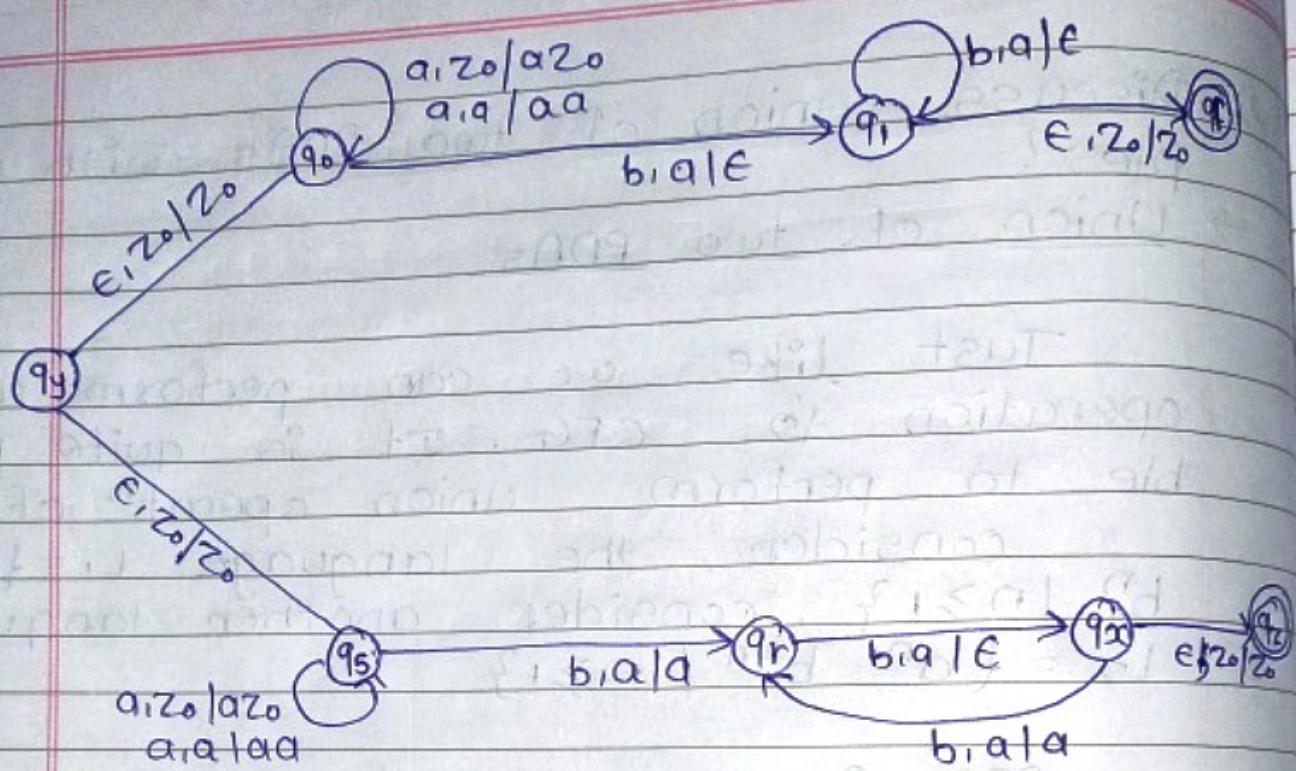
PDA  $P_1$  can be drawn for  $L_1$  as follows



Another PDA  $P_2$  to accept CFL  $L_2$  can be drawn as



Now, this two machine can be put under the union as shown below.



Q.3) Explain the language acceptance by PDA with formal notations.

→ PDA accept CFL by any of following 2 ways

i) Language acceptance by PDA by  $\epsilon$ -Stack approach.

ii) Language acceptance by PDA by final state mechanizime.

i) Acceptance by final state -

Upon processing <sup>each</sup> of every string of a language from initial state, if PDA successfully reaches at the final state then it is said by concern language has been accepted by PDA by final state mechanism.

Let,  $P$  is the PDA then  $L_f(P)$  is the language accepted by PDA with final state approach.

Formally, we can define  $L_f(P)$  as follows

$$L_f(P) = \{ w / (q_0, w, z_0) \xrightarrow{*} (q_f, \epsilon, z_0) \text{ where } q_f \in F \}$$

That means, for initial ID  $(q_0, w, z_0)$  PDA should obtain final ID  $(q_f, \epsilon, z_0)$  consisting any of the final state  $q_f$  such as  $(q_f, \epsilon, z_0)$  in same finite number of steps.

