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Nuclear Energy (3)

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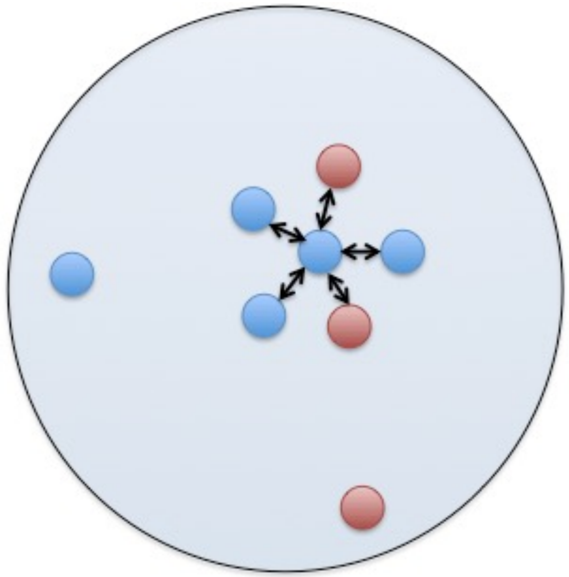
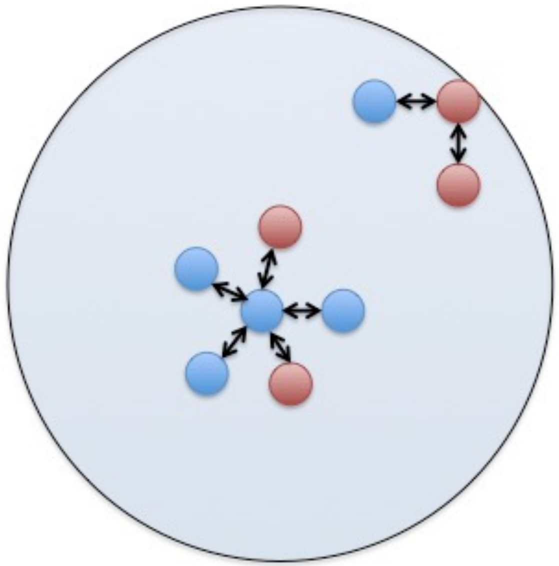
A model for nuclear fission

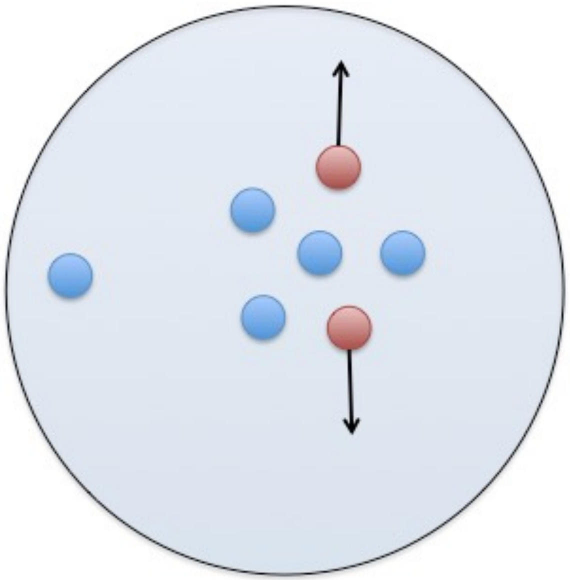
Many of the features of fission can be understood with the liquid drop model of the nucleus, based on the analogy between a nucleus and a charged liquid drop.



