Practical - 2

Implement Union, Intersection, Complement and difference operations on fuzzy sets. Also create ~~for~~ fuzzy relations by cartesian products of any two fuzzy sets and perform max-min composition on any two fuzzy relations.

→ What are fuzzy sets?

- Unlike clasical sets (where elements are either in or out i.e 0 or 1), fuzzy sets allow degree of membership.
- Each element has a membership value $\mu(x)$ in $[0, 1]$.
  - 0 means no membership ~~for~~
  - 1 means full membership.
  - Any value in between means partial membership.

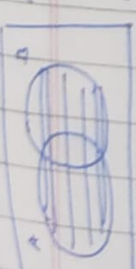
" How much it belongs seem is called membership value.

- Eg: In a fuzzy set of "Tall people":
  - A person of height 5'2" might have $\mu = 0.2$.
  - A person of height 6'1" might have $\mu = 0.9$.

Operations on fuzzy sets:

1) Union $(A \cup B)$:-

- $\mu_{A \cup B}(x) = \max(\mu A(x), \mu B(x))$ (set → element)
- An element belons to union if it belongs to either A or B.
- Used when we want to combine possibilities.



All elements from A & B
eg $A = \{1, 2\}$ $B = \{2, 3\}$
⇒ $A \cup B = \{1, 2, 3\}$

2) Intersection $(A \cap B)$:-

- $\mu_{A \cap B}(x) = \min(\mu_A(x), \mu_B(x))$.
- An element is in the intersection only if it's present in both, with the least confidence.
- Same elements from both set, i.e elements that are present both in A & B.
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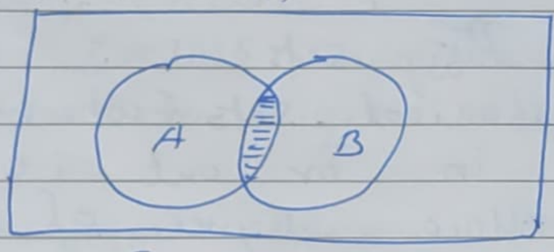

eg $A = \{1, 2\}$, $B = \{2, 3\}$
$A \cap B) = \{2\}$

- Used when we want common agreement.

3) Complement $(\sim A)$:-

- $\mu_{\sim A}(x) = 1 - \mu_A(x)$
- Measures how much a element is not in the set.
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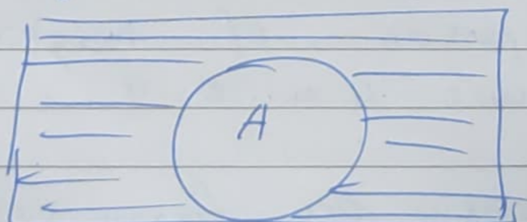
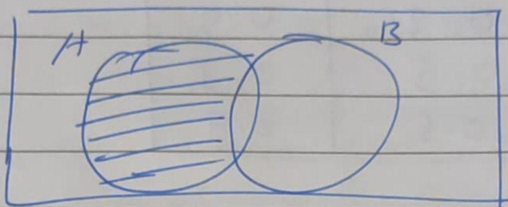


4) Difference set $(A - B)$:-

- $\mu_{A-B}(x) = (\mu_A(x), 1 - \mu_B(x))$
- Part of A thats not in B.
- Eg:- $A = \{1, 2, 3\}$ & $B = \{3, 5\}$ then

$A - B = \{1, 2\}$

- Note $A - B = A \cap B^c$



→ Fuzzy Relations

Cartesian product is a new set formed by taking all possible ordered pairs where the first element of each pair comes from set A & second from B.

1) Cartesian Product of fuzzy sets $(A \times B)$

- Just like crisp set theory, we can take the cartesian product of two fuzzy sets A and B, but instead of creating just pairs (a, b) we also assign a membership value to each pair.
- This creates a fuzzy relation that connects elements of A to elements of B based on how strongly they are present in their own sets.
- Formula:
  For two fuzzy sets A and B:
  - $A = \{a_1 : \mu A(a_1),\ a_2 : \mu A(a_2), \dots\}$
  - $B = \{b_1 : \mu B(b_1),\ b_2 : \mu B(b_2), \dots\}$
- Then the cartesian product $A \times B$ is:
  Each pair (a, b) will have
  $$\mu_{A \times B}(a, b) = \min(\mu_A(a), \mu_B(b))$$
- Eg:- $A = \{x1 : 0.2,\ x2 : 0.5\}$
  $B = \{y1 : 0.3,\ y2 : 0.9\}$

Then the cartesian product $A \times B$ is:

Pair	$\mu A(x)$	$\mu B(y)$	$\mu(x,y) = \min(\mu A(x), \mu B(y))$
(x_1, y_1)	0.2	0.3	0.2
(x_1, y_2)	0.2	0.9	0.2
(x_2, y_1)	0.5	0.3	0.3
(x_2, y_2)	0.5	0.9	0.5

So this becomes a fuzzy relation R from A to B.

