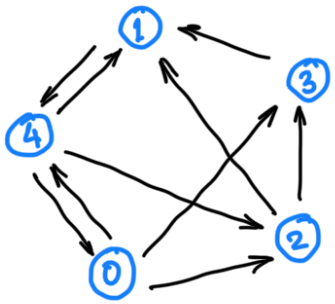
SI 671 HW 3

1. Page Rank Computation [10 Points]



	0	1	2	3	4
in	1	3	2	2	2
out	3	1	2	1	3

$$y_0 = \frac{y_4}{3}$$

$$y_1 = \frac{y_2}{2} + \frac{y_3}{1} + \frac{y_4}{3}$$

$$y_2 = \frac{y_0}{3} + \frac{y_4}{3}$$

$$y_3 = \frac{y_0}{3} + \frac{y_2}{2}$$

$$y_4 = \frac{y_0}{3} + \frac{y_1}{1}$$

$$\begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & \frac{1}{2} & 1 & \frac{1}{3} \\ \frac{1}{3} & 0 & 0 & 0 & \frac{1}{3} \\ \frac{1}{3} & 0 & \frac{1}{2} & 0 & 0 \\ \frac{1}{3} & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} y_0 \\ y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}$$

Iteration 0:

$$\begin{bmatrix} r_0 \\ r_1 \\ r_2 \\ r_3 \\ r_4 \end{bmatrix} = \begin{bmatrix} 1/5 \\ 1/5 \\ 1/5 \\ 1/5 \\ 1/5 \end{bmatrix}$$

Iteration 1:

$$\begin{bmatrix} r_0 \\ r_1 \\ r_2 \\ r_3 \\ r_4 \end{bmatrix} = \begin{bmatrix} 1/15 \\ \frac{1}{5}\left(\frac{1}{2}+1+\frac{1}{3}\right) \\ \frac{1}{5}\left(\frac{2}{3}\right) \\ \frac{1}{5}\left(\frac{1}{3}+\frac{1}{2}\right) \\ \frac{1}{5}\left(\frac{1}{3}+1\right) \end{bmatrix} = \begin{bmatrix} 2/30 \\ 11/30 \\ 4/30 \\ 5/30 \\ 8/30 \end{bmatrix} = \begin{bmatrix} 1/15 \\ 11/30 \\ 2/15 \\ 1/6 \\ 4/15 \end{bmatrix}$$

Iteration 2:

$$\begin{bmatrix} r_0 \\ r_1 \\ r_2 \\ r_3 \\ r_4 \end{bmatrix} = \begin{bmatrix} \frac{1}{3}\cdot\frac{4}{15} \\ \frac{1}{2}\cdot\frac{2}{15}+\frac{1}{6}+\frac{1}{3}\cdot\frac{4}{15} \\ \frac{1}{3}\cdot\frac{1}{15}+\frac{1}{3}\cdot\frac{4}{15} \\ \frac{1}{3}\cdot\frac{1}{15}+\frac{1}{2}\cdot\frac{2}{15} \\ \frac{1}{3}\cdot\frac{1}{15}+\frac{11}{30} \end{bmatrix} = \begin{bmatrix} \frac{4}{45} \\ \frac{6}{90}+\frac{15}{90}+\frac{8}{90} \\ \frac{5}{45} \\ \frac{1}{45}+\frac{3}{45} \\ \frac{1}{45}+\frac{11}{30} \end{bmatrix} = \begin{bmatrix} \frac{8}{90} \\ \frac{29}{90} \\ \frac{10}{90} \\ \frac{8}{90} \\ \frac{35}{90} \end{bmatrix} = \begin{bmatrix} \frac{4}{45} \\ \frac{29}{90} \\ \frac{1}{9} \\ \frac{4}{45} \\ \frac{7}{18} \end{bmatrix}$$

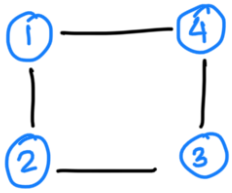
After 2 iterations Node 4: has highest Page Rank

Page Rank (Node 4) = 7/18

2. Network Density Computation [10 Points]

$$\text{Network Density} = \frac{\text{Actual Connections}}{\text{Potential Connections}}$$

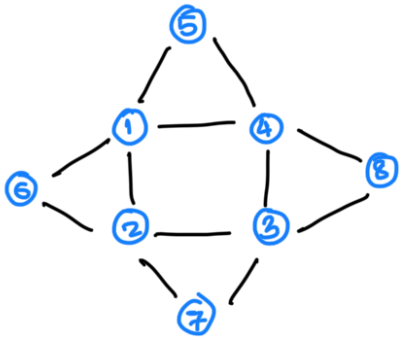
$$\text{Potential Connections} = \frac{n(n-1)}{2}$$



Actual conn = 4

Potential conn $= \frac{4(3)}{2} = 6$

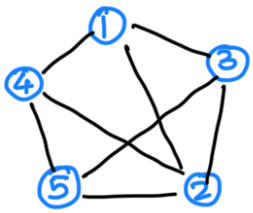
Network density $= \frac{2}{3} = 66.67\%$



Actual conn = 12

Potential conn $= \frac{8(7)}{2} = 28$

Network density $= \frac{3}{7} = 42.85\%$



Actual conn = 8

Potential conn $= \frac{5(4)}{2} = 10$

Network density $= \frac{4}{5} = 80\%$