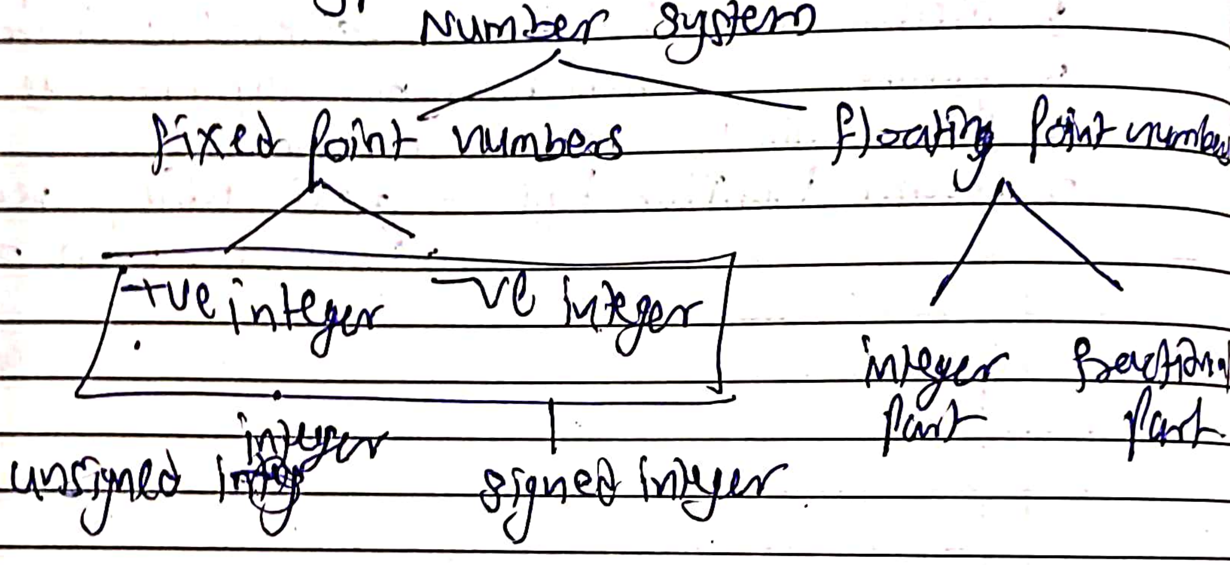
UNIT II

Data path unit

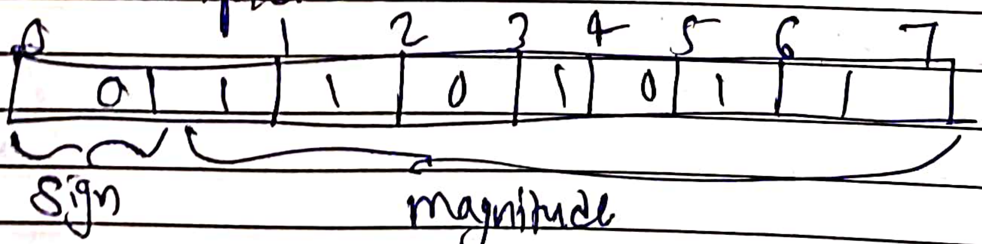
1 Scalar data types



Sign magnitude Representation

A 1's complement

B 2's complement.

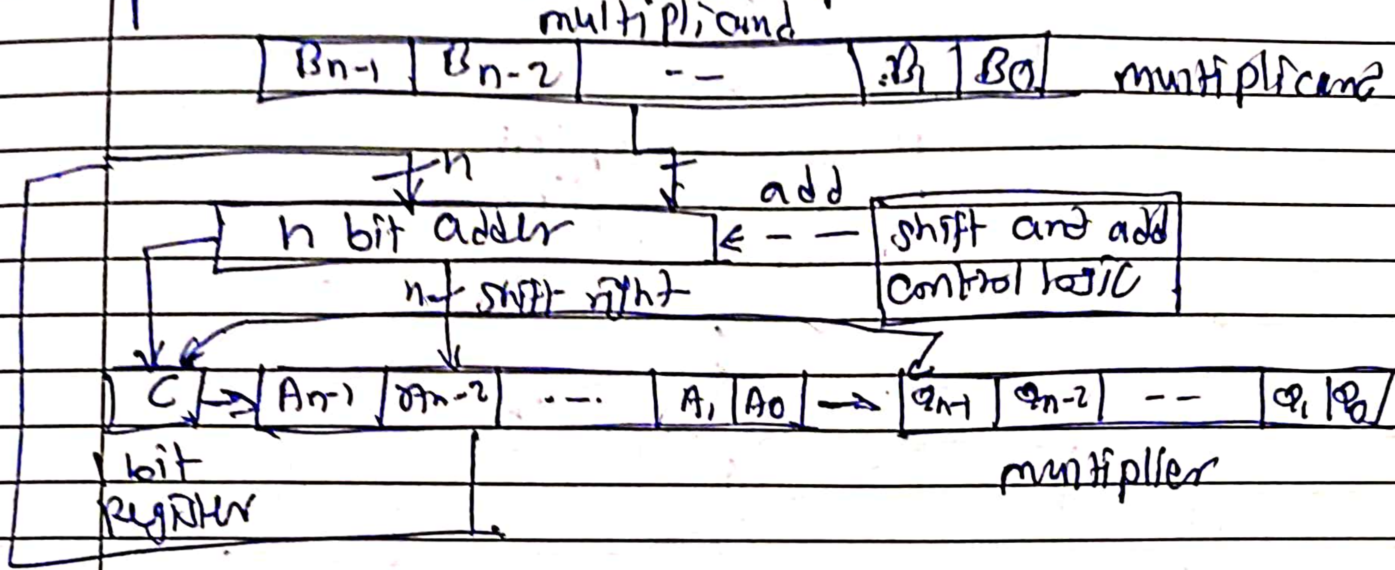


A 1's complement

- In 1's complement representation -ve numbers are obtained by complementing each bit of corresponding +ve number.

