

UNIT 3- Context Free grammar

Grammar (Generation)



Language



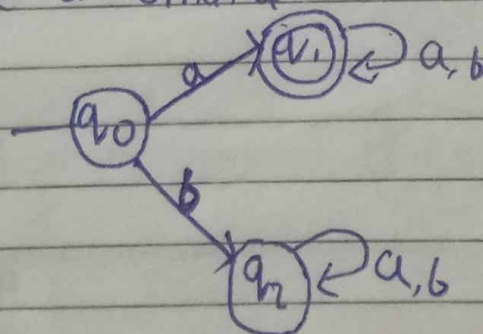
Automata (Accepts)

$L = \{a, aa, ab, aba, aabaa, abb\}$

This language is infinite

Because we have infinite combination of string **a** & **b**.

~~Diagram~~ Diagram of finite automata of infinite state automata



This is a machine of Finite representation of infinite language

$$L(M) = \{w \mid w \in \Sigma^*, \delta^*(q_0, w) \in q_f\}$$

for every w

'L' of 'M' is equal to such that 'w' belongs to transition closure.

only capital letters can be rewritten

$$S \rightarrow aA$$

S can be rewrite as 'a' & 'A'

'A' can be rewrite as ($A \rightarrow aA / bA / \epsilon$)

Non-terminal = capital symbols

Terminal = small case symbols / letters

String
is not
terminated

String
is
terminated

Small case symbols are terminal, it is nothing but sigma or a, b, c...

* Production Rule (We can rewrite any symbol)

$$\alpha \rightarrow \beta$$

non-terminal

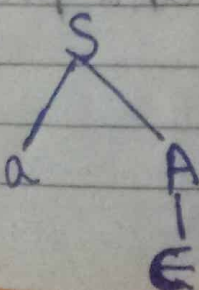
It can be sential from a tree form.

Derivation means from start symbol when we try to reach string symbol's or string generation.

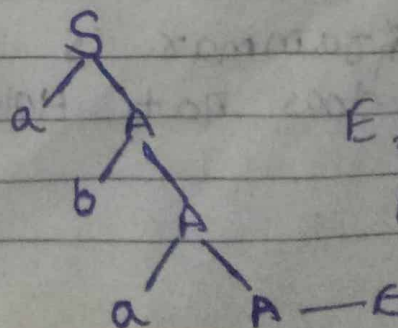
Ex

$$\begin{aligned} S &\rightarrow aA \\ A &\rightarrow aA / bA / \epsilon \end{aligned}$$

Tree form for S



or



After we use ~~it~~ it ends

is used to terminate

$$L(G) = \{w \mid w \in \Sigma^*, s^* \rightarrow w\}$$

This means \rightarrow Taking any no. of production from start

For Grammar is defined by quadruple

$$G = \{\Sigma, V_n, P, S\} \text{ 4 Tuple}$$

where,

$\Sigma \rightarrow$ Known as finite set of terminals
small case letters, ex a, b

$V_n \rightarrow$ Finite non-empty set of non-terminals / upper case ex. S, A

$P \rightarrow$ Production Rule

Finite non-empty set of production Rule

$S \rightarrow$ S stands for start symbol.

Note:

EX $a \rightarrow B$
 \swarrow
non terminal

If we not have the alpha then the Grammar is no invalid and production Rule does not holds.

$a \rightarrow b$
 \downarrow
 Tends to
 can be re-write as

Ex to solve

$\rightarrow w = bbaababab$

Consider the grammar

$S \rightarrow bB / aA$

$A \rightarrow b / bS / aAA$

$B \rightarrow a / aS / bBB$

For the String

$w = bbaababab$

S

Left most Derivation.

$S \rightarrow bB$

$\rightarrow b bBB$ (using $B \rightarrow bBB$)

$\rightarrow b b aSB$ (using $B \rightarrow aS$)

$\rightarrow b b a aAB$ (using $S \rightarrow aA$)

$\rightarrow b b a a bB$ (using $A \rightarrow b$)

$\rightarrow b b a a b aS$ (using $B \rightarrow aS$)

$\rightarrow b b a a b a bB$ (using $S \rightarrow bB$)

$\rightarrow b b a a b a b a$ (using $B \rightarrow a$)

Right most derivation

$S \rightarrow bB$

$\rightarrow b b BB$ (using $B \rightarrow bBB$)

$\rightarrow b b B a$ (using $B \rightarrow a$)

