

**Example:** The given table shows the scores obtained by different players in a match. What is mean, median and mode of the given data?

S.No	Name	Runs Scored
1	Sachin	80
2	Yuvraj	52
3	Virat	40
4	Schwag	52
5	Rohit	70
6	Harbhajan	1
7	Dhoni	6

**Solution:**

i) The mean is given by:

$$\bar{x} = \frac{\sum_{i=1}^n x_i}{n}$$

$$\bar{x} = \frac{80+52+40+52+70+1+6}{7}$$

$$\bar{x} = 43$$

The mean of the given data is 43.

ii) To find out the median let us first arrange the given data in ascending order

Name	Harbhajan	Dhoni	Virat	Yuvraj	Schwag	Rohit	Sachin
Runs	1	6	40	52	52	70	80

As the number of items in the data is odd. Hence, the median is  $[(n+1)/2]^{\text{th}}$  observation.

$$\Rightarrow \text{Median} = [(7+1)/2]^{\text{th}} \text{ observation} = 52$$

iii) Mode is the most frequent data, which is 52.

## Relation of Mean Median Mode

The relation between mean, median and mode that means the three measures of central tendency for moderately skewed distribution is given the formula:

$$\text{Mode} = 3 \text{ Median} - 2 \text{ Mean}$$

This relation is also called an empirical relationship. This is used to find one of the measures when the other two measures are known to us for certain data. This relationship is rewritten in different forms by interchanging the LHS and RHS.

## Range

In statistics, the range is the difference between the highest and lowest data value in the set. The formula is:

**Range – Highest value – Lowest value**

### Solved Problem

**Question:** Find the mean, median, mode and range for the given data:

90, 94, 53, 68, 79, 94, 53, 65, 87, 90, 70, 69, 65, 89, 85, 53, 47, 61, 27, 80

**Solution:**

Given,

90, 94, 53, 68, 79, 94, 53, 65, 87, 90, 70, 69, 65, 89, 85, 53, 47, 61, 27, 80

Number of observations = 20

Mean = (Sum of observations) / Number of observations

$$= (90 + 94 + 53 + 68 + 79 + 94 + 53 + 65 + 87 + 90 + 70 + 69 + 65 + 89 + 85 + 53 + 47 + 61 + 27 + 80) / 20$$

$$= 1419 / 20$$

$$= 70.95$$

Therefore, mean is 70.95.

**Median:**

The ascending order of given observations is:

27, 47, 53, 53, 53, 61, 65, 65, 68, 69, 70, 79, 80, 85, 87, 89, 90, 90, 94, 94

Here,  $n = 20$

Median =  $1/2 [(n/2) + (n/2 + 1)]$ th observation

$$= 1/2 [10 + 11] \text{th observation}$$

$$= 1/2 (69 + 70)$$

$$= 139 / 2$$

$$= 69.5$$

Thus, the median is 69.5.

**Mode:**

The most frequently occurred value in the given data is 53.

Therefore, mode = 53

Range = Highest value – Lowest value

$$= 94 - 27$$

$$= 67$$

### Practice Questions

- The points scored by a Kabaddi team in a series of matches are as follows:  
17, 2, 7, 27, 15, 5, 14, 8, 10, 24, 48, 10, 8, 7, 18, 28  
Find the mean, median and mode of the points scored by the team.
- The following observations have been arranged in ascending order. If the median of the data is 63, find the value of  $x$ . 29, 32, 48, 50,  $x$ ,  $x + 2$ , 72, 78, 84, 95
- A survey conducted on 20 households in a locality by a group of students resulted in the following frequency table for the number of family members in a household:

family size	1 – 3	3 – 5	5 – 7	7 – 9	9 – 11
Number of families	7	8	2	2	1

- Find the mode of this data.
- Olivia is studying for a test, and she wonders if her friend, Laney, is also studying for the test. She calls Laney and asks her how long she has been studying. Laney has been studying for her test all week, approximately 8 hours total. Olivia has only been studying for her test for a couple of hours. The next week, Olivia and Laney get their test scores back. Laney got an A on her test, and Olivia got a C. Olivia wonders if there is a correlation between the number of hours spent studying and the grade a student earns. Take a look at the data Olivia collected from her classmates, and see if you can find a correlation.

X	Y
8	98
2	74
6	87
4	82
2	72

- How did you do? Are you stuck? Here's a hint if you need help. Start by making a table like this:

X	Y	XY	X <sup>2</sup>	Y <sup>2</sup>
8	98	784	64	9,604
2	74	148	4	5,476
6	87	522	36	7,569
4	82	328	16	6,724
2	72	144	4	5,184
22	413	1,926	124	34,557
SUMS				

- I've already done all of the summations for you. Can you use this information to insert into the equation and solve? Try it!
- Here's the table and the top and bottom of the formula with the appropriate values. Keep going for a breakdown of the solution.