In final state approach the contents of stack are irrelevant. PDA successfully halt some final state.

## ii) Acceptance by Empty Stack -

It is another way to accept the CFL. In this approach final state don't exist.

That means  $F = \emptyset$ .

In this approach PDA mainly imphsis-es on making stack completely empty.

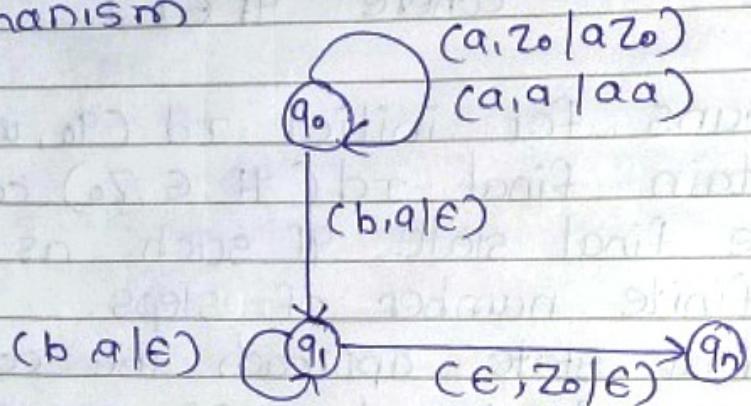
That means more prisly upon processing. upon processing each on every string of the language if PDA is completely empty then, it is said that concerned language accepted by PDA with empty stack approach.

Let  $P$  is the PDA then  $L_E(P)$  is language accepted by push-down automata by empty stack mechanism.

Formally  $L_N(P)$  can be defined as,

$$L_N(P) = \{ w \mid (q_0, w, z_0) \xrightarrow{*} (q_n, \epsilon, \epsilon) \}$$

For example, consider the language  $L = \{a^m b^m \mid m \geq 1\}$  by empty stack mechanism



Observing above transition diagram upon processing enter I/P string from initial state  $q_0$  machine is reaching to the non-final state  $q_n$  by making stack completely empty.

∴ It can be concluded that above PDA accepts CFL -  $L = \{a^m b^m \mid m \geq 1\}$  by empty stack mechanism.

- Q.4) What is NPDA? Explain the non deterministic behavior of machine with example.
- - It is another form of PDA.
  - NPDA is an extension of  $\epsilon$ -NFA.
  - If we include 1 stack along with  $\epsilon$ -NFA resulting structure is called as a NPDA.
  - NPDA shows non-deterministic behaviour.

- Every DPDA is NPDA. However, every NPDA may or may not be DPDA.
- by default PDA is NPDA.
- NPDA are used to accept NCFL or simply CFL.

consider the  $L = \{ww^R \mid w \in \{a,b\}^*\}$

Above language is consisting all even length palindrome.

let's just identify some of the string in these languages

$\{\epsilon, aa, bb, abba, \dots\}$

In contrast to odd length palindrome we easily predict the centre in even length palindromes.

we cannot locate the centre.

since, none of the fixed character resides at the centre.

so, in this case machine consider empty character  $\epsilon$  to predict the centre.

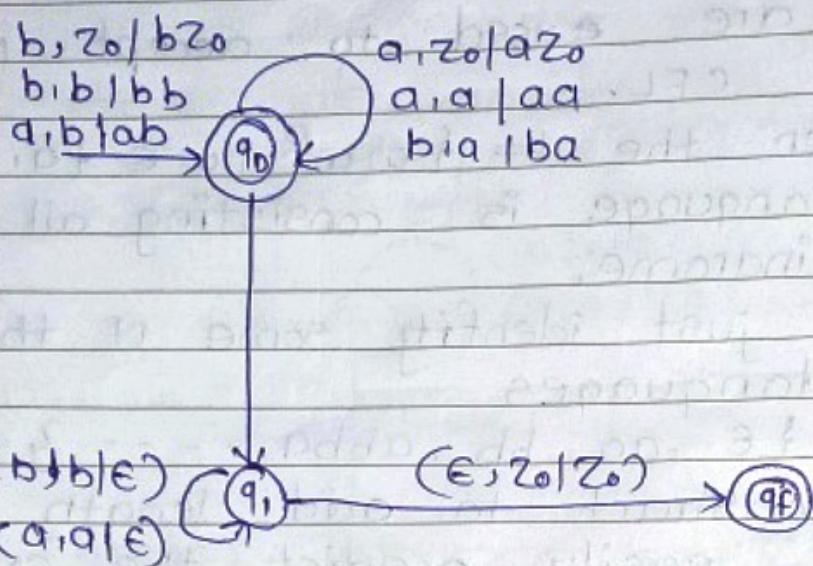
However  $\epsilon$  can appear anywhere in the I/p String.