$\rightarrow b b a S a$ (using $B \rightarrow aS$)

$\rightarrow b b a a A a$ (using $S \rightarrow aA$)

$\rightarrow b b a a b S a$ (using $A \rightarrow bS$)

$\rightarrow b b a a b a A a$ (using $S \rightarrow aA$)

$\rightarrow b b a a b a b a$ (using $A \rightarrow B$)

problem - 2 :

consider the grammar

$$S \rightarrow A \mid B$$

$$A \rightarrow OA \mid E$$

$$B \rightarrow OB \mid IB \mid E$$

For the string $W = 00101$ find -

Leftmost derivation

Rightmost derivation

Parse Tree

Left most derivation.

$$\begin{aligned} S &\rightarrow A \mid B \\ &\rightarrow OA \mid B \quad (\text{using } A \rightarrow OA) \\ &\rightarrow OOA \mid B \quad (\text{using } A \rightarrow OA) \\ &\rightarrow OOE \mid B \quad (\text{using } A \rightarrow E) \\ &\rightarrow OOE \mid OB \quad (\text{using } B \rightarrow OB) \\ &\rightarrow OOE \mid O \mid B \quad (\text{using } B \rightarrow IB) \\ &\rightarrow OOE \mid O \mid E \quad (\text{using } B \rightarrow E) \\ &\rightarrow 00101 \end{aligned}$$

Rightmost derivation

$$\begin{aligned} S &\rightarrow A \mid B \\ &\rightarrow A \mid OB \quad (\text{using } B \rightarrow OB) \\ &\quad A \mid O \mid B \quad (\text{using } B \rightarrow IB) \\ &\quad A \mid O \mid E \quad (\text{using } B \rightarrow E) \\ &\quad OA \mid O \mid E \quad (\text{using } A \rightarrow OA) \\ &\quad OOA \mid O \mid E \quad (\text{using } A \rightarrow OA) \\ &\quad OOE \mid O \mid E \quad (\text{using } A \rightarrow E) \end{aligned}$$

00101

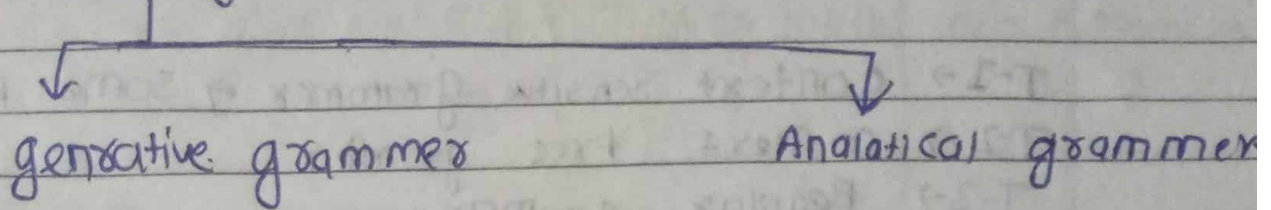
CFL = Context Free Language

T-0 \rightarrow unrestricted grammar \rightarrow ~~very~~ very few rules

T-1 \rightarrow context.