Volume term



Surface term



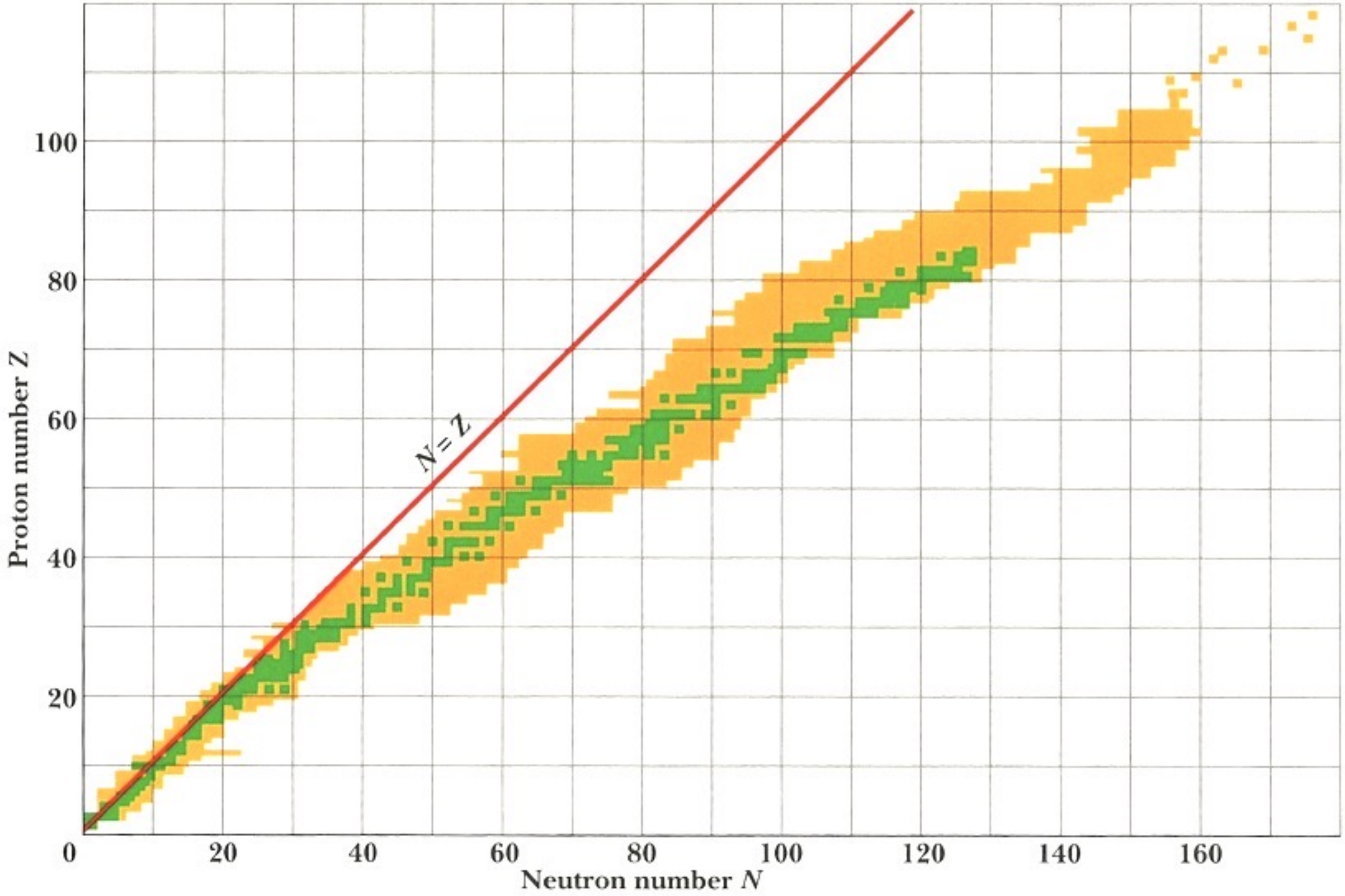
Coulomb term

Fission and the liquid drop model

A significant feature of the Segré chart is that there are no stable nuclei above Z = 83 (bismuth). Heavier nuclei are unstable and decay mainly by α decay.

As we have seen in our previous example using ^{238}U, such nuclei could (in principle) gain stability by breaking up into two smaller fragments. Although this form of decay, called **spontaneous fission**, becomes energetically possible for nuclei with $A \geq 90$, it is not, in fact, observed for nuclei with $Z < 92$.

Even for such heavy nuclei, spontaneous fission does not compete successfully with the α decay, and does not become an important decay process until we get to nuclei of A >250 – e.g. for ^{238}U, $T_\alpha = 4.5 \times 10^9$ y, while the partial half-life for spontaneous fission is $\sim 10^{16}$ y.