- 4 0100 } 1's complement
→ 1011

Positive numbers multiplication



H/W implementation of unsigned binary multiplication

multiplicand → 13 × 9 → multiplier

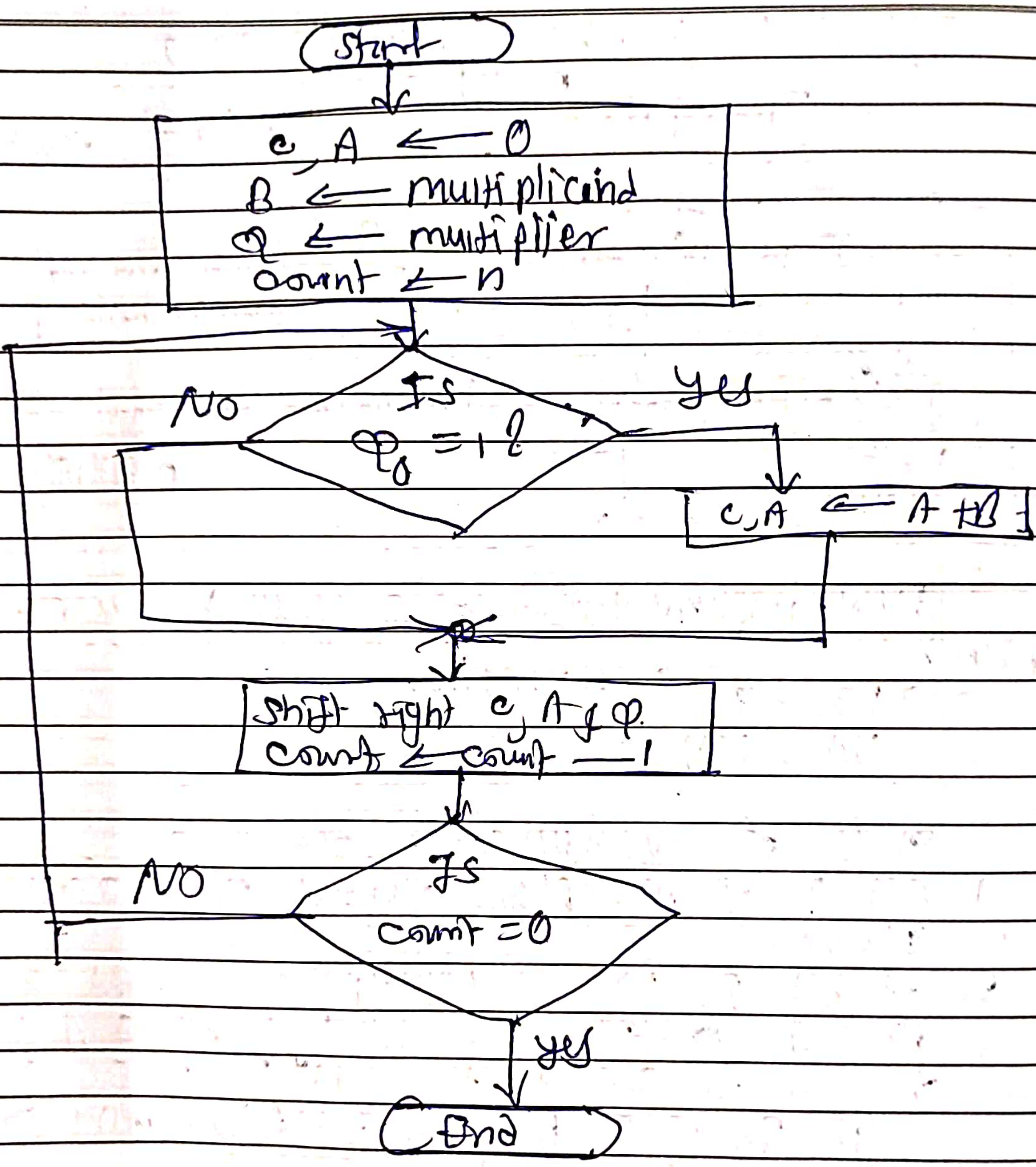
1101 × 1001

Steps of multiplication operation

1 Bit 0 of multiplier operand q_0 of Q register is checked

2. if bit q_0 is = 1 multiplicand and partial product added and all bits of C, A & Q register are shifted to right one bit.
if q_0 is 0 then no addition. only shift operation is carried out