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Formal grammar



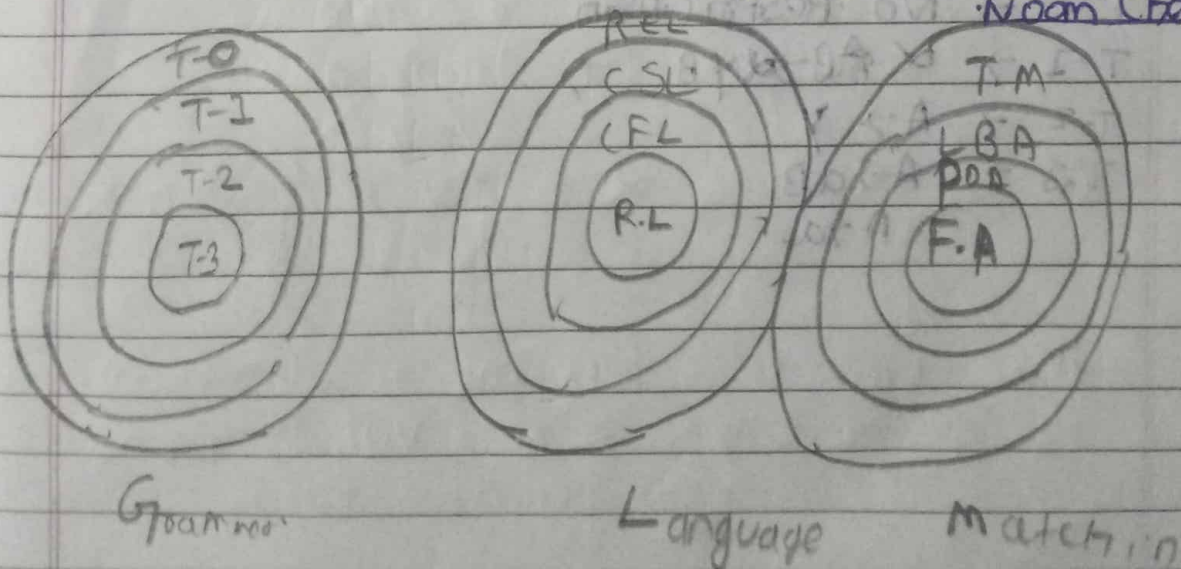
* Context Free grammar

In formal language theory, A context free language is a language generated by some context free grammar.

When the set of all CFL is identical to the set of languages accepted by pushdown automata.

Chomsky Hierarchy of languages by

Noam Chomsky in 1958



T-0 \rightarrow unrestricted Grammar

Very Few Restriction

T-1 \rightarrow Context Sensitive Grammar \rightarrow some Restriction

T-2 \rightarrow Context Free Grammar

T-3 \rightarrow Regular Grammar

REL - Recursive Enumerable Language

CSL - Context ^{Sensitive} Free language

CFL - context Free Language

RL - Regular Language

T.M - Turing machine

LBA - Least Bounded Automata

PDA - Pushdown Automata

FA - Finite Automata

Production Rules

T-0 \rightarrow No Restriction

T-1 $\rightarrow \alpha AB \rightarrow \alpha \gamma B$

T-2 $\rightarrow A \rightarrow \gamma$

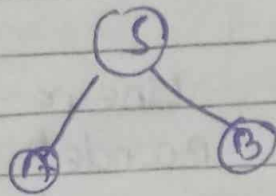
T-3 $\rightarrow A \rightarrow aB$

$A \rightarrow a$

Grammar	Language	Automation	Machine
Type-0	Recursively Enumerable unrestricted of all languages	Turing machine	No Restriction
Type-1	Context - Sensitive	Linear - Bounded non deterministic Turing machine	$\alpha AB \rightarrow \alpha \gamma \beta$ Grammar with some Restrictions
Type-2	Context Free	Non-Deterministic Pushdown Automata	$A \rightarrow \gamma$ Grammar
Type-3	Regular	Finite set Automata	$A \rightarrow aB$ $A \rightarrow a$

* = closure

Sentential Form

 $S \rightarrow AB \quad / \quad A \rightarrow Baa \quad / \quad A \rightarrow \epsilon$ 

when you have S as Root Node

If a partial derivation tree contains the root 'S' it is called sentential form and above subtree is a sentential tree.

$G = (V, T, P, S)$ be a CFG
 then any string w in $(V \cup T)^* / S \xrightarrow{*} w$
 is a Sentential form

If $S \xrightarrow{*} \alpha$ where $\alpha \in (V \cup T)^*$, then
 α is a sentential form

$V =$ is set of Variable

$T =$ is set of Terminals

If $S \xrightarrow{*}_{lm} \alpha$ where $\alpha \in (V \cup T)^*$, then α is a
 left-sentential
 form.

Here $_{lm}$ is left most derivation
 If $S \xrightarrow{*}_{rm} \alpha$ where $\alpha \in (V \cup T)^*$, then α is a
 right sentential form.

Derivation in sentential form

ex. Grammar Given

$$S \rightarrow A1B$$

$$A \rightarrow 0A, \epsilon$$

$$B \rightarrow 0B, 1B, \epsilon$$

String 1001

L.M

①	$S \rightarrow A1B$	
②	$\rightarrow \epsilon 1B$	(Using $A \rightarrow \epsilon$)
③	$\rightarrow \epsilon 10B$	(Using $B = 0B$)
④	$\rightarrow \epsilon 100B$	(Using $B = 0B$)
⑤	$\rightarrow \epsilon 100B1B$	(Using $B = 1B$)
⑥	$\rightarrow \epsilon 1001 \epsilon$	(Using $B \rightarrow \epsilon$)

1001

R.M.