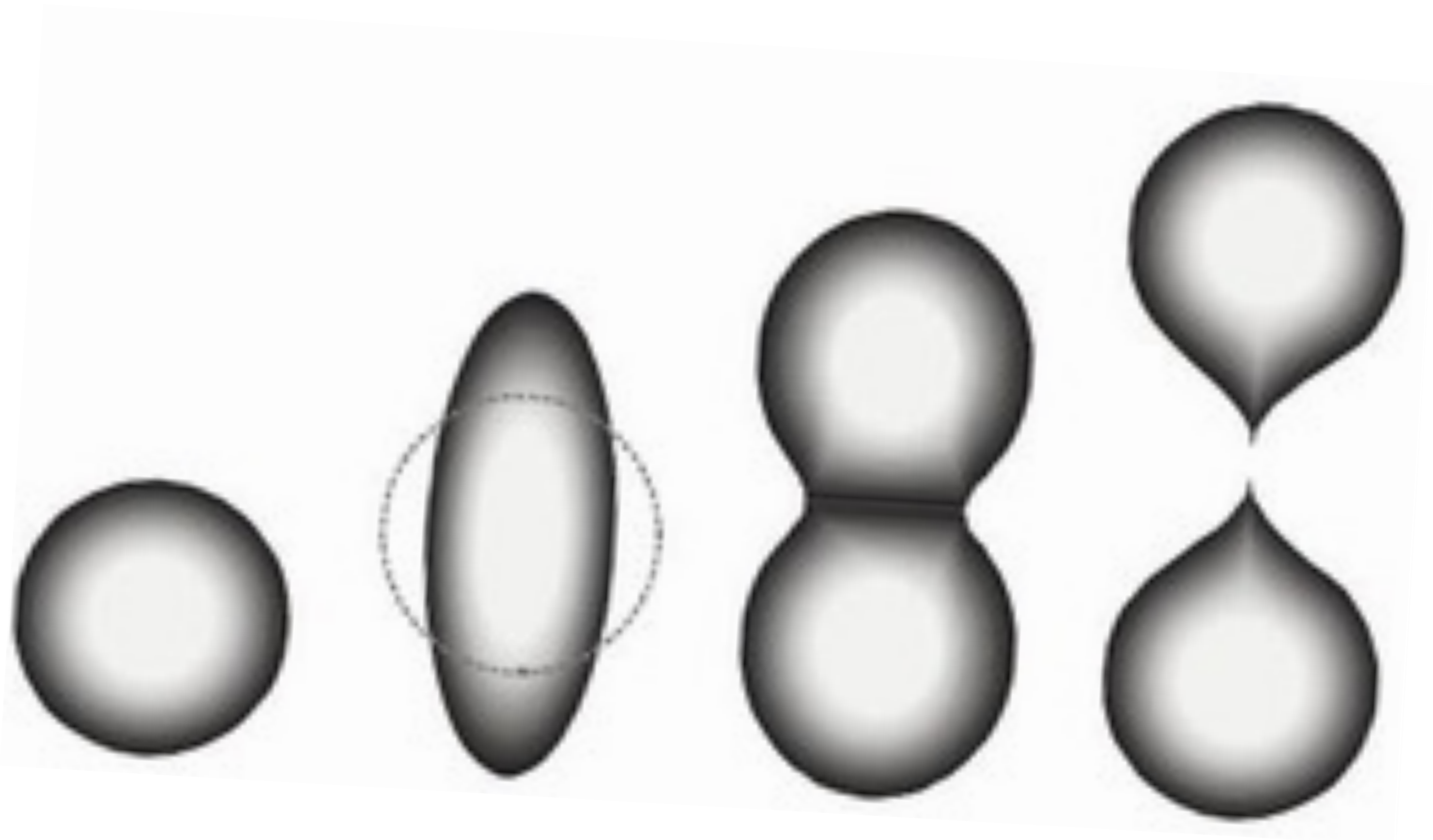
Nuclide stability diagram. The green shading identifies the band of stable nuclides; the orange shading identifies unstable nuclides

The reason why spontaneous fission is so rare is that such nuclei are stable with respect to a **small** deformation from their equilibrium shape.

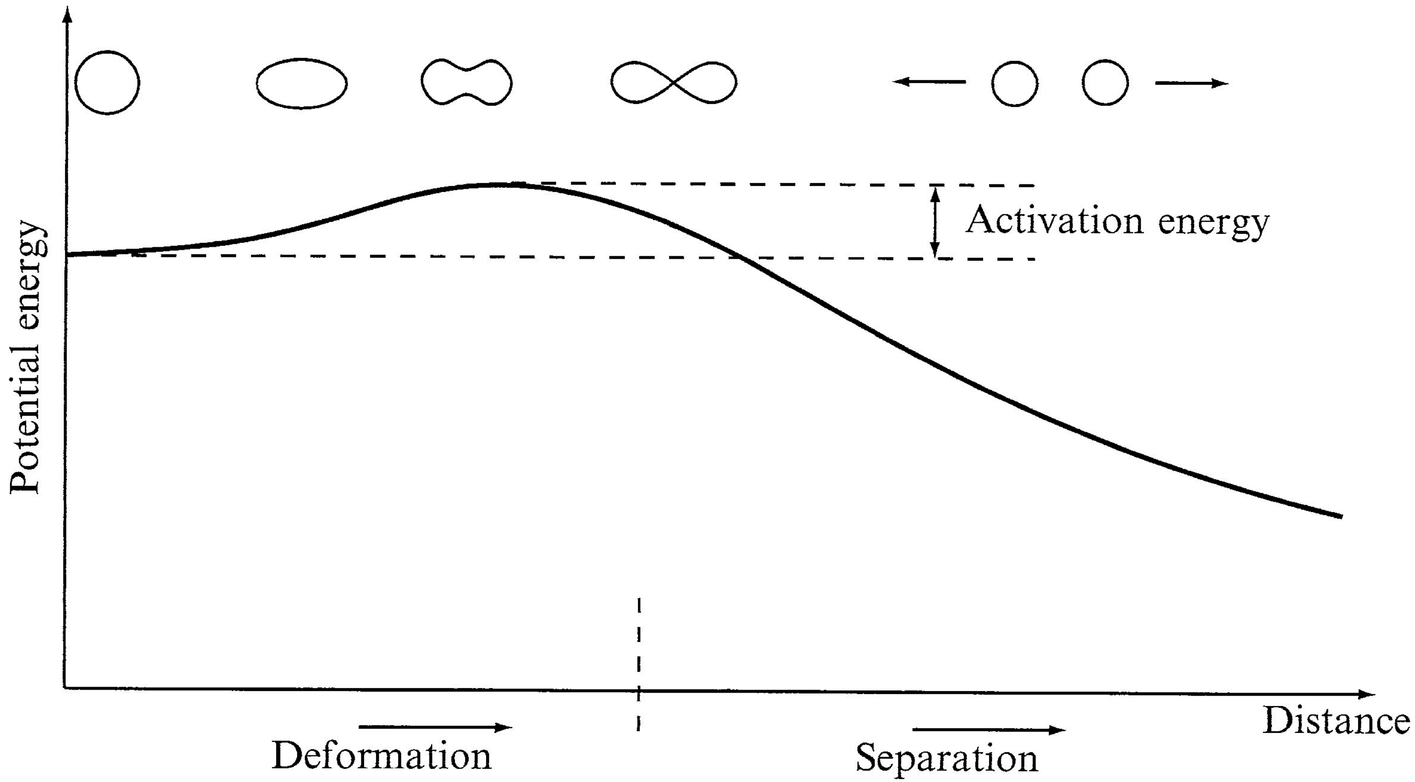
In order for fission to occur, a nucleus must first become deformed from its equilibrium shape. This increases the surface area, which requires energy (decrease in binding energy) according to the liquid drop model.