3 Step 1 & 2 repeated n times to get the desired result. in A & Q. registers

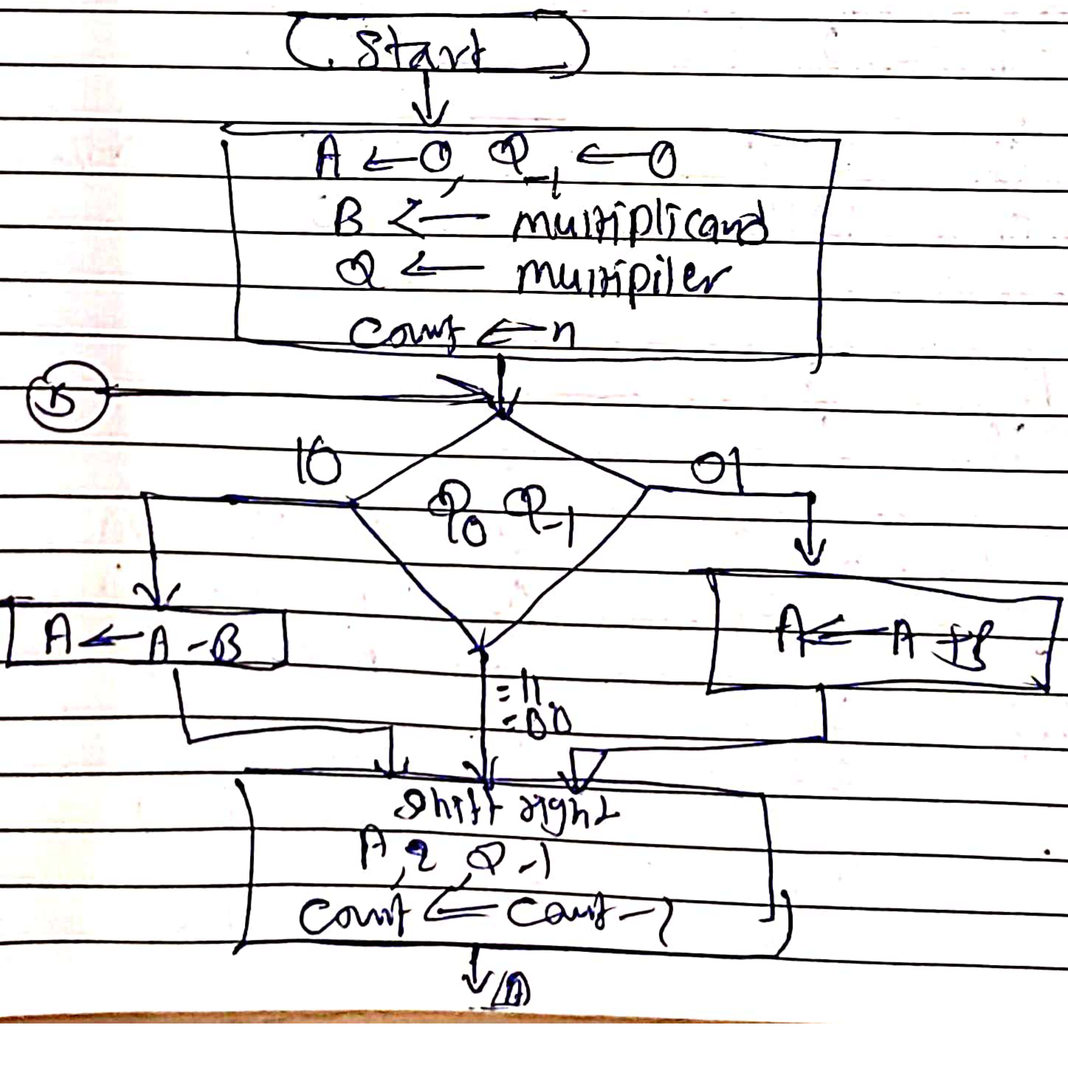


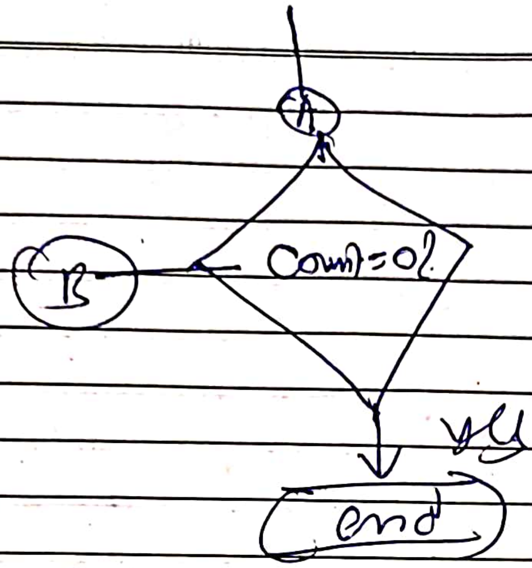
Flowchart of multiplication operation.

m 7 0111
Q ×3 ×0011

A	Q	M		
0000	0011	0111		
1001	0011	0111	A ← A + m	0111
1100	1001		shift	+0011
1110	0100		shift	1010
0101	0100		A = A + m	
0010	1010	0	shift	
0001	0101	0	shift	