①	$S \rightarrow A1B$	
②	$\rightarrow A10B$	Using ($B \rightarrow 0B$)
③	$\rightarrow A100B$	Using ($B \rightarrow 0B$)
④	$\rightarrow A1001B$	Using ($B \rightarrow 1B$)
⑤	$\rightarrow A1001\epsilon$	Using ($B \rightarrow \epsilon$)
⑥	$\rightarrow \epsilon 1001 \epsilon$	Using ($A \rightarrow \epsilon$)

1001

L.M.D \rightarrow left most derivation

2 or more parse tree

Ambiguity \rightarrow \rightarrow 2 or more parse tree
of grammar

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\rightarrow Only one

Ambiguous Grammar

Unambiguous Grammar

\star Ambiguous Grammar \rightarrow A grammar is said to be ambiguous if there exist more than one left most derivations or more than one Right most Derivation or more than one parse tree for the given input string.

Let us consider a string w generated by the grammar $w = id + id \times id$

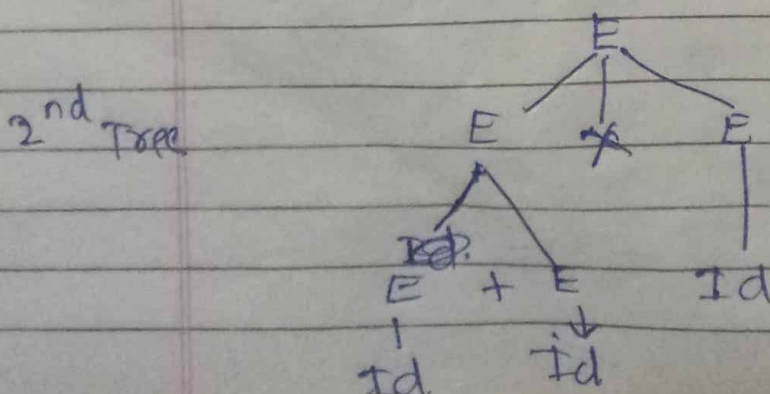
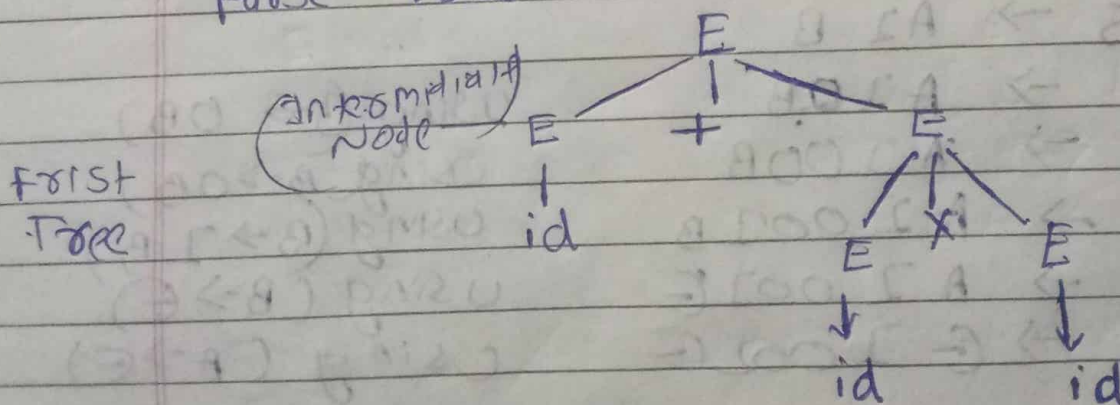
Grammar
 $E \rightarrow E + E / E \times E / id$

(Reason \rightarrow)

L.M.D for above example.

L.M.D

Parse Tree



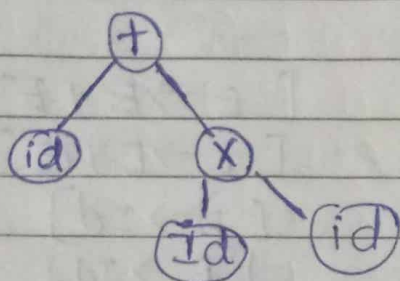
se tree

Syntax tree

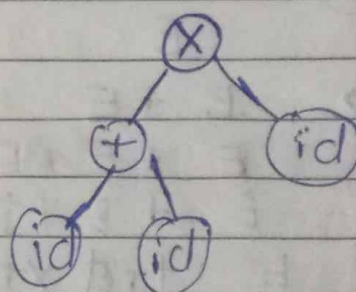
Since 2 parse tree exist (in L.M.D) for string w Therefore the grammar is ambiguous.