The increase of surface energy is offset to some extent by a binding energy gain from the Coulomb repulsion between protons, which decreases with deformation.

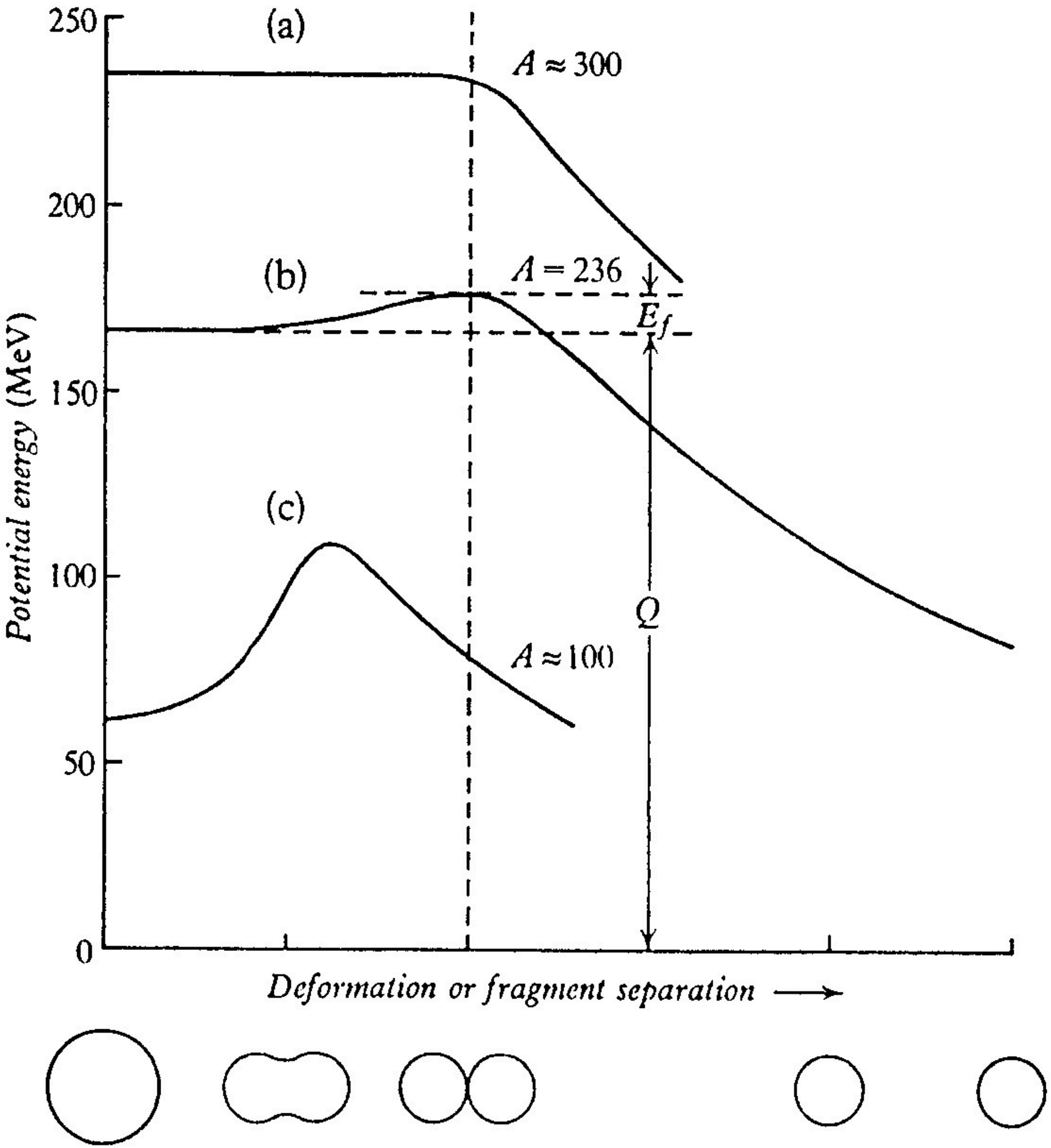
The stability of a heavy nucleus, therefore, depends on the balance between these surface and Coulomb effects.