Booth's Algorithm for signed multiplication



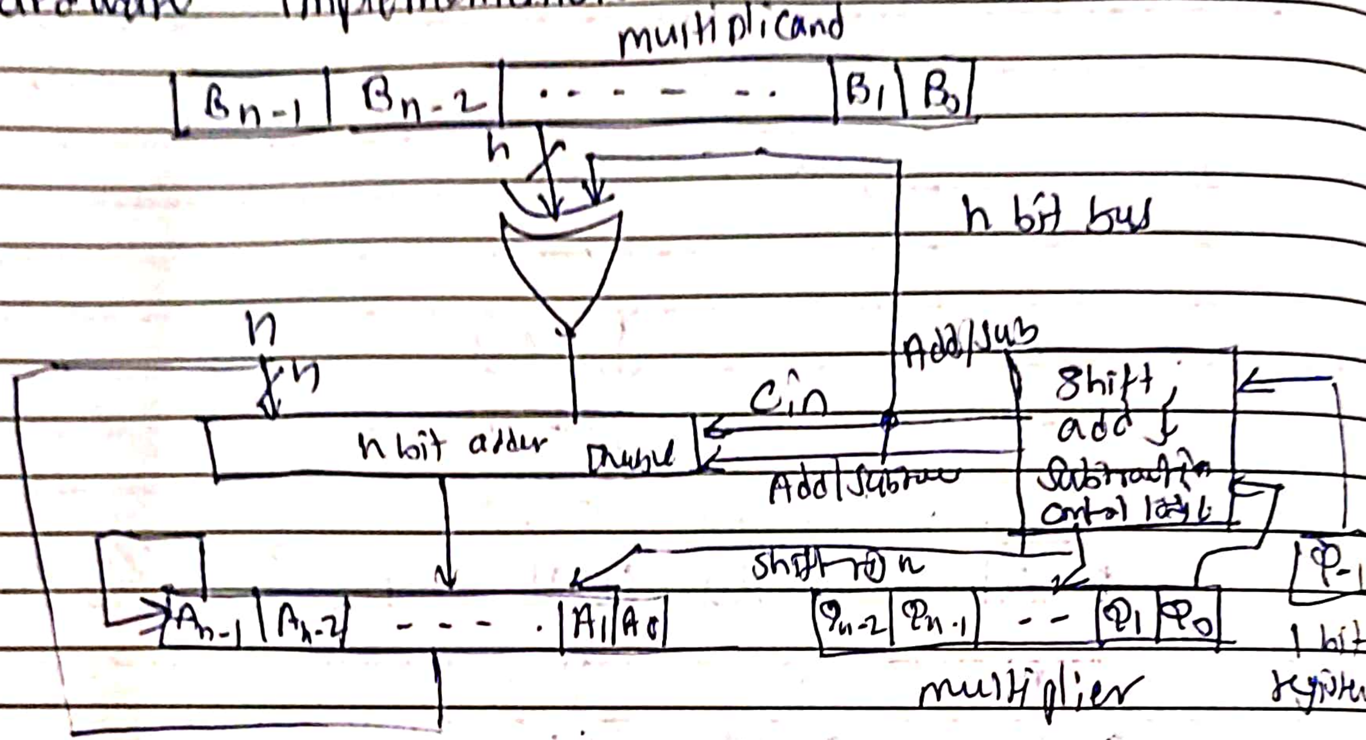


Case +ve numbers 5x4

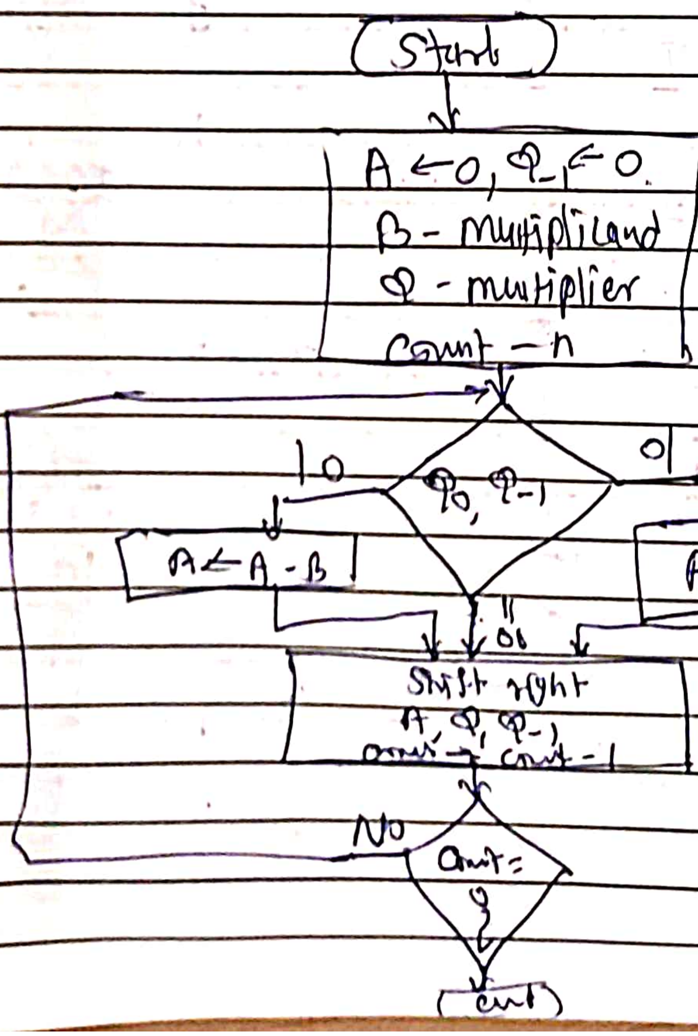
multiplicand (B) 0101 multiplier(Q) 0100

A	Q	Q_{-1}	Operation
0000	0100	0	Initial
0000	0010	0	shift right
0000	0001	0	shift right
			A ← A-B
1011	0001	0	0000 1011
			- 0101 - 0001
			1010
0 1101	0000	0	
1011			shift right
1101	1000	1	
			A ← A+B
0010	1000	1	1101
			+ 1000
0001	0100	0	1 0101
			Shift right

Hardware implementation



$A = 0 \quad Q_{-1} = 0$



5×4

multiplicand (B) 0101 (5) multiplier (Q) 0100 (4)

A/B	Q /A	Q_{-1}	
0000	0100	0	Initial
			1 shift right
0000	0010	0	2 shift right
0000 (0101)	0001	0	3 A ← A − B
1011	0001	0	shift right
1101	1000	1	
0101	1000	1	4 A ← A + B
0010	1000	1	shift right
0001	0100	0	