### PRACTICE PROBLEM 1

X	Y	XY	X <sup>2</sup>	Y <sup>2</sup>
8	98	784	64	9,604
2	74	148	4	5,476
6	87	522	36	7,569
4	82	328	16	6,724
2	72	144	4	5,184
Σ 22	413	1,926	124	34,557

$$\begin{aligned}
 & \frac{n(\sum xy) - (\sum x)(\sum y)}{n\sum x^2 - (\sum x)^2} \\
 &= \frac{5(1,926) - (22)(413)}{5(124) - (22)^2} \\
 &= \frac{9,630 - 9,086}{620 - 484} \\
 &= \frac{544}{136} \\
 &= 4
 \end{aligned}$$

1. Number of Ordered Pairs  
 2. Summation  
 3. X times Y  
 4. X values  
 5. Y values  
 6. Summation of x values, squared  
 7. Summation of y values, squared

Formula with the appropriate values

- Let's start with the top of the equation.
- Notice that I work from left to right using the order of operations to solve this equation. Most often, you will get an answer wrong for one of two reasons: either you didn't use the order of operations correctly or you made an incorrect calculation somewhere in the equation. Always make sure to go back and check your table and each step in the equation to verify accuracy.

### PRACTICE PROBLEM 1

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1. Number of Ordered Pairs  
 2. Summation  
 3. X times Y  
 4. X values  
 5. Y values

Starting with the top of the equation



# PRACTICE PROBLEM 1

X	Y	XY	X <sup>2</sup>	Y <sup>2</sup>
8	98	784	64	9,604
2	74	148	4	5,476
6	87	522	36	7,569
4	82	328	16	6,724
2	72	144	4	5,184
Σ 22	413	1,926	124	34,557

$$\begin{aligned}
 &= \sqrt{[5(124) - (22)^2] [5(34,557) - (413)^2]} \\
 &= \sqrt{[620 - (22)^2] [5(34,557) - (413)^2]} \\
 &= \sqrt{[620 - 484] [5(34,557) - (413)^2]} \\
 &= \sqrt{[620 - 484] [172,785 - 170,569]} \\
 &= \sqrt{[620 - 484] [172,785 - 170,569]} \\
 &= \sqrt{[136] [2,216]} \\
 &= \sqrt{[301,376]} \\
 &= 548.98
 \end{aligned}$$

$$\begin{aligned}
 &= 544/548.98 \\
 &= .99
 \end{aligned}$$

2. This time, Olivia is talking to a friend about a new television show. She notices that some of her classmates aren't familiar with the show, but they are carrying around a lot of library books. She wonders if the more library books a student owns, the less television they watch. She collects data from her friends about how many library books they own and how much television they watch. Can you use her data to find a correlation?

X	Y
3	4
2	5
1	8
4	4
6	2

1. During a survey, 6 students were asked the number hours per day they give time to their studies on an average? The answers of the students are as follows: 2, 6, 5, 3, 2, 3. Calculate the standard deviation.

**Solution:**

**Step 1:** Calculate the mean value of the given data

$$= 2 + 6 + 5 + 3 + 2 + 3 = 21$$

$$= 21/6 = 3.5$$

$$= 3.5$$

**Step 2: Construct a table for the above given data**

$x_i$	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
2	-1.5	2.25
6	2.5	6.25
5	1.5	2.25
3	-0.5	0.25
2	-1.5	2.25
3	-0.5	0.25
$\sum (x_i - \bar{x})^2$		13.50

**Step 3 : Now, use the standard dev formula.**

**Sample Standard Deviation Formula -**

$$\begin{aligned}
 s &= \sqrt{\frac{\sum (x_i - \bar{x})^2}{n-1}} = \sqrt{\frac{13.50}{6-1}} \\
 &= \sqrt{\frac{13.50}{5}} = \sqrt{2.7} \\
 &= 1.643
 \end{aligned}$$

2. A Hen lays eight eggs. The weight of each egg laid by hen is given below.

Weight of an Egg (X)	60 gms	56 gms	61 gms	68 gms	51 gms	53 gms	69 gms	54 gms
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**Solution:**

**Step 1:** Let us first calculate the mean of the above data

Mean =

$$\begin{aligned} & \frac{\sum X}{N} \\ & = \frac{60+56+61+68+51+53+69+54}{8} \\ & = \frac{472}{8} = 59 \end{aligned}$$

= 59

**Step 2:** Construct a table for the above - given data

$x_i$	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
60	1	1
56	-3	9
61	2	4
68	9	81
51	-8	64

53	-6	36
69	10	100
54	-5	25
472		320

**Step 3 :** Now, use the standard dev formula

Standard Deviation Formula

$$\begin{aligned}
 &= \frac{\sum (x_i - \bar{x})^2}{n} = \frac{3208}{40} = 80.2 \\
 &= \sqrt{80.2} = 8.955 \text{ grams}
 \end{aligned}$$

### Quiz Time

1. Find the Standard Deviation for the Given Data  
5, 10, 7, 12, 10, 20, 15, 22, 8, 2

- a. 6.89
- b. 10.01
- c. 7.26
- d. 9

e. **Question:** Find the standard deviation of the numbers given (3, 8, 6, 10, 12, 9, 11, 10, 12, and 7).

f. **Answer: Step 1:** First, we need to find the mean of those ten given values.



g.  $\bar{x} = (3 + 8 + 6 + 10 + 12 + 9 + 11 + 10 + 12 + 7) / 10 = 88 / 10 = 8.8$

h.

i. **Step 2:** You need to make a table as below with three columns. One column holds the values of  $x$ , the second column holds the deviations, and the third has squared deviations.

j.

k. (Note: You may not need a table when computing a lesser number of values.)

l.