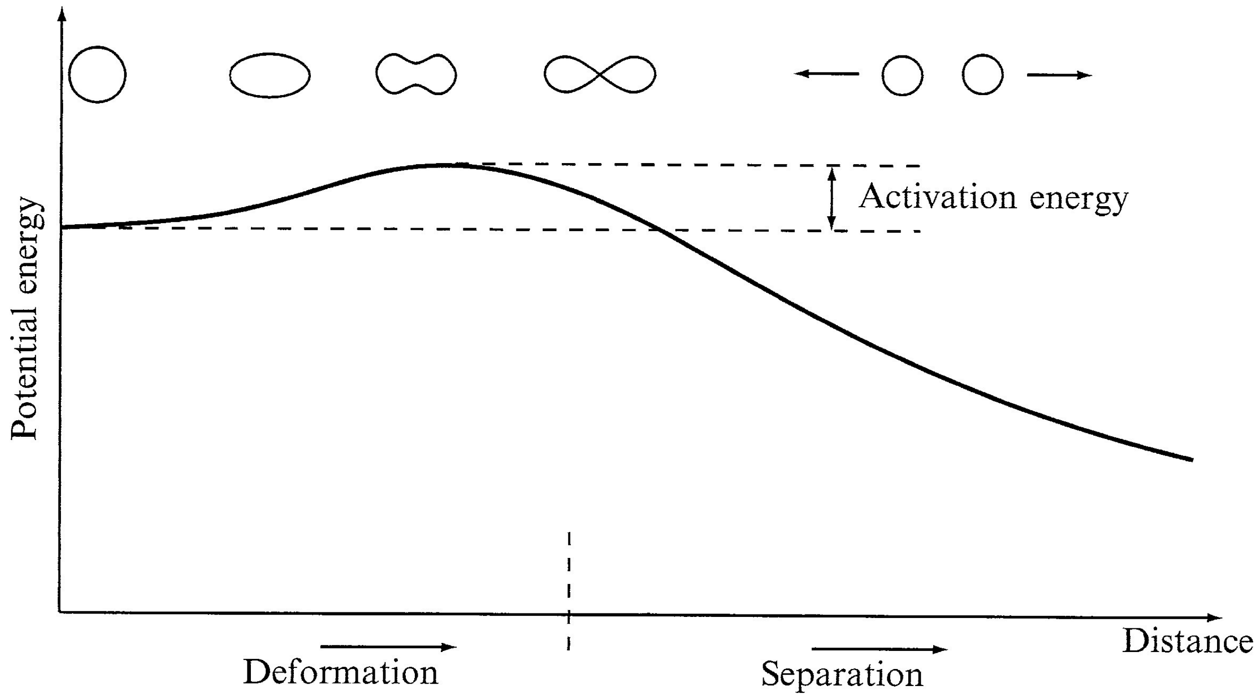
For all naturally-occurring nuclei, the surface effects dominate, and the nucleus exists in a state of stable equilibrium, as shown schematically below.



Schematic plot of the potential energy of a nucleus, first, as a function of deformation from a spherical shape and then as a function of separation after fission has occurred. The activation energy (fission barrier) is the energy required to overcome the barrier preventing fission from taking place spontaneously (Lilley, *Nuclear Physics: Principles and Applications*, 2001).


250
200
150
100
50
0
(a)
A ≈ 300
(b)
A = 236
E_f
(c)
A ≈ 100
Q
Potential energy (MeV)
Deformation or fragment separation

For some nuclei, however, absorption of a relatively small amount of energy, such as from a low-energy neutron or photon, results on the formation of an intermediate state that is at or above the barrier of this intermediate state, so that **induced fission** occurs readily, competing successfully with other decay modes of the compound nucleus.



In a typical event, a ^{235}U nucleus absorbs a thermal neutron, producing a compound ^{236}U nucleus in a highly excited state.