- Why do we use min() in ~~the~~ cartesian product of fuzzy sets?
  - A fuzzy relation is just a way to say:
    - How strong is the pair?
    - This is done using cartesian product.
    - We match every student in A with every student in B.

Eg: set A : how much student like math?

John	0.9
Mary	0.2

& Set B : how much student like science?

John	0.5
Mary	0.7

~~Now we take~~ cartesian product.

Set A: Liking for math

$A = \{$"John" 0.9, "Mary" : 0.2$\}$

Set B: Liking for science

$B = \{$"John" 0.5, "Mary" : 0.7$\}$

Now we take Cartesian product:

- Pair (John, John) → min (0.9, 0.5) = 0.5
- Pair (John, Mary) → min (0.9, 0.7) = 0.7
- Pair (Mary, John) → min(0.2, 0.5) = 0.2
- Pair (Mary, Mary) → min(0.2, 0.7) = 0.2

- Min is used because in fuzzy logic:
The relation(a, b) can be as strong as the weakest of ~~these~~ two elements.

∴ Eg Two friend carrying a heavy box. One is strong & other is weak. The box can only be lifted as much as the weaker person can handle.

2) Min-max Composition ($R \circ S$)

- Combining two fuzzy realtions R (from A to B) and S(from B to C) to form a relation from A to C.
- For each pair (a, c):-
$\mu(a, c) = \max(\min(R(a, b), S(b, c)))$ for all b.

- Used in control systems, recommendation logic.
- "How strong does A relate to C through B"?

→ For each intermediate b, take $\min(R(a, b), S(b, c))$ → weakest link
→ Then take the max of all those paths → best possible connection.

Eg:- • R (A → B)
$R = \{(x_1, y_1) = 0.2, (x_1, y_2) = 0.2, (x_2, y_1) = 0.3, (x_2, y_2) = 0.5\}$

• S (B → C)
$S = \{(y_1, z_1) = 0.4, (y_1, z_2) = 0.6, (y_2, z_1) = 0.7, (y_2, z_2) = 0.9\}$

• Lets compute ~~A~~ $R \circ S (A \to C)$:
we want to compute each (x, z):

• (x_1, z_1)

$= \max(\min(R(x_1, y_1), S(y_1, z_1)), \min(R(x_1, y_2), S(y_2, z_2)))$

$= \max(\min(0.2, 0.4), \min(0.2, 0.7))$

$= \max(0.2, 0.2)$

$= \underline{0.2}$

• $(x_1, z_2) = 0.2$

• $(x_2, z_1) = \underline{0.5}$

• $(x_2, z_2) = \underline{0.5}$

→ Real Life Applications:-

1) Smart traffic control:-
- Sensors detect traffic density in fuzzy terms:
  'Heavy', 'Moderate', 'Low' → fuzzy values.
- Controllers use fuzzy union/intersection to decide when to turn signals green/red.

2) Fuzzy control system (temp hinch fan speed High, low → low).

3) Recommendation Systems
- Union/Intersection helps comine similar & items or users.
- User preferences and items features are fuzzy.

~~Practical-3~~

1) Fuzzy Union

In fuzzy set union means taking the maximum membership value between two sets for each element.

2) Fuzzy intersection

In fuzzy set, intersection means taking the minimum membership value between two sets for each element.

3) Fuzzy complement

In fuzzy set, complement means how much an element is NOT in the set. It is calculated as 1 − membership value.

Eg:- $A = \{'x_1' : 0.2, 'x_2' : 0.5, 'x_3' \ 0.7\}$

∴ for $x1 : 1 - 0.2 = 0.8$

$x_2 : 1 - 0.5 = 0.5$

$x_3 : 1 - 0.7 = 0.7$

4) Fuzzy ~~Di~~ difference

Difference (A − B) in fuzzy set means, take elements from A but remove membership part contributed by B.