0101
1010
+1
1011

1101
1000
[1] 0101

5 × −4

multiplicand (0101) 5 multiplier (Q) = 1100 (−4)

0100
1011
+1
1100

A	Q	Q_{-1}	
0000	1100	0	initial
0000	0110	0	shift right
			shift right
0000 (0101)	0011	0	A ← A − B
0101	0011	0	
1011	0011	0	shift right
1101	1001	1	shift right
1110	1100	1	

0000
1111
+1
[1] 0000

3) -5×4

multiplicand → 1011 (−5) multiplier (Q) 0100 (4)

A	Q	Q_{-1}	
0000	0100	0	initial
0000	0010	0	shift right
0000	0001	0	shift right
2's(1011) 0101			
0101	0001	0	$A \leftarrow A - B$
0010	1000	1	shift right
1101	1000	1	$A \leftarrow A + B$
1110	1100	0	shift right

Take original value A − B (+5)

Take (+5) −5 value

1011 + 0001 = 1100

0010 + 1000 = 1010

-5×-4

multiplicand (-5)
0101
1010
+1
1011

multipiler (-4)
0100
1011
+1
1100

	A	B	Q_{-1}	
	0000	1100	0	initial
	0000	0110	0	Shift right.
(-5) +5	0000	0011	0	Shift right
	0101			$A \leftarrow A - B$
	0101	0011	0	
	0010	1001	1	Shift right
	0001	0100	1	Shift right

Booth Restoring division Algorithm

11 $10 \div 3$

Dividend (Q) $10 = 1010$ Divisor (3) (B) $= 0011$

Operation	Q_0	A	Q	(B)
initial value	0	0000	1010	
Shift A, Q.	0	0001	010 □	00011
$A = A - B$	1	1101		11100
				+1
Set Q_0	1	1110		11101
Restore			010 $\boxed{0}$	
$A = A + B$		11100		
		+ 00011	010 $\boxed{0}$	
		$\boxed{1}$ 00000		
Shift A, Q		00010	100 $\boxed{0}$ □	
$A = A - B$		11101		
		1 1111		
Restore A + B		+ 00011	10 $\boxed{0}$ $\boxed{0}$	
		00010		
Shift A, Q.		00101	0 $\boxed{0}$ $\boxed{0}$ □	
$A = A - B$		+ 11101		
Set Q_0		$\boxed{0}$ 0010	$\boxed{0}$ $\boxed{0}$ $\boxed{0}$ $\boxed{1}$	
Shift A, Q.		00100	$\boxed{0}$ $\boxed{0}$ $\boxed{1}$ □	
$A = A - B$		+ 11101		
		$\boxed{0}$ 0001 Reminder.	$\boxed{0}$ $\boxed{0}$ $\boxed{1}$ $\boxed{1}$ quotient	

2) 9 ÷ 4

Dividend (9)10 = 1001 Divisor(4) = 0100

Operation	Q_0	A	Q
initial value	0	0000	1001
Shift A, Q	0	0001	001☐
A = A − B	+	1100	
		① 1101	
Set Q_0		11101	001 $\boxed{0}$
Restore A = A + B		+ 00100	
		① 00001	001 $\boxed{0}$
Shift A, Q		00010	01 $\boxed{0}$ ☐
A = A − B		+ 11100	
		① 1110	
Set Q_0			
Restore A = A + B		+ 00100	01 $\boxed{0}$ $\boxed{0}$
		① ⓪ 0010	
Shift A, Q		00100	$\boxed{0}$ $\boxed{0}$ ☐
A = A + B		+ 00100	
Set Q_0 = 1		⓪ 1000	
			$\boxed{0}$ $\boxed{0}$ $\boxed{0}$ $\boxed{1}$
Shift A, Q		10001	$\boxed{0}$ $\boxed{0}$ $\boxed{0}$ $\boxed{1}$
A = A − B		+ 11100	
		$\boxed{1}$ $\boxed{0}$ 1101	
			$\boxed{0}$ $\boxed{0}$ $\boxed{1}$ $\boxed{0}$

B (4) = 00100
11011
+1
(−B) 11100

and 00100 1 $\boxed{0}$ $\boxed{0}$ $\boxed{1}$
A−B 11100
$\boxed{1}$ $\boxed{0}$ 1000
0001 $\boxed{0}$ $\boxed{0}$ $\boxed{1}$ $\boxed{0}$

7 ÷ 2

Dividend (7) - 0111 = Q   Divisor (2) = 0010 (B)

Operation Q_0 A Q

Initial 0 0000 0111

Shift A, Q 0 0000 111 $\boxed{\ }$ 00010 +B

11101

+1

11110 (-B)

A=A-B +1 1110

Set Q_0 1 1110

1 1110 111 $\boxed{0}$

A=A+B + 0 0010

1 $\boxed{1}$ 0 0000

Shift A, Q 00001 11 $\boxed{0}$ $\boxed{\ }$

A=A-B + 11110

Set Q_0 1 1111

A=A+B + 00010 11 $\boxed{0}$ $\boxed{0}$

1 $\boxed{1}$ 0 0001

Shift A, Q 00011 1 $\boxed{0}$ $\boxed{0}$ $\boxed{\ }$

Set Q_0

A=A-B + 11110

1 $\boxed{1}$ 0 0 0 1

Set Q_0

1 $\boxed{0}$ $\boxed{0}$ $\boxed{1}$

A=A+B + [illegible]