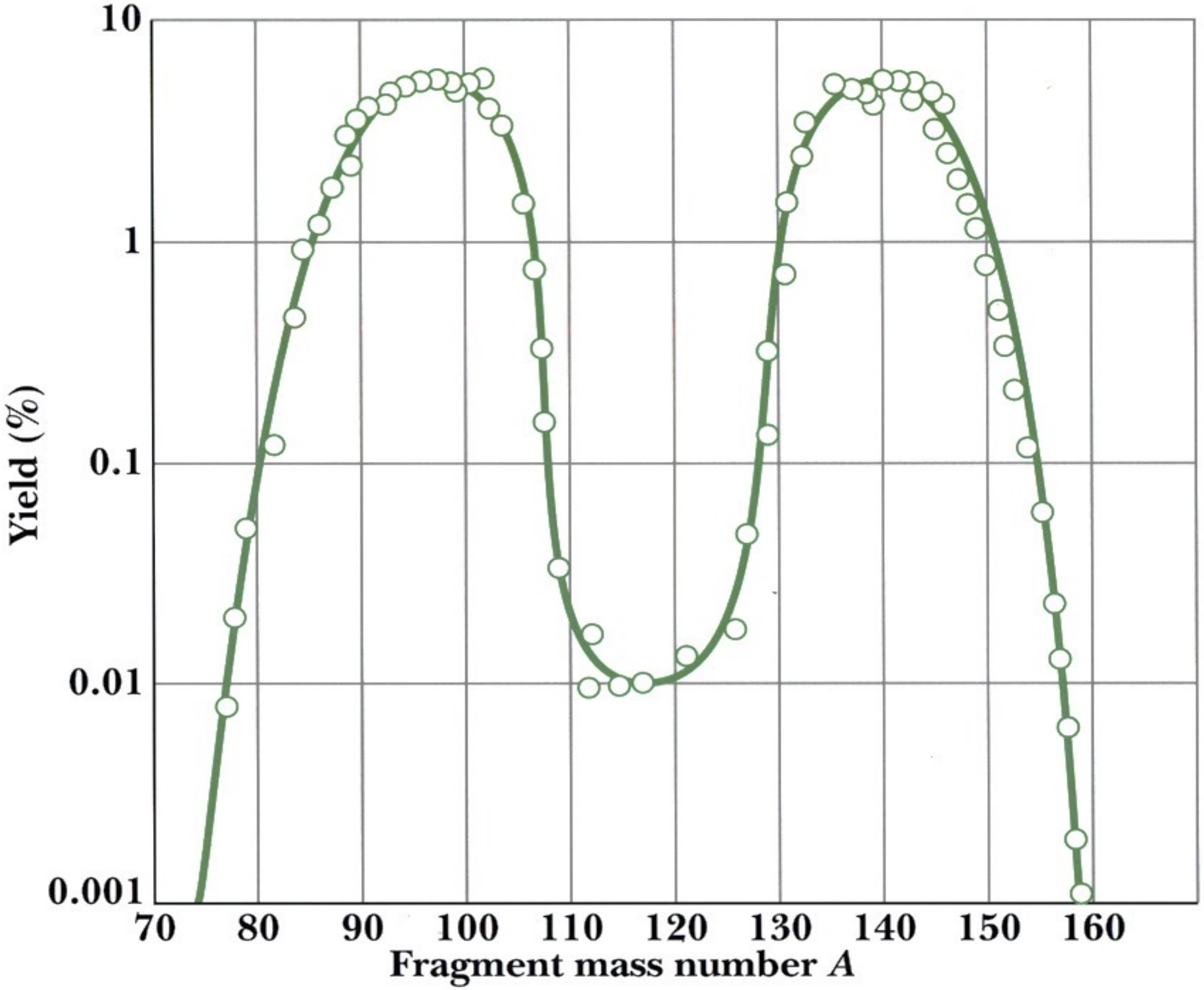
It is *this* nucleus that actually undergoes fission, splitting into two fission fragments.

These fragments (between them) rapidly emit two prompt neutrons, leaving (in a typical case) ^{140}Xe (Z = 54) and ^{94}Sr (Z =38) as fission fragments.

Thus, the overall fission equation for this event is

$$^{235}\mathrm{U} + \mathrm{n} \rightarrow {}^{236}\mathrm{U}^{*} \rightarrow {}^{140}\mathrm{Xe} + {}^{94}\mathrm{Sr} + 2\mathrm{n} \;\; .$$

The fragments ^{140}Xe and ^{94}Sr are both highly unstable, undergoing beta decay until each reaches a stable end product.



Distribution by mass number of the fragments produced when ^{235}U is bombarded with thermal neutrons.

