An Adapted Cat and Mouse Based Optimizer for Solving the Capacitated Vehicle Routing Problem

Duvvuri Prathamesh, Neeraj Ramprasad, SN Omkar

ICOA 2025

September 14, 2025

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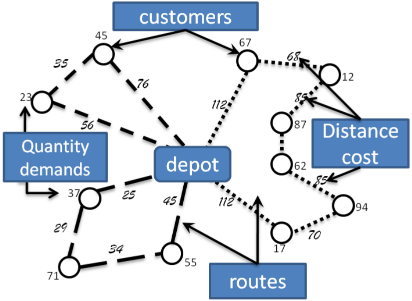
Introduction

Capacitated Vehicle Routing Problem

1. CVRP : Extends the Traveling Salesman Problem to multiple vehicles with capacity limits, starting and ending at a single depot.
2. The objective is to design minimum-cost delivery routes that serve all customers while respecting vehicle capacities.
3. The main challenge is that CVRP is NP-hard; exact methods struggle with large instances, motivating heuristic and metaheuristic approaches.



Problem Description and Mathematical Modelling

Capacitated Vehicle Routing Problem (CVRP):

- Set of customers with known demand, served by a fleet of identical vehicles.
- Each vehicle starts/ends at a depot and cannot exceed capacity Q.
- Objective: minimize the **total travel cost (distance)**.

Mathematical Formulation:

$$\text{Minimize: } Z = \sum_{i=0}^{n} \sum_{j=0, j \neq i}^{n} \sum_{k=1}^{m} c_{ij} x_{ij}^{k}$$

$$\text{s.t. } \sum_{k=1}^{m} \sum_{i=0, i \neq j}^{n} x_{ij}^{k} = 1, \quad \forall j \text{ (each customer visited once)}$$

$$\sum_{j=1}^{n} \sum_{i=0, i \neq j}^{n} q_j x_{ij}^{k} \leq Q, \quad \forall k \text{ (capacity constraint)}$$

$$\sum_{i \in S} \sum_{j \in S, i \neq j} x_{ij}^{k} \leq |S| - 1, \quad \forall S \subset V \text{ (sub-tour elimination)}$$

Literature Review

Literature Review – Metaheuristics for CVRP

- Metaheuristics gained prominence for CVRP since the 1990s.
- Population-based, nature-inspired algorithms dominate the field.
- Key approaches:
  - PSO (1995/2009) – Models bird flocking, strong exploration, but weak diversity later
  - ACO (1999/2000) – inspired by ant foraging, effective with local search, but has poor scalability
  - Hybrids (2010s–2020s) - Firefly, Grey Wolf Optimizer, Enhanced Artificial Bee Colony

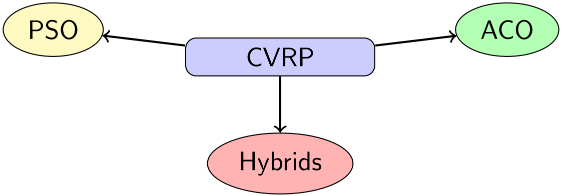


Figure: Major metaheuristic methodologies applied to CVRP

Cat and Mouse Optimization (CMBO) and Challenges for CVRP

What is CMBO?

- Population-based metaheuristic inspired by predator–prey dynamics.
- **Cats:** explore the search space by chasing solutions.
- **Mice:** exploit by escaping to safe havens (local refinement).
- Iterative process: update cats and mice $\rightarrow$ evaluate $\rightarrow$ best solution.

Challenges for CVRP:

- Originally designed for *continuous domains*.
- CVRP is *discrete* + *combinatorial* $\rightarrow$ route construction and vehicle assignment.
- Struggles with feasibility (capacity constraints) and avoiding sub-tours.

Improvements Required:

- Discrete solution representation (routes + vehicle loads).
- Feasibility-ensuring decoding mechanism.
- Problem-specific local search (e.g., 3-opt, route exchanges).
- Mechanisms to maintain diversity and escape local optima.

Adapted Cat and Mouse Based Optimizer

Population Initialization & Customer Clustering

Population Initialization

- Population size:

$$n_{population} = \max\left(50,\ \min(100,\ 2 \times n_{customers})\right)$$

- Each solution encoded as a vector:
  - **Customer Priorities** $\rightarrow$ order of service.
  - **Vehicle Reference Points** $\rightarrow$ coordinates guiding route assignment.
- Random uniform initialization ensures diverse starting solutions.

Customer Clustering

- Customers grouped with **K-Means**, number of clusters:

$$K = \lceil \sqrt{n} \rceil$$

- Reduces combinatorial complexity by focusing on sub-routes.
- **Dynamic update:** Clusters adjusted every 50 iterations to avoid stagnation.

