Department of Mechanical Engineering
Indian Institute of Technology Madras

Finite Element Analysis (ME5204)

A4 - Scalar valued problems

Date: 11-Oct-2024 Maximum Marks: 40

General instructions:

- Typeset the assignment in LaTeX 2_ε or MS word
- Handwritten document will NOT be accepted unless the assignment specifies
- Upload the code and the report to Google classroom
- Do NOT upload zip files. If zip files are uploaded, the assignments will **NOT** be graded
- Report file name: Rollnumber_A#.pdf (# indicates assignment number)
- Other supporting files should be appropriately named
- Reports/codes found copied, will be assigned '0' marks

1. Somehow the information is shared that ME5204 is putting all the best brains to estimate extent of heat, so the dates are now revised. It is likely to arrive a week ahead, also, the base radius of the heat source is now 5 times the radius of the outer circle (c.f. Assignment 3). $\boxed{20}$

   From your heat transfer course, you learnt that the governing differential equation for the transient heat conduction is given by:

$$\rho C_p \frac{\partial \theta(\mathbf{x},t)}{\partial t} = \nabla \cdot (\kappa \nabla \theta(\mathbf{x},t)) + Q_0(\mathbf{x})$$

   where, ρ, C_p and κ are the density (kg/m^3), specific heat capacity (J/kgK) and thermal conductivity (W/mK), respectively and are assumed to be functions of temperature, θ, $Q_0(\mathbf{x})$ is the volumetric heat generation due to the laser heat source, which is a function of laser power, P and t is the temporal variable. For the present study, the volumetric heat source term is assumed to be a Gaussian distribution, centered at the GC, $\mathbf{x}_o$, given by:

$$Q_0(\mathbf{x}) = \frac{P}{2\pi\sigma^2} exp\Big(- \frac{\|\mathbf{x} - \mathbf{x}_o\|}{2\sigma^2}\Big) \tag{1}$$

   where P = 500 W/mm^2 is the laser power according to the sources and σ is the spot size. For simplicity, we ignore convection and radiation boundary conditions. Identify the other boundary conditions that are required and how would the change of the boundary conditions influence the temperature distribution.

   IIT Madras has a serene campus with both buildings and trees with slow moving traffic ☺. It also has a lake and couple of other water bodies. The thermal conductivity, heat capacity and density in these regions are different. Assume the thermal conductivity of water is 0.6 W/mK[1]. Based on experimental data, the thermal conductivity of trees are in the range 0.116 to 0.128 in W/mK at the air-dry condition (use the maximum value for the calculations).

   For computational purposes, model the academic zone, hostel zone and the residential as without trees. However, if the density of trees from google satellite picture is high, model that region with trees. For regions without trees, use 10% lower properties to that of the trees. Assume ambient temperature to be 36°C. By solving the transient heat conduction equation using the Bubnov-Galerkin finite element method, estimate how long it takes for the temperature to reach 46°C in the residential zone, academic zone and hostel zone so that phased evacuation can be planned.

[1]We know that the thermal conductivity of water is a function of temperature, for present calculations, we assume it is independent of temperature

For temporal discretization use both implicit and explicit scheme and discuss the observations. Make sure you do a mesh convergence study and a time convergence study before presenting the results. Details of mesh convergence study (by measuring the temperature at the above locations) should be reported.

Hint: for doing a mesh convergence, first solve the static problem for a prescribed boundary conditions and arrive at an optimal mesh. Then, for this mesh, do a temporal convergence.

2. Every Saturday as we know, a movie is screened in OAT. With the new improved sound system installed, the quality of sound did improve but it has also increased the range over which the sound is heard. Before the movie starts, Indian National Anthem is played and as a citizen of India, we are supposed to stand up for the National Anthem, provided we are able to hear it. Every week you start at the same time from hostel to go to OAT to watch a movie (assume you watch every week), but on your way you are stopped due to the song. You would like to know how far you should be away from OAT when the National Anthem is being played, so you can continue to walk/ride the cycle ☺. So you decided to measure the sound level at the following places: main entrance of OAT, students gate, newly built fountain outside the OAT near to the road and near the badminton mud court. Based on these measurements you decide where you should be. $\boxed{20}$

In ME5204, you learnt that the governing differential equation for the acoustic pressure distribution is given by:

$$\nabla^2 p' - \frac{1}{c^2}\frac{\partial^2 p'}{\partial t^2} = 0 \qquad \text{in} \quad \Omega$$

where p' is the acoustic pressure, c is the speed of sound, Δ and t represents the Laplace operator and time, respectively. Assuming harmonic solution $p' = pe^{i\omega t}$, write the steady state equation, also know as the Helmholtz equation and identify the boundary conditions both natural and essential.

The sound system at the OAT is modelled as a point source along the wall of the screen with $v = \dfrac{\partial p}{\partial n} = 0.1$ unit as its strength. With this information, you are curious to put it to test and decided to compute the pressure distribution due to sound system at OAT by solving the Helmholtz equation using finite elements. Next question is choice of the computational domain, since you have no clue how small is small, you do a trial and error to see how far should you consider such that the pressure is "**small**" enough. Details of mesh convergence study (by measuring the pressure at the above locations) should be reported. From your FE calculations, you measure the sound level at the same locations as you did the measurements. Tabulate the values of pressure measured and those computed using FE. How do they compare with each other.?

Hint: for doing a mesh convergence, first solve the eigenvalue problem and arrive at an optimal mesh. Then, for this mesh do the pressure calculation.

Acknowledgement: Thanks to Prof. Mouli for the discussions related to question 2.