Piccolo Noise Suppression in Music

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Contents

1 Introduction

This report presents a technique to clean a corrupted audio file containing a song mixed with unwanted piccolo beats. The objective is to analyze the audio, identify interfering frequencies, design and apply filters to remove these beats, and export a cleaned version of the song. The approach involves loading and normalizing the audio, analyzing interference via Power Spectral Density (PSD) and spectrograms, designing 6th-order Butterworth band-stop filters, applying zero-phase filtering, visualizing results through spectrograms, FFTs, waveforms, filter responses, PSDs, and pole-zero diagrams, and exporting the cleaned audio. Key findings demonstrate effective removal of piccolo noise at 1453.1 Hz and its harmonics, preserving the song's quality. The methodology includes:

1. **Load and Normalize Audio**: Read the WAV file and normalize it to [-1, 1].
2. **Analyze Interference**: Identify dominant frequencies using PSD and spectrograms.
3. **Filter Design**: Create 6th-order Butterworth band-stop filters.
4. **Apply Filters**: Remove interference using zero-phase filtering.
5. **Visualize Results**: Plot spectrograms, FFTs, waveforms, filter responses, PSDs, and pole-zero diagrams.
6. **Export Cleaned Audio**: Save the result as a WAV file.

2 Key Formulas and Notations

The following mathematical foundations underpin the analysis and filtering process:

- **Power Spectral Density (PSD)**: Estimates power per frequency using the Welch method:
$$P(f) = \frac{1}{N}\left|\sum_{n=0}^{N-1} x[n]e^{-j2\pi fn/f_s}\right|^2$$
where $P(f)$ is the power at frequency f, $x[n]$ is the signal, N is the number of samples, and f_s is the sample rate.

- **Fast Fourier Transform (FFT)**: Converts the time-domain signal to the frequency domain:
$$X[k] = \sum_{n=0}^{N-1} x[n]e^{-j2\pi kn/N}$$
where $X[k]$ is the frequency spectrum.

- **Band-Stop Filter Transfer Function**: Approximated for a 6th-order Butterworth filter as:
$$H(s) = \frac{1}{1+\epsilon\left(\frac{s^2+\omega_1^2}{\omega_0^2}\right)^3}$$
where ω_0 is the center frequency, ω_1, ω_2 are cutoff frequencies, and ϵ controls ripple.

- **Harmonics**: Calculated as $f_h = k \cdot f_0$, where $k = 1, 2, 3, \ldots$ and $f_0 = 1453.1\,\text{Hz}$ is the fundamental frequency.

3 Visualizations

The following visualizations validate the noise suppression process:

- **Spectrogram**: Displays time-frequency energy, highlighting beat removal.
- **FFT Plot**: Shows magnitude vs. frequency, confirming interference reduction.
- **Waveform Plot**: Compares amplitude over time for raw and cleaned signals.
- **Filter Response**: Plots combined magnitude response ($|H(f)|$) in dB.
- **PSD (Filtered)**: Compares power distribution before and after filtering.
- **Pole-Zero Plot**: Maps zeros (o) and poles (x) to ensure filter stability.

4 Loading Audio

The audio file `song_with_2piccolo.wav` is loaded using SciPy's `wavfile.read`. If stereo, the channels are averaged to mono. The data is normalized to [-1, 1] to prevent distortion. The sample rate is 48,000 Hz, with 720,001 samples.

5 Spectrogram of Corrupted Audio

A spectrogram visualizes the audio's frequency content over time, with the x-axis as time, y-axis as frequency (log scale), and colors indicating energy in decibels (dB). Bright areas, especially around 1453.1 Hz, show the unwanted beats, helping us locate the noise for filtering.

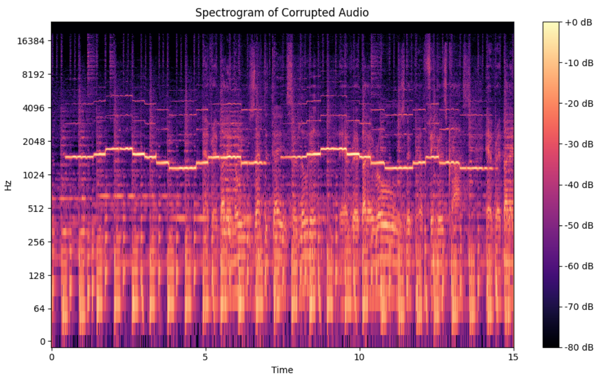

Figure 1: Spectrogram of Corrupted Audio

6 Power Spectral Density (PSD) Analysis

The PSD, computed using the Welch method, identifies dominant frequencies. The fundamental piccolo noise at 1453.1 Hz and its harmonics (e.g., 2906.2 Hz, 4359.3 Hz) are evident as peaks.