Fitness Evaluation & Sorting

Fitness Function

- Measures quality of each solution through 2 parameters: total route distance for a solution, penalties for each unserved customer in a solution.

$$Fitness = Total\_distance + 10000 \times (\text{unserved customers})$$

- Ensures all customers are served within vehicle capacity.

Sorting into Roles

- Rank population by fitness (best $\rightarrow$ worst).
- **Mice (Top 50%)**: high-quality solutions, used for exploitation.
- **Cats (Bottom 50%)**: weaker solutions, explore new areas.

Fitness Sharing

- Adjusts fitness to penalize similar solutions within clusters.
- Maintains **diversity** of solutions.
- Prevents premature convergence to poor local optima.

F. Solution Update and Integration & G. Iteration Transition

Solution Update (Core Phase):

- **Update Cats:** Cluster-based refinement of subroutes.

$$s_{\text{new}}[l] = s_{\text{cat}}[l] + r \cdot (s_{\text{mouse}}[l] - l_{\text{rand}} \cdot s_{\text{cat}}[l]) \cdot \left(1 + \alpha \frac{t}{n_{\text{iter}}}\right)$$

  - $r \in [0, 1]$, $l_{\text{rand}} \in \{1, 2\}$, $\alpha = 0.5$
  - Random swaps & temperature-based acceptance $\Rightarrow$ maintain diversity

- **Solution Integration:** Aggregate cats from clusters $\Rightarrow$ unified population:

$$S = S_{\text{mice}} \cup S_{\text{cats}}$$

  Preserves cluster-optimized subroutes while retaining elite mice.

- **Update Mice:** High-quality solutions refined:

$$s_{\text{new}} = s_{\text{mouse}} + r \cdot (pop[l] - l_{\text{rand}} \cdot s_{\text{mouse}}) \cdot sign$$

  Accepted only if fitness improves; mutations (prob. p_m) avoid stagnation.

H. Local Search Operators

Variable Neighborhood Descent (VND)

- Refines routes using 6 operators.
- Ensures thorough neighborhood search
- Accepted only if distance reduces and capacity remains feasible.
- **Intra-route:**
  - Insert: relocate a customer.
  - Swap: exchange two customers.
  - Reverse: flip a segment.
  - 3-Opt: three-edge exchange.
- **Inter-route:**
  - Insert: move customer to another route.
  - Swap: exchange across routes.

Destroy-and-Repair

- **Destroy:** Remove up to $k = \max(5, \lfloor n/10 \rfloor)$ customers, guided by correlation:

$$R_{ij} = \frac{1}{(d_{ij}/\text{maxd}) + V_{ij} + 10^{-6}}$$

where d_{ij} = distance, maxd = max pairwise distance, $V_{ij} = 0$ (same route), 1 (different routes). Targets closely located customers across routes for disruption.

- **Repair:** Reinsert customers via cheapest insertion heuristic, ensuring feasibility. If infeasible, retain original; adjust vehicle count if needed.

Results and Analysis

Results: Evaluation Setup

- **Parameter Settings:**
  - Iterations: $n_{iterations} = 1500$, Restart limit: $n_{restart} = 100$, VND calls: $n_{vnd} = 40$.
  - Control parameters: $\alpha = 0.5$, $i_{restart} = 5$.
  - Problem-specific: Capacity, number of vehicles (m), and customers taken directly from CVRPLIB.
- **Implementation Environment:** Python 3.13 in VS Code, Windows 11 (64-bit), Intel Core i7-10510U @ 1.80 GHz, 8 GB RAM.
- **Evaluation Protocol:**
  - Each instance tested across 10 independent runs.
  - Statistical measures: *Best*, *Worst*, and *Average* objective function values.
  - Quality metric: Percentage Deviation (*Dev.%*) from Best-Known Solutions (BKS).
- **Comparison:** Compared ACMBO performance with the best solutions obtained by ACO and PSO over 10 runs across diverse CVRP instances, highlighting ACMBO's competitiveness against other state-of-the-art algorithms.