Value ( $x$ )	$X - \bar{x}$	$(x - \bar{x})^2$
3	-5.8	33.64
8	-0.8	0.64
6	-2.8	7.84
10	1.2	1.44
12	3.2	10.24
9	0.2	0.04
11	2.2	4.84
10	1.2	1.44
12	3.2	10.24
7	-1.8	3.24
Total	0	73.6

m.

n. **Step 3:** Since the data is not in the form of sample data, you need to use the formula of the population variance.

o. The standard deviation formula is,  $\sigma = \sqrt{\sum_{i=1}^n (x_i - \bar{x})^2 / N}$ .

p. Now, you have  $\sigma = \sqrt{73.6 / 10} = \sqrt{7.36}$ .

q. Finally, the standard deviation you get is 2.71.

A machine produces a component of a product with a standard deviation of 1.6 cm in length. A random sample of 64 components was selected from the output and this sample has a mean length of 90 cm. The customer will reject the part if it is either less than 88 cm or more than 92 cm. Does the 95% confidence interval for the true mean length of all the components produced ensure acceptance by the customer?

**Solution:**

Here  $\mu$  is the mean length of the components in the population.

The formula for the confidence interval is

$$\bar{x} - Z_{\alpha/2} \frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

Here  $\sigma = 1.6$ ,  $Z_{\alpha/2} = 1.96$ ,  $\bar{x} = 90$  and  $n = 64$

$$\text{Then } S.E. = \frac{\sigma}{\sqrt{n}} = \frac{1.6}{\sqrt{64}} = 0.2$$

Therefore,  $90 - (1.96 \times 0.2) \leq \mu \leq 90 + (1.96 \times 0.2)$

$$(89.61 \leq \mu \leq 90.39)$$

This implies that the probability that the true value of the population mean length of the components will fall in this interval (89.61, 90.39) at 95%. Hence we concluded that 95% confidence interval ensures acceptance of the component by the consumer.

### Example 8.12

A sample of 100 measurements at breaking strength of cotton thread gave a mean of 7.4 and a standard deviation of 1.2 gms. Find 95% confidence limits for the mean breaking strength of cotton thread.

**Solution:**

Given, sample size = 100,  $\bar{x} = 7.4$ , since  $\sigma$  is unknown but  $s = 1.2$  is known.

In this problem, we consider  $\tilde{\sigma} = s$ ,  $Z_{\alpha/2} = 1.96$

$$S.E. = \frac{\tilde{\sigma}}{\sqrt{n}} = \frac{s}{\sqrt{n}} = \frac{1.2}{\sqrt{100}} = 0.12$$

Hence 95% confidence limits for the population mean are

$$\bar{x} - Z_{\alpha/2} \frac{\sigma}{\sqrt{n}} < \mu < \bar{x} + Z_{\alpha/2} \frac{\sigma}{\sqrt{n}}$$

$$7.4 - (1.96 \times 0.12) \leq \mu \leq 7.4 + (1.96 \times 0.12)$$

$$7.4 - 0.2352 \leq \mu \leq 7.4 + 0.2352$$

$$7.165 \leq \mu \leq 7.635$$

This implies that the probability that the true value of the population mean breaking strength of the cotton threads will fall in this interval (7.165, 7.635) at 95% .

### Example 8.13

The mean life time of a sample of 169 light bulbs manufactured by a company is found to be 1350 hours with a standard deviation of 100 hours. Establish 90% confidence limits within which the mean life time of light bulbs is expected to lie.

*Solution:*

Given:  $n = 169$ ,  $\bar{x} = 1350$  hours,  $s = 100$  hours, since the level of significance is  $(100-90)\% = 10\%$  thus  $\alpha$  is 0.1, hence the significant value at 10% is  $Z_{\alpha/2} = 1.645$

$$S.E. = \frac{\sigma}{\sqrt{n}} = \frac{100}{\sqrt{169}} = 7.69$$

Hence 90% confidence limits for the population mean are

$$\bar{x} - Z_{\alpha/2} SE < \mu < \bar{x} + Z_{\alpha/2} SE$$

$$1350 - (1.645 \times 7.69) \leq \mu$$

$$1337.35 \leq \mu \leq 1362.65$$

Hence the mean life time of light bulbs is expected to lie between the interval (1337.35, 1362.65)

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