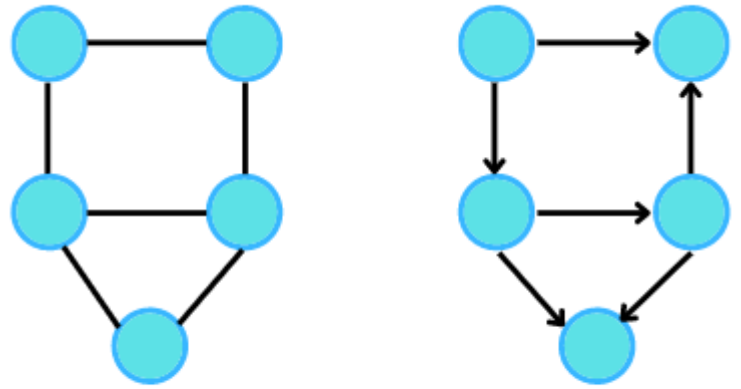
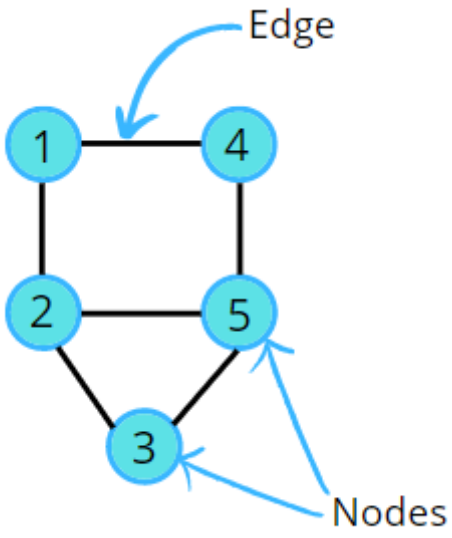
Graph Theory

A graph is a non-linear data structure consisting of nodes that have data and are connected to other nodes through edges.



Nodes are circles represented by numbers. Nodes are also referred to as vertices. They store the data. The numbering of the nodes can be done in any order, no specific order needs to be followed.

In the following example, the number of nodes or vertices = 5

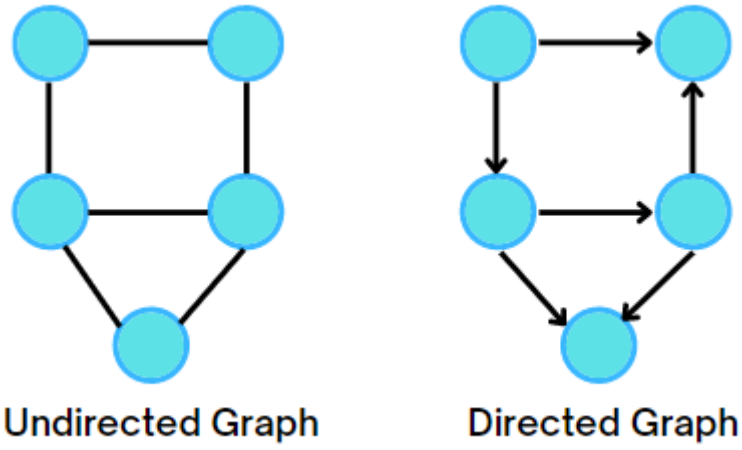


Two nodes are connected by a horizontal line called **Edge**. Edge can be directed or undirected. Basically, pairs of vertices are called edges.

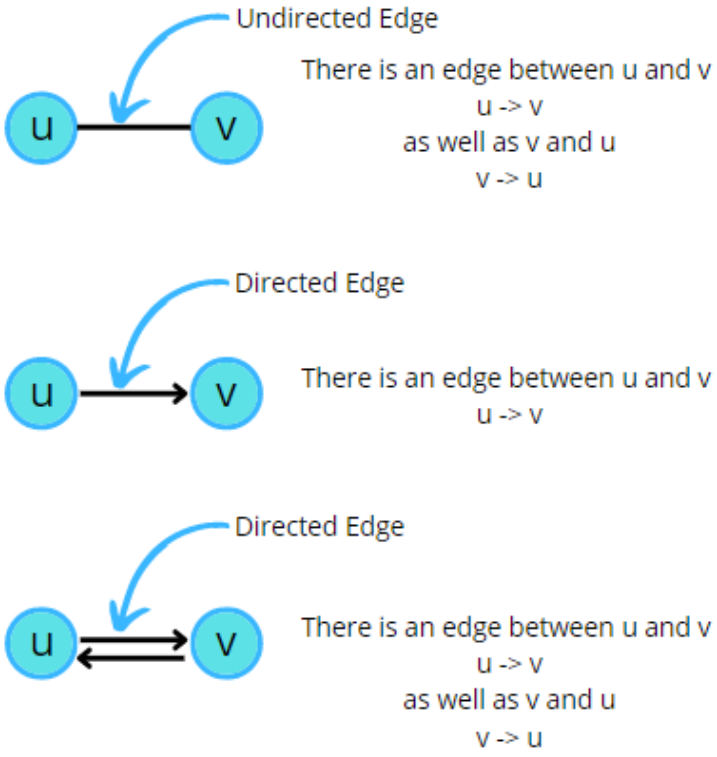
In the above example, the edge can go from 1 to 4 or from 4 to 1, i.e. a bidirectional edge can be in both directions, hence called an **undirected edge**. Thus, the pairs (1,4) and (4,1) represent the same edge.

Types of Graphs

- **An undirected graph** is a graph where edges are bidirectional, with no direction associated with them, i.e, there will be an undirected edge. In an undirected graph, the pair of vertices representing any edge is unordered. Thus, the pairs (u, v) and (v, u) represent the same edge.
- **A directed graph** is a graph where all the edges are directed from one vertex to another, i.e, there will be a directed edge. It contains an ordered pair of vertices. It implies each edge is represented by a directed pair <u, v>. Therefore, <u, v> and <v, u> represent two different edges.



There can be multi-directed edges, hence bidirectional edges, as shown in the example below.



Note:

- Graphs may or may not include cycles(paths that lead to the starting edge).
- Graphs that do not contain any cycles are also called trees.

Degree of Graph

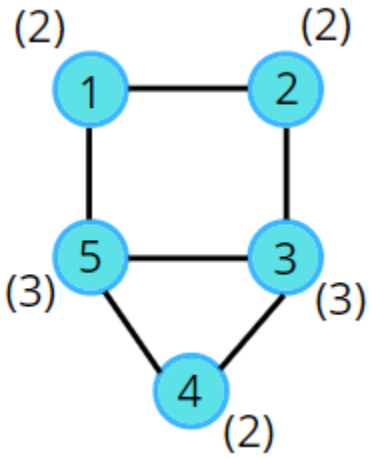
It is the number of edges that go inside or outside that node.

For **undirected graphs**, the degree is the number of edges attached to a node.

Example,

D(3) = 3

D(4) = 2



In directed graphs we use two metrics for a node, namely,

- **Indegree:** refers to the number of edges pointing to the particular node.
- **Outdegree:** refers to the number of nodes this node points to.