Shift A, Q   0 0 0 1 1   0 0 1 □

A = A − B   + 1 1 1 1 0

Set Q_0 1 (1) 0 0 0 1   0 0 1 1

Shift A, Q   1 0 0 1 1   1 0 1 1

0 0 1 1 1   0 0 1 □

A = A − B   + 1 1 1 1 0

Set Q_0 1 (0) 0 1 0 1

0 0 1 1 1   0 0 1 1

3) 4 ÷ 2

Dividend (Q) (4) = 0100   Divisor (B) (2) = 0010

Operation	Q_0	A	Q
initial value	0	0000	0100
Shift A, Q	0	0000	100□
A = A − B	1	1110	
Set Q_0	(1)	1110	100[0]
A = A + B	0	0010	
	1	0000	
Shift A, Q	0	0001	00[0]□
A = A − B	+ 1	1110	
	(1)	1111	
Set Q_0			00[0][0]
A = A + B	+ 0	0010	
	1 [1]	0001	

(B) 2 = 00010
↓
11101
+1
11110

A

	A	Q
	0 0 0 0 1	0 0 1 0 1 0 1
Shift A, Q	0 0 0 1 0	0 0 1 0 1 0
A=A-B	+ 1 1 1 1 0	
	1 0 0 0 0 0	
Set Q_0		
	+ 0 0 0 1 0	0 0 1 0 1 1
A=A+B	0 0 0 1 0	
Shift A, Q	0 0 0 0 0	0 0 1 1
A=A-B	+ 1 1 1 1 0	
	1 1 1 1 0	
Set Q_0		0 0 1 0
A=A+B	+ 0 0 0 1 0	
	1 0 0 0 0 0	

Parity $\boxed{1011}$

$$\boxed{S} \longrightarrow \boxed{D}$$

generator   detector

generator

A	B	Epg
0	0	0
0	1	1
1	0	1
1	1	0

A	B	Opg
0	0	1
0	1	0
1	0	0
1	1	1

A \ B	0	1
0	0	1 1
1	1 2	3

A \ B	0	1
0	1 0	1
1	2	1 3

$Epg = \bar{A}B + A\bar{B}$
$= A \oplus B$

$Opg = \bar{A}\bar{B} + AB$
$= \overline{A \oplus B}$

Detector

A	B		Epg	Epd
0	0	0	0	0
0	0	1	1	1
0	1	2	0	1
0	1	3	1	0
1	0	4	0	1
1	0	5	1	0
1	1	6	0	0
1	1	7	1	1

A	B	Opg	Opd
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

A \ B Epg	00	01	11	10
0	0	1 1	0 3	1 2
1	1 4	0 5	1 7	6

$Epd = A \oplus B \oplus Epg$

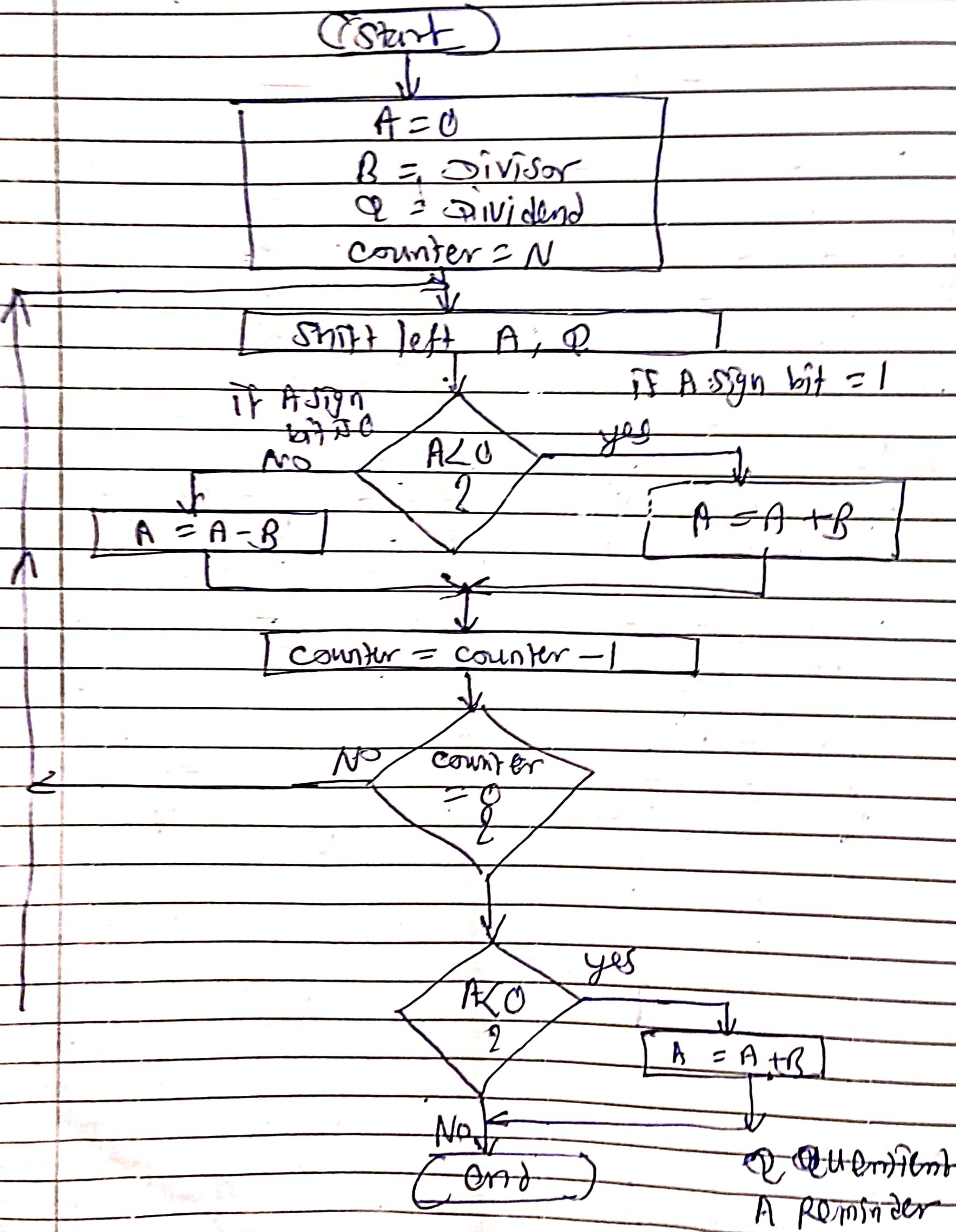
A \ B Opg	00	01	11	10
0	0	1	3	2
1	4	5	7	6