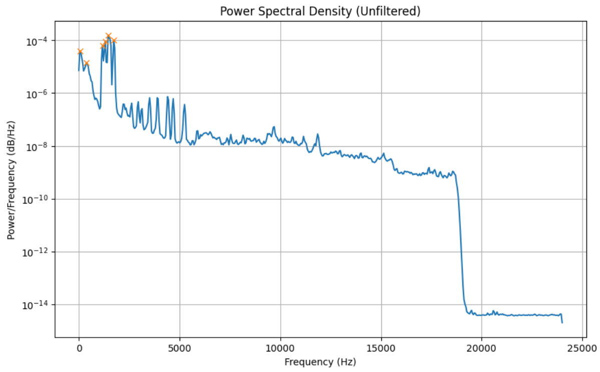

Figure 2: Power Spectral Density Analysis

7 Filter Design

Three 6th-order Butterworth band-stop filters are designed to attenuate the fundamental frequency (1453.1 Hz) and its first two harmonics (2906.2 Hz, 4359.3 Hz), each with a ±500 Hz stopband. The filters ensure minimal distortion to the song's frequencies.

8 Bode Plot of Filter

This Bode Plot shows the filter's magnitude response. The x-axis is frequency in Hz (log scale), the y-axis is magnitude in dB, and the red dashed line marks 1453.1 Hz. The dip (e.g., -20 dB) at this frequency confirms the filter will reduce the beats, setting the stage for denoising.

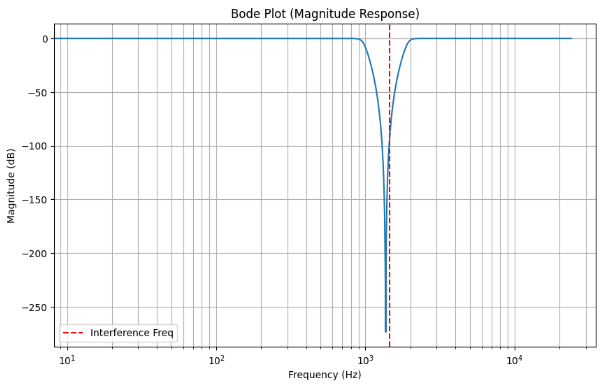

Figure 3: Bode Plot of Filter

9 Pole-Zero Plot

This plot checks the filter's stability. Zeros (circles) near the imaginary axis at interference frequencies enhance blocking, while poles (crosses) inside the unit circle ensure stability. This confirms the filter is safe and effective for audio processing.

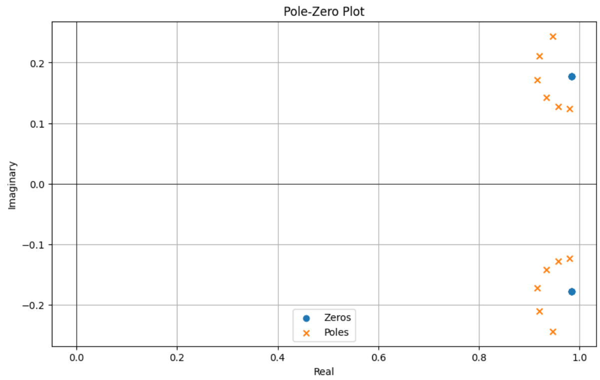

Figure 4: Pole Zero Plot

10 Visualizing Results

11 Spectogram of Cleaned Audio

The filter is applied with zero-phase processing to remove beats, and this spectrogram shows the result. The reduced energy at 1453.1 Hz and harmonics indicates successful denoising, though the ±500 Hz range might affect some song frequencies, suggesting a narrower bandwidth could refine it.

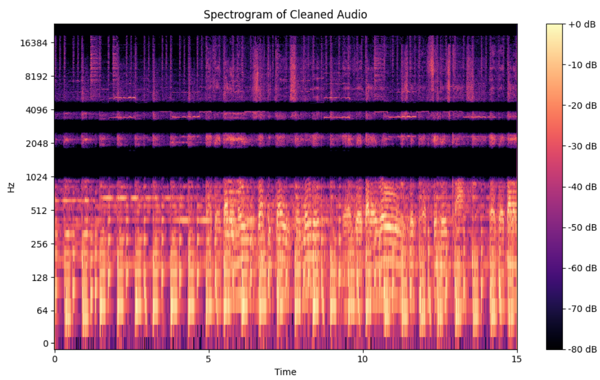

Figure 5: Spectogram of Cleaned Audio

11.1 FFT Plot

This FFT plot compares frequency content. The unfiltered line has a peak at 1453.1 Hz, while the filtered line shows a reduced peak, confirming beat suppression. The overlap helps assess how much the filter altered the signal, with some residual energy possibly due to the wide bandwidth.