For xenon and strontium, the decay chains are:

$$^{140}\mathrm{Xe} \rightarrow {}^{140}\mathrm{Cs} \rightarrow {}^{140}\mathrm{Ba} \rightarrow {}^{140}\mathrm{La} \rightarrow {}^{140}\mathrm{Ce}\ \text{(stable)}$$

$$^{94}\mathrm{Sr} \rightarrow {}^{94}\mathrm{Y} \rightarrow {}^{94}\mathrm{Zr}\ \text{(stable)}$$

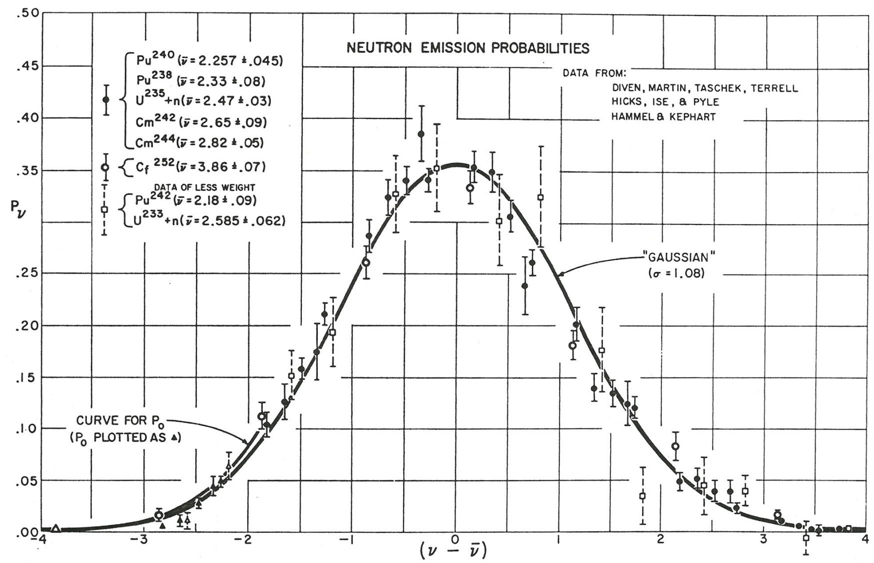
The average number of prompt neutrons, ν, is characteristic of the particular fission process.

For thermal-neutron induced fission, the experimentally observed values are

$\bar{\nu}$ = 2.48 for ^{233}U

$\bar{\nu}$ = 2.42 for ^{235}U, and

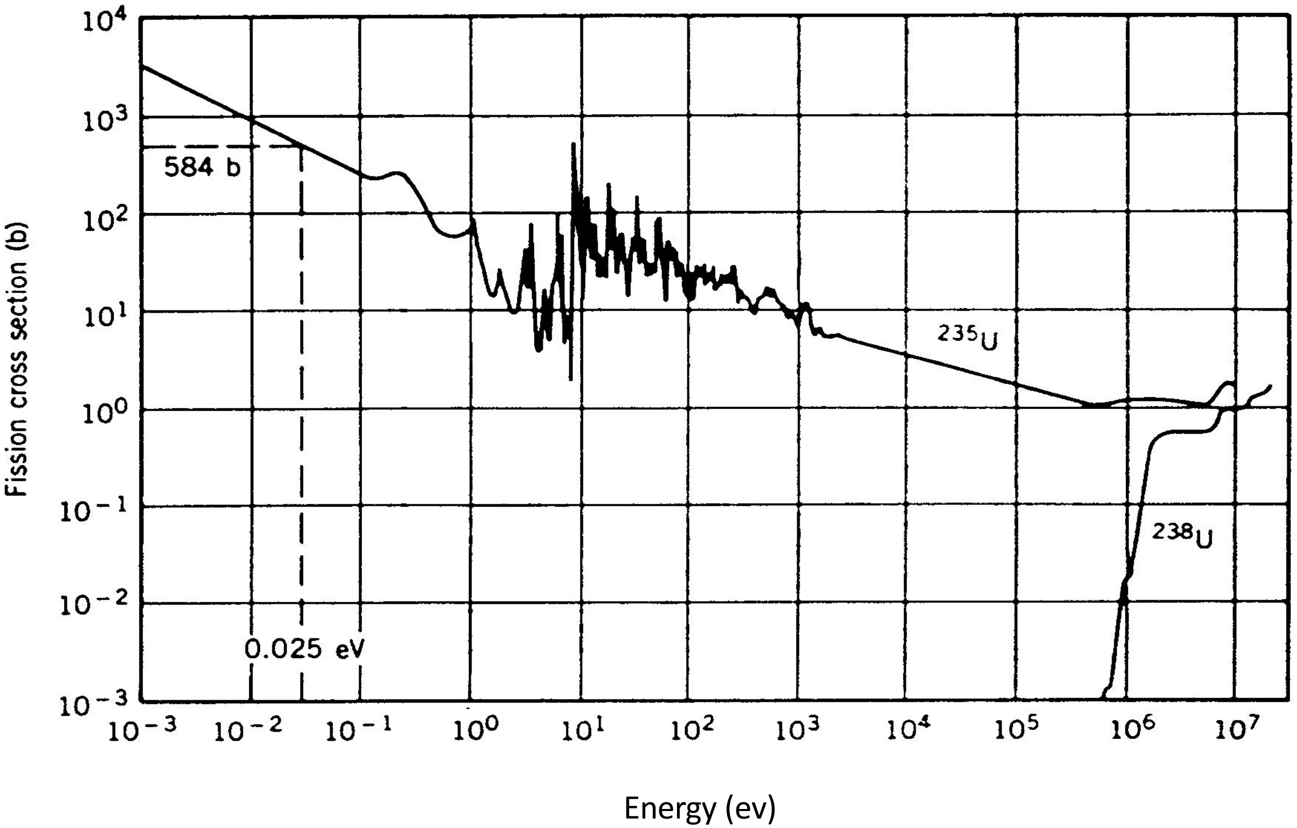
$\bar{\nu}$ = 2.86 for ^{239}Pu .