$Opd = A \oplus B \oplus Opd$

Q4)

Booth's Algorithm

Non-Restoring Division



1) $10 \div 3$

Dividend (Q) = 10 = 1010 Divisor (B) = 3 = 0011

Operation	Q_0	A	Q
initial value	0	0000	1010
Shift left A, Q	0	0001	010□
A = A − B	+ 1	1101	
	1	1110	010$\boxed{0}$
Shift Left A, Q	1	1100	100$\boxed{0}$□
A = A + B	+ 0	0011	
	1	1111	10$\boxed{0}\boxed{0}$
shift left A, Q	1	1111	0$\boxed{0}\boxed{0}$□
A = A + B	+ 0	0011	
	$\boxed{1}$	$\boxed{0}$0000	0$\boxed{0}\boxed{0}\boxed{1}$
Shift left A, Q	0	0000	$\boxed{0}\boxed{0}\boxed{1}$□
A = A − B	+	11101	
	$\boxed{1}$	00001	$\boxed{0}\boxed{0}\boxed{1}\boxed{1}$
		Reminder	Quotient

00011 → 11100 +1 = 11101

II] $11 \div 5$

Dividend (Q) = 11 = 1011   Divisor (B) = 5 = 0101

Operation	Q_0	A	Q
Initial value	0	0000	1011
Shift left A, Q	0	0001	011☐
A = A − B	+ 1	1011	
	1	1100	011 0
Shift left A, Q	1	1000	110☐
A = A + B	+ 0	0101	
	1	1101	1100
Shift left A, Q	1	1011	100☐
A = A + B	0	0101	
	1 0	0000	1001
Shift left A, Q	0	0001	001☐
A = A − B	+ 1	1011	
	1	1100	0010

00101 → 11010 + 1 = 11011

No Carry → Reminder is −ve   Quotient

$$
\begin{array}{r}
11100 \\
+\ 00101 \\
\hline
\boxed{1}\,00001
\end{array}
$$

Reminder

III) 4 ÷ 2

Dividend (Q) = 0100 = 4   Divisor (B) = 0010 = 2

Operation	Q_0	A	Q	
Initial Value	0	0000	0100	
Shift left A, Q	0	0000	100☐	
A = A − B	+ 1	1101		00010 → 11101 + 1 = 11110 (2's)
	1	1101	100$\boxed{0}$	
Shift left A, Q	1	1011	00$\boxed{0}$☐	
A = A + B	+ 0	0010		
	1	1101	00$\boxed{0}\boxed{0}$	
Shift left A, Q	1	1010	0$\boxed{0}\boxed{0}$☐	
A = A + B	+ 0	0010		
	1	1100	$\boxed{0}\boxed{0}\boxed{0}\boxed{0}$	
Shift left A, Q	1	1000	$\boxed{0}\boxed{0}\boxed{0}$☐	
A = A + B	+ 0	0010		
	1	1010	$\boxed{0}\boxed{0}\boxed{0}\boxed{0}$	

Q

iv) $7 \div 2$

Dividend (7) = (Q) = 0111 Divisor (2) = (B) = 0010

Operation	Q_0	A	Q
initial operation	0	0000	0111
shift left A, Q	0	0000	111□
A = A − B	+ 1	1110	
	1	1110	111[0]
shift left A, Q	1	1101	11[0]□
A = A + B	+ 0	0010	
	1	1111	11[0][0]
shift left A, Q	1	1111	1[0][0]□
A = A + B	+ 0	0010	
	[1] 0	0001	1[0][0][1]
shift left A, Q		00011	[0][0][1]□
A = A − B		+ 11101	
	[1]	00000	[0][0][1][1]

00010 → 11101, +1, 11110

Quotient

Add carry [1] 00000

+ 1

00001

Reminder

v) $4 \div 2$

Dividend (Q) = 4 = 0100   Divisor (B) = 2 = 0010

Operation	A_0	A	Q	
Initial value	0	0000	0100	
Shift left A, Q	0	0000	100 $\square$	00010
A = A − B	+ 1	1110		↓
	1	1110	100 $\boxed{0}$	11101
				+1
				11110 (−B)
Shift left A, Q	1	1101	00 $\boxed{0}$ $\square$	
A = A + B	+ 0	0010		
	1	1111	00 $\boxed{0}$ $\boxed{0}$	
A = A + B	+ 0	0010		
	1	00001	0 0 $\boxed{0}$ $\boxed{0}$	
Shift left A, Q	0	0010	0 $\boxed{0}$ $\boxed{0}$ $\square$	
A = A − B	+ 1	1110		
	1	0000	0 $\boxed{0}$ $\boxed{0}$ $\boxed{1}$	
Shift left A, Q	0	0000	$\boxed{0}$ $\boxed{0}$ $\boxed{1}$ $\square$	
A = A − B	+ 1	1100		
if carry	1	1110	$\boxed{0}$ $\boxed{0}$ $\boxed{1}$ $\boxed{0}$	