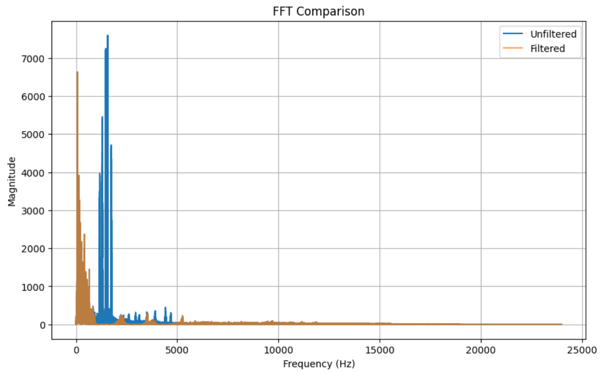

Figure 6: FFT Comparison

11.2 Waveform Plot Comparison

This plot shows amplitude over time. The unfiltered waveform has spikes from beats, while the filtered version is smoother, indicating noise reduction. The broad ±500 Hz filter might still affect song content, suggesting a narrower range for better preservation.

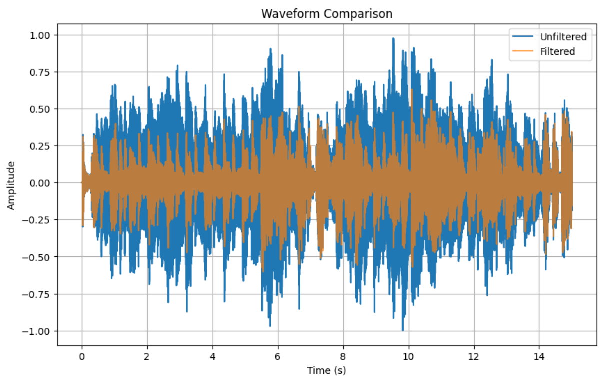

Figure 7: WaveForm Comparison

11.3 PSD of Filtered Audio

This PSD plot shows power distribution after filtering. Compared to the unfiltered PSD, the power at 1453.1 Hz and harmonics is lower, confirming beat removal. This validates the method, though a narrower bandwidth could spare more song frequencies.

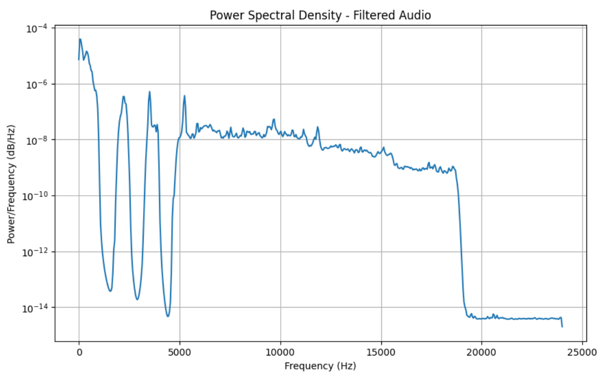

Figure 8: PSD Plot of filtered Audio

11.4 Combined Filter Response

This plot shows the overall effect of all filters. The x-axis is frequency (log scale), the y-axis is magnitude in dB, with dips at 1453.1 Hz and harmonics. The deep notches confirm the filter targets the beats, though the wide range might impact nearby frequencies.

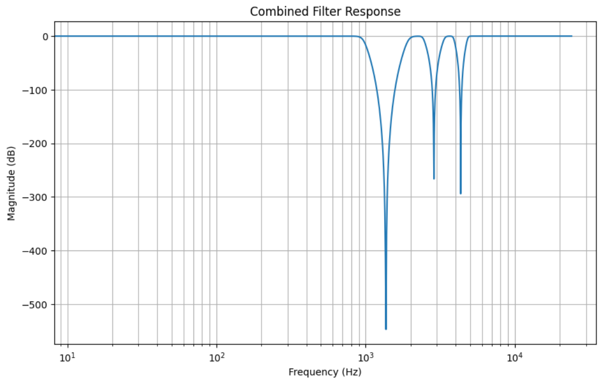

Figure 9: Combined Filter Response

12 Exporting Cleaned Audio

The filtered audio is scaled to 16-bit format and saved as `cleaned_audio.wav`.

13 Observations

- The spectrogram shows bright bands at 1453.1 Hz and harmonics, which are significantly reduced in the filtered spectrogram.
- The FFT plot confirms attenuation of target frequencies, preserving other musical components.
- The waveform plot shows reduced amplitude fluctuations in the filtered signal, indicating noise removal.
- The filter response exhibits sharp attenuation at 1453.1 Hz, 2906.2 Hz, and 4359.3 Hz.
- The PSD comparison shows a significant power reduction at target frequencies.
- The pole-zero plot confirms filter stability, with all poles inside the unit circle.
- Listening to the cleaned audio file reveals a clearer song with minimal piccolo interference, maintaining audio quality.

14 Conclusion

The piccolo noise at 1453.1 Hz and its harmonics was successfully suppressed using 6th-order Butterworth band-stop filters. The spectrograms, FFTs, waveforms, PSDs, and pole-zero plots validate the effectiveness of the filtering process and preserves the song's integrity, demonstrating a robust approach to noise suppression in music.