$\therefore$  machine cannot able to find out the exact instance of  $\epsilon$  which will appear at the centre.

Due to which in each of the move machine goes on processing empty character  $\epsilon$ , as well as actual character from the I/p string.

Due to which behaviour of the string due to non-deterministic.

The particular non-deterministic machine NPDA as drawn can be sketched as,



Now, we shall demonstrate the process of I/p string abba with respect to abba.

Machine can visualize as  $\epsilon \cdot a \cdot \epsilon \cdot b$ ,  $\epsilon \cdot b \cdot \epsilon \cdot a \cdot \epsilon$ .

Machine can't locate the exact instance of  $\epsilon$  to divide I/p in two & half.

$\therefore$  Machine goes on processing  $\epsilon$  character ' $\epsilon$ ' blindly along with actual character in the I/p & this is main clause of the non-deterministic behaviour.

For initial I/O machine goes on processing empty character of  $\epsilon$  along with actual character of I/p string.

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so, actual part of acceptance for the I/p string abba w.r. to above machi-

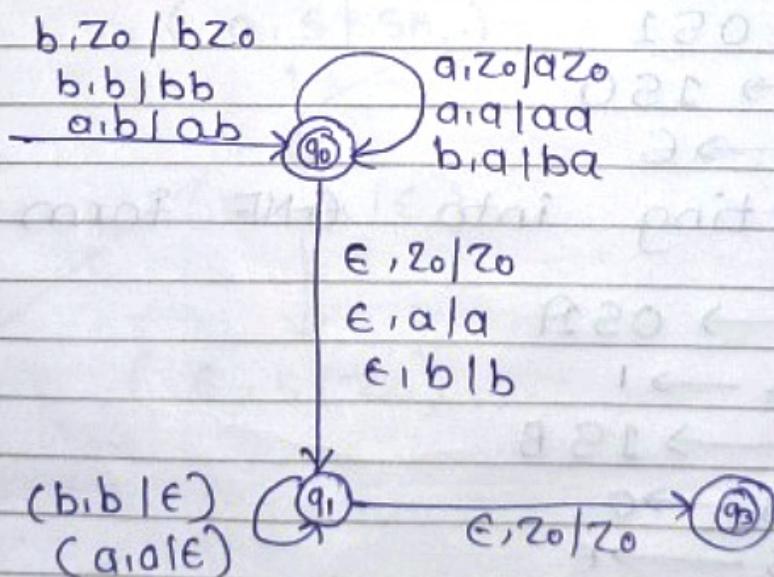
ne can be interpreted in the form of sequence of ID.

$$\begin{array}{c}
 (\text{q}_0, abba, z_0) \xleftarrow{} (\text{q}_0, bba, a z_0) \xleftarrow{} (\text{q}_0, ba, ba z_0) \\
 \xleftarrow{} (\text{q}_1, ba, ba z_0) \xleftarrow{} (\text{q}_1, a, a z_0) \xleftarrow{} (\text{q}_1, \epsilon, z_0) \\
 \xleftarrow{} (\text{q}_F, \epsilon, z_0)
 \end{array}$$