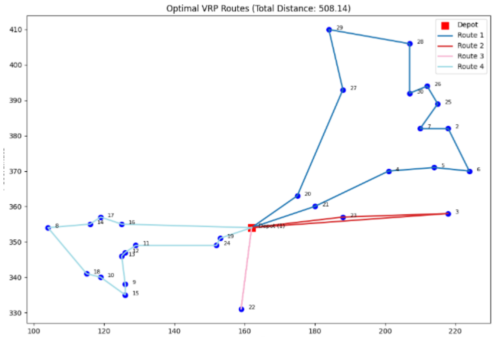


Figure: E-n30-k4 with 30 customers

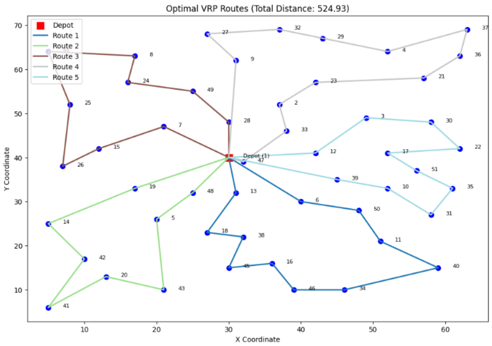


Figure: E-n51-k5 with 51 customers

In both these cases, ACMBO achieves equal or better solutions than the best-known solutions (BKS) from CVRPLIB, demonstrating that the algorithm performs very effectively for instances with uniformly distributed customers.

Results: Large-scale instances with Non-uniformly distributed Customers

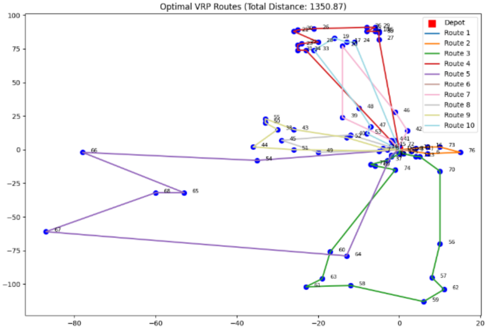


Figure: Tai75b with 75 customers

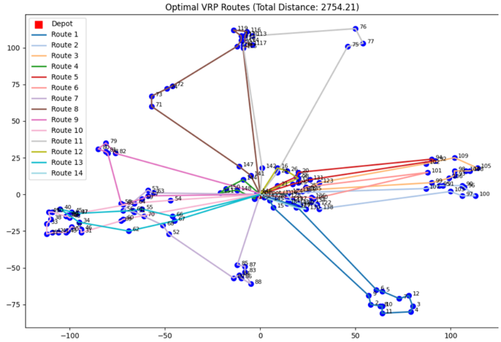


Figure: Tai150b with 150 customers

For large-scale instances with non-uniformly distributed customers, the graphs demonstrate that the ACMBO algorithm consistently achieves solutions within a 1% deviation from the best-known solutions.

Result Analysis of ACMBO

- Across 36 benchmark instances, ACMBO obtains the **optimal BKS** in 7 cases and achieves **improved solutions** in 2 cases with deviations of –2.49% and –4.87%.
- For **small-scale instances** ($n < 25$), deviations remain at 0.00%, indicating consistent recovery of benchmark values.
- For **medium-sized problems** (30–70 customers), deviations are mostly contained within 0.2–2.0%, with a few cases extending to 3.52%.
- For **large-scale instances** ($n \geq 100$), deviations range between 1.0% and 2.61%, confirming stable performance across increasing instance size.
- In comparative evaluation over 35 instances, ACMBO produces the **best or equal-best solutions** in 29 cases, corresponding to approximately 83% coverage.
- Relative to ACO, ACMBO consistently yields **lower objective values**, with notable gaps such as 92 units in B-n67-k10 and 442.77 units in Tai75a.
- Relative to PSO, ACMBO outperforms or matches in 32 cases; PSO records marginally better solutions in only 3 cases, with differences limited to 3–5 units.

Conclusion

Conclusion

- The ACMBO algorithm demonstrates strong effectiveness in solving the Capacitated Vehicle Routing Problem (CVRP), consistently delivering **high-quality and near-optimal solutions**.
- It successfully **achieves or surpasses best-known solutions (BKS)** in multiple benchmark instances, while maintaining **low deviations** across small, medium, and large problem sizes.
- In non-uniform clustered customer scenarios (Tai series), ACMBO outperforms other algorithms significantly, demonstrating its adaptability to irregular and challenging problem structures.
- Comparative analysis confirms ACMBO's **superior stability and accuracy** over established metaheuristics such as PSO and ACO, establishing its competitive advantage.
- The algorithm's adaptive mechanisms, customer segmentation, and local search strategies contribute to its **scalability and robustness**, making it effective across diverse benchmark sets.

Future Work

Future Work

- **Refinement for Large-Scale CVRP:**
  Enhance scalability and precision to further reduce deviations in very large customer instances, ensuring consistent near-optimal solutions.

- **Extension to VRP Variants:**
  Apply ACMBO to *electric, time-window, multi-depot, and heterogeneous vehicle routing problems* to evaluate adaptability under diverse and realistic logistical constraints.

- **Real-World Applications:**
  Test ACMBO in domains like *e-commerce delivery optimization* and *urban waste collection*, incorporating real-time data and dynamic constraints to validate operational utility.

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Thank You