Department of Mechanical Engineering
Indian Institute of Technology Madras

Finite Element Analysis (ME5204)

A3 - L^2 projection

Date: 13-Sep-2024 Maximum Marks: 30

General instructions:

- Typeset the assignment in LaTeX 2_ε or MS word
- Handwritten document will NOT be accepted unless the assignment specifies
- Upload the code and the report to Google classroom
- Do NOT upload zip files. If zip files are uploaded, the assignments will **NOT** be graded
- Report file name: Rollnumber_A#.pdf (# indicates assignment number)
- Other supporting files should be appropriately named
- Reports/codes found copied, will be assigned '0' marks

1. Let $\mathcal{V}$ be the space of all continuous piecewise linear polynomials on $I = [0, 3]$. Determine the interpolating function (L^2 projection) to $f(x) = exp(\sin(0.25\pi x^2))$. Find the number of points required such that the error in the L^2 norm is less than 1×10^{-5}. Plot the error as a function of mesh size and compute the rate of convergence. Also include for a particular mesh size, an overlay of the actual function and the interpolated function. Please note that actual function should be evaluated at 'more' points. [10]

2. Intelligence unit at IIT Madras received a tip from the secret government agency that monitors extraterrestrial life on Earth, that a giant heat source, (see Figure 1(a) for the description of the heat source) is likely to be arrive on 8-Nov-2024. The center of the heat source will be focused at the center of the Gajendra circle (see Figure 1(b)) with the base radius equal to the diameter of the inner circle (marked with black circle). [20]

(a)

(b)

Figure 1: (a) Heat source from outer space and (b) Gajendra circle. Note that the pictures were taken from Google search.

Your objective as a finite element expert is to estimate the extent of heat distribution *a priori* so evacuation can be planned accordingly. However, the power of the heat source will be known *only* by 2-Oct-2024.

The current task is get an L^2 projection of the heat source with an appropriate FE mesh. As not much information is available at this stage, let us assume that the heat source follows a Gaussian distribution with center of the source at center of black circle (c.f. Figure 1(b)). Plot the error as a function of mesh size and compute the rate of convergence. Also include for a particular mesh size, the actual function and

the interpolated function in a contour plot. Shown them side by side and also an error plot by taking the difference of them.

Error in L^2 norm:

$$||f(x) - \Pi_h f(x)|| = \sqrt{\int_\Omega (\mathbf{u} - \mathbf{u}^h) \cdot (\mathbf{u} - \mathbf{u}^h) \, \mathrm{d}\Omega} \tag{1}$$

The report to have the following:

- Description of the equations
- Mesh details
- Discussion on the rate of convergence (the convergence plot between mesh size and error should be in a log-log plot)