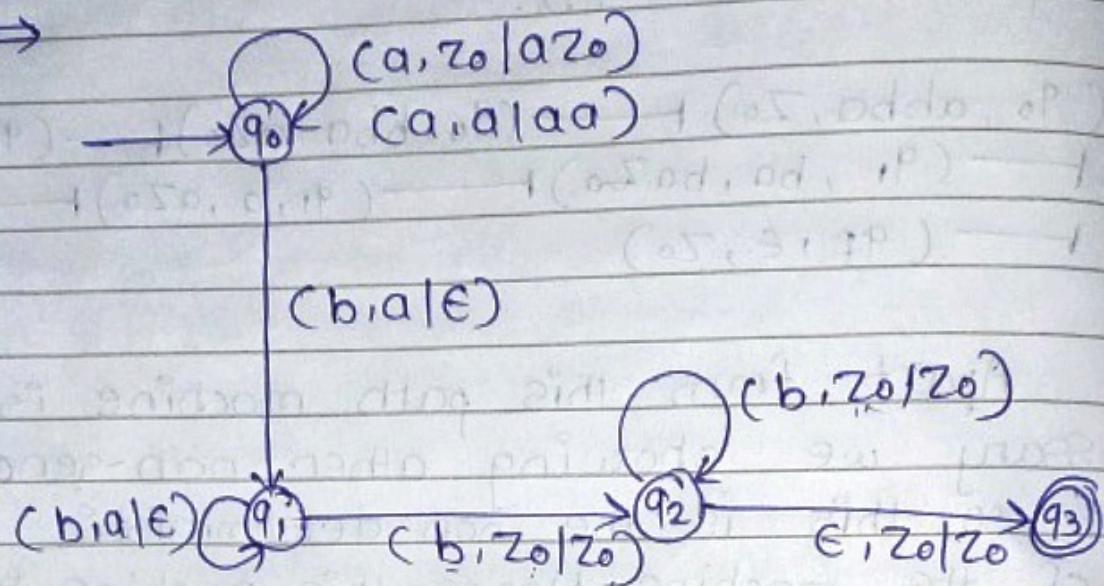
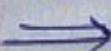
Apart from this path machine is unnecessary we showing other non-sense moves. so, this is the non-deterministic behaviour of the machine. Hence this machine is known as NPDA.

Q.5) construct the PDA for:

$$\text{i)} L = \{ww^R \mid w \in \{a, b\}^*\}$$



$$iii) L = \{a^m b^{n-m} \mid m < n\}$$



Q. 6) Convert following grammar in PDA

$$S \rightarrow OS1 \mid 1SO \mid \epsilon$$

And simulate the derivation of IP string 0101 with above PDA.

→  $S \rightarrow OS1$

$S \rightarrow 1SO$

$S \rightarrow \epsilon$

Converting into GNF form

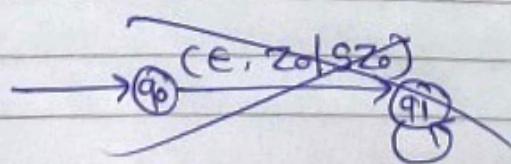
$$S \rightarrow OSA$$

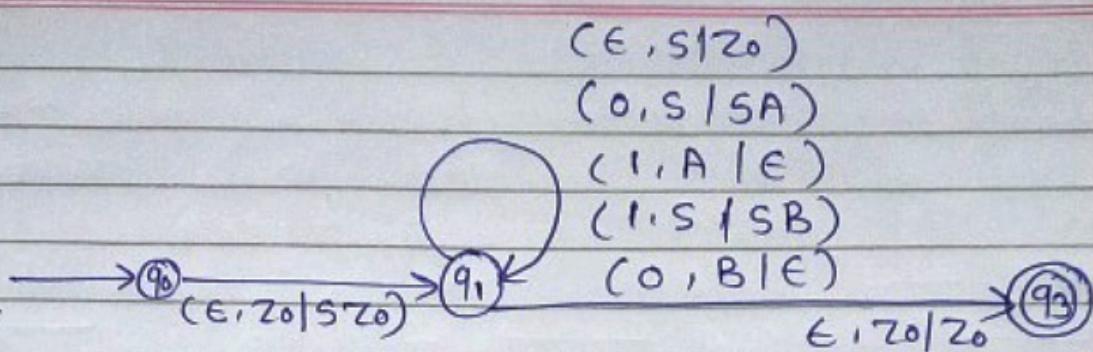
$$A \rightarrow I$$

$$S \rightarrow 1SB$$

$$B \rightarrow O$$

$$S \rightarrow \epsilon$$





Input string 0101 -

$(\epsilon, z_0 | S | z_0)$

$\epsilon \quad \boxed{S \\ z_0}$

$(0, S | SA)$

$0 \quad \boxed{A \\ S \\ z_0}$

$(1, A | \epsilon)$

$1 \quad \boxed{S \\ z_0}$

$(0, S | SA)$

$0 \quad \boxed{A \\ S \\ z_0}$

$(1, A | \epsilon)$

$1 \quad \boxed{S \\ z_0}$

$(\epsilon, S | z_0)$

$\epsilon \quad \boxed{z_0}$

Q

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