Reason: 02

Let us draw the syntax trees for the string w



Syntax Tree (1)



Syntax Tree (2)

Again 2 Trees are formed for same Grammar.

So Ambiguous

Reason 03

Let us consider a string w generated by the grammar $w = id + id * id$

First write down L.M.D.

L.M.D

$E \rightarrow E + E$

$E \rightarrow E + E$
 $\rightarrow id + E$
 $\rightarrow id + E * E$
 $\rightarrow id + id * E$
 $\rightarrow id + id * id$

Using $E \rightarrow E + E$
Using $E \rightarrow id$
Using $E \rightarrow E * E$
Using $E \rightarrow id$
Using $E \rightarrow id$

L.M.D ②

$E \rightarrow E \times E$
 $\rightarrow E + E \times E$
 $\rightarrow id + E \times E$
 $\rightarrow id + id \times E$
 $\rightarrow id + id \times id$

US

$[E \rightarrow E \times E]$
 $[E \rightarrow E + E]$
 $[E \rightarrow id]$
 $[E \rightarrow id]$
 $[E \rightarrow id]$

R.M.D ①

$E \rightarrow E + E$
 $\rightarrow E + E \times E$
 $\rightarrow E + E \times id$
 $\rightarrow E + id \times id$
 $\rightarrow id + id \times id$

$[E \rightarrow E + E]$
 $[E \rightarrow E \times E]$
 $[E \rightarrow id]$
 $[E \rightarrow id]$
 $[E \rightarrow id]$

R.M.D ②

$E \rightarrow E \times E$
 $\rightarrow E \times id$
 $\rightarrow E + E \times id$
 $\rightarrow E + id \times id$
 $\rightarrow id + id \times id$

$[E \rightarrow E \times E]$
 $[E \rightarrow id]$
 $[E \rightarrow E + E]$
 $[E \rightarrow id]$
 $[E \rightarrow id]$

Since 2 R.M.D. & 2 L.M.D. are exist
 For string w
 \therefore The grammar is ambiguous

Example 2

$$S \rightarrow aSb \mid SS$$

$$S \rightarrow \epsilon$$

For the string $aabb$ the above grammar generates two parse tree check whether the given Grammar is ambiguous or not.

$$w = aabb$$

LSB ①

RSB ①

$$\begin{aligned} S &\rightarrow aSb \\ &\quad aSb \\ &\quad aab \\ &\quad aabb \end{aligned}$$

$$\begin{aligned} E \cdot S &\rightarrow aSb \\ S &\rightarrow aSb \\ S &\rightarrow E \end{aligned}$$

LSB ②

$$\begin{aligned} S &\rightarrow aSb \\ &\quad aSSb \\ &\quad aasb \\ &\quad aabb \end{aligned}$$

$$\begin{aligned} S &\rightarrow aSb \\ S &\rightarrow aSS \\ S &\rightarrow aSb \\ S &\rightarrow E \end{aligned}$$

LSB ③

RSB ②

LSB ③

$$\begin{aligned} S &\rightarrow aSb \\ S &\rightarrow aSSb \\ S &\rightarrow aESb \\ S &\rightarrow aasb \\ S &\rightarrow aabb \end{aligned}$$

$$S \rightarrow aSb$$

$$S \rightarrow aSb$$

$$[S \rightarrow aSb]$$

$$[S \rightarrow SS]$$

$$[S \rightarrow aSb]$$

$$[S \rightarrow aSb]$$

$$[S \rightarrow aSb]$$

RS6(2)

$S \rightarrow asb$
 $\rightarrow assb$
 $\rightarrow asasbb$
 $\rightarrow asaebb$
 $\rightarrow aeaebb$

$[S \rightarrow asb]$
 $[S \rightarrow ss]$
 $[S \rightarrow asb]$
 $[S \rightarrow \epsilon]$
 $[S \rightarrow \epsilon]$

so

aa bb

RS6(3)

~~$S \rightarrow asb$~~
 ~~$a ssb$~~
 ~~$asasbsb$~~
 ~~aea~~

RS6(3)

$s \rightarrow asb$
 $assb$
 $aseb$
 $asabb$
 $aa bb$

$(S \rightarrow asb)$
 ~~$(S \rightarrow ss)$~~
 ~~$(S \rightarrow \epsilon)$~~
 $(S \rightarrow asb)$