→ Reminder is −ve.   Quotient

1 1110
↓ 1's com complement
00001
−1
00000 Reminder

vi) 8 ÷ 3

Dividend (Q) = 8 = 1000   Divisor (B) = (3) = 0011

Operation	Q_0	A	Q
initial value	0	0000	1000
Shift left A, Q	0	0001	000□
A = A − B	+ 1	1101	
	1	1110	000$\boxed{0}$
Shift left A, Q	1	1100	00$\boxed{0}$□
A = A + B	+ 0	0011	
	1	1111	00$\boxed{0}\boxed{0}$
A = A + B	+ 0	0011	
	$\boxed{1}$ 0	0010	00$\boxed{0}\boxed{0}$
Shift left A, Q	0	0100	0$\boxed{0}\boxed{0}$□
A = A − B	+ 1	1101	
	$\boxed{1}$ 0	0001	0$\boxed{0}\boxed{0}\boxed{1}$
Shift left A, Q	0	0010	$\boxed{0}\boxed{0}\boxed{1}$□
A = A − B	+ 1	1101	
	1	1111	$\boxed{0}\boxed{0}\boxed{1}\boxed{0}$

Reminder   Quotient

No carry
Reminder is -ve

B 0011
↓
1100
+ 1
1101

vii) 10 ÷ 3

Dividend (Q) = 10 = 1010 Divisor (B) = 3 = 0011

Operation	Q_0	A	Q
initial Value	0	0000	1010
Shift left A, Q	0	0001	010☐
A = A − B	+ 1	1101	
	1	1110	0100
Shift left A Q	1	1100	100☐
A = A + B	+ 0	0011	
	1	1111	1000
Shift left A, Q	1	1111	000☐
A = A + B	+ 0	0011	
	1	0 0010	0001
Shift left A, Q	0	0100	001☐
A = A − B	+ 1	1101	
2's and carry	1	0 0001	0011

B
0 0011
⇓
11100
+1
11101

IEEE Standard format for binary Floating point Arithmematics.

Single precision | 32bit

double precision | 64 bit

I Single precision

S	E'	M
Sign of Number = 1bit 0 = +ve 1 = -ve	8bit Exponents Excess 127 presentation	23 bit mantissa fraction

value represented = $\pm 1.M \times 2^{E'-127}$

unsign integer = $E' = E + 127$

1) $(571.25)_{10}$

Step 1 - convert decimal to binary

$(571.25)_{10} = (1000111011.01)_2$

$= 1.00011101101 \times 2^9$

S = 0 +ve E = 09 M = 00011101101

$E' = E + 127$

$= 9 + 127 = (136)_{10}$

$E' = (136)_{10} = (10001000)_2$

Single precision format

0	10001000	00011101101 - - - -
S	E'	M

2) $(1259.125)_{10} = (\quad)_2$

$= 10011101011 + 0.001$

$= 10011101011.001$

$= 1.0011101011001 \times 2^{10}$

S = 0 E = 10 M = 0011101011001

$E' = E + 127$

$= 10 + 127$

$= (137)_{10}$

$= 10001001$

0	10001001	001110101100 - - - -
S	E'	M

3) $(-307.1875_{10}) = (\qquad)_2$

$= -100110011 + 0.0011_2$

$= -1.00110011 0011 \times 2^8$

$S = 1 \quad E = 8 \quad M = 00110011 0011$

$E' = E + 127$

$= 8 + 127 = (135)_{10}$

$E' = (10000111)_2$

1	10000111	0011001100 1100
S	E'	M

iv) $(0.0625)_{10} = (0.0001)_2$

$= 0.0001$

$= 1.0 \times 2^{-4}$

$S = 0 \quad E = -4 \quad M = 0001$

$E' = E + 127$

$= -4 + 127 = 123_{10}$

$= (01111011)_2$

0	01111011	0001 - - - - - -

double precision

S	E'	M
1 bit	11 bit excess 1023 exponent	52 bit mantissa fraction

1) $(1259.125)_{10} = 10011101011.001$

$= 1.0011101011001 \times 2^{10}$

$S = 0$ $E = 10$ $M = 0011101011001$

$E' = E + 1023$

$= 10 + 1023$

$= (1033)_{10}$

$= (10000001001)_2$

0	10001001	0011101011001

2) $(-307.1875)_{10} = (\quad)_2$

$= -100110011.0011$

$= -1.001100110011 \times 2^8$

$S = 1$ $E = 8$ $M = 001100110011$

$E' = E + 1023$

$= 8 + 1023$

$= (1031)_{10} = (10000000111)_2$

1	10000000111	001100110011

3) $(0.0625)_{10} = (\quad\quad)_2$

$= (0.0001)_2$

$= 1.0 \times 2^{-4}$

$S = 0 \quad E = -4 \quad M = 0001$

$E' = E + 1023$

$= -4 + 1023$

$= (1019)_{10}$

$= (01111111011)_2$

| 0 | 01111111011 | 0001 . . . . . . |