Distribution of fission neutrons (Terrell, 1965)

Fission cross-sections

The cross-sections for neutron-induced fission of ^{235}U and ^{238}U are shown in the next figure (note: 1 b = 10^{-28} m^2)



Cross-sections for neutron-induced fission of ^{235}U and ^{238}U (K.S. Krane, *Introductory Nuclear Physics*, 1988)

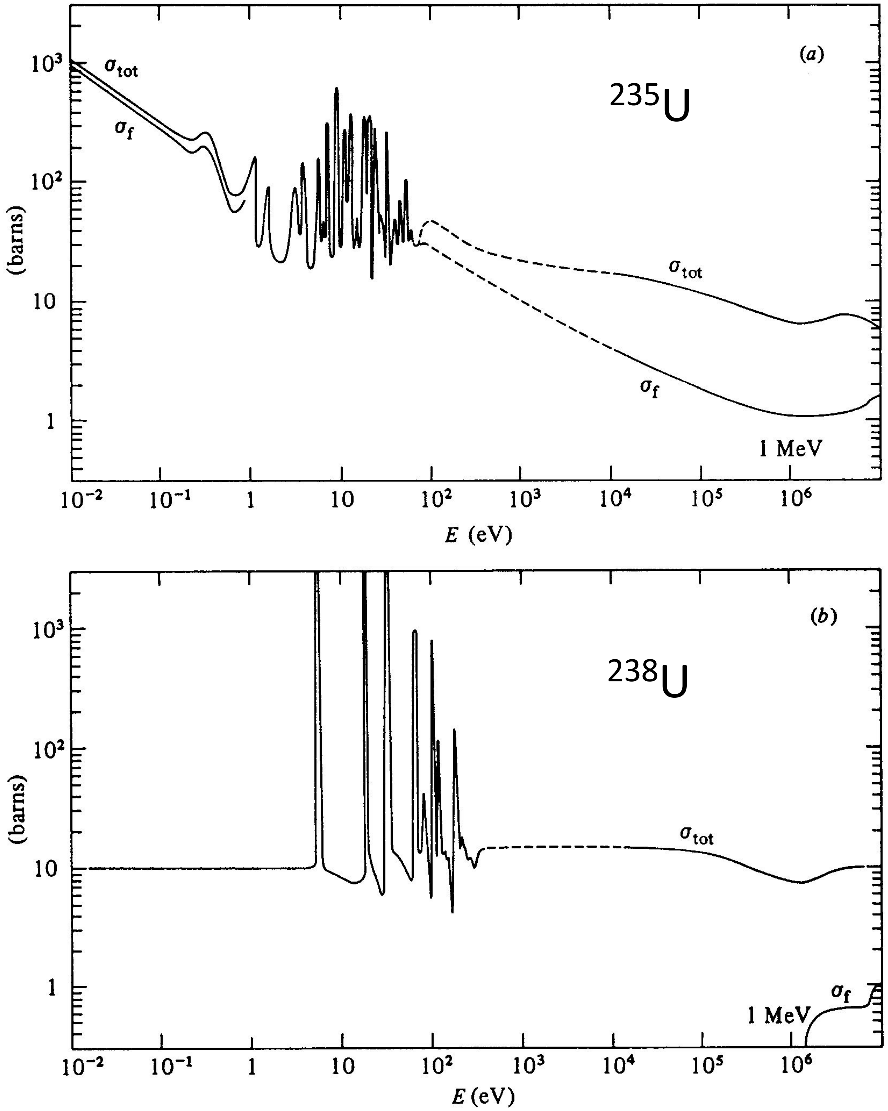
Of course, neutron-induced fission is not the only possible process for neutrons. In addition to fission, **scattering** and **radiative capture** processes can also take place. The total cross-section σ_T is given by

$$\sigma_T = \sigma_s + \sigma_c + \sigma_f$$

where the subscripts *s*, *c* and *f* refer to scattering, capture and fission, respectively.

The magnitude and contribution of each of these processes is highly dependent on the energy of the incoming neutron and the target nucleus.

The contribution of neutron-induced fission to the total cross-sections as a function of energy for ^{235}U and ^{238}U is illustrated in the next figure.



Total and neutron-induced fission cross-sections as a function of energy for neutrons incident on (a) ^{235}U, and (b) ^{238}U. Note that both the horizontal and vertical scales are logarithmic (Garber & Kinsey, 1976)

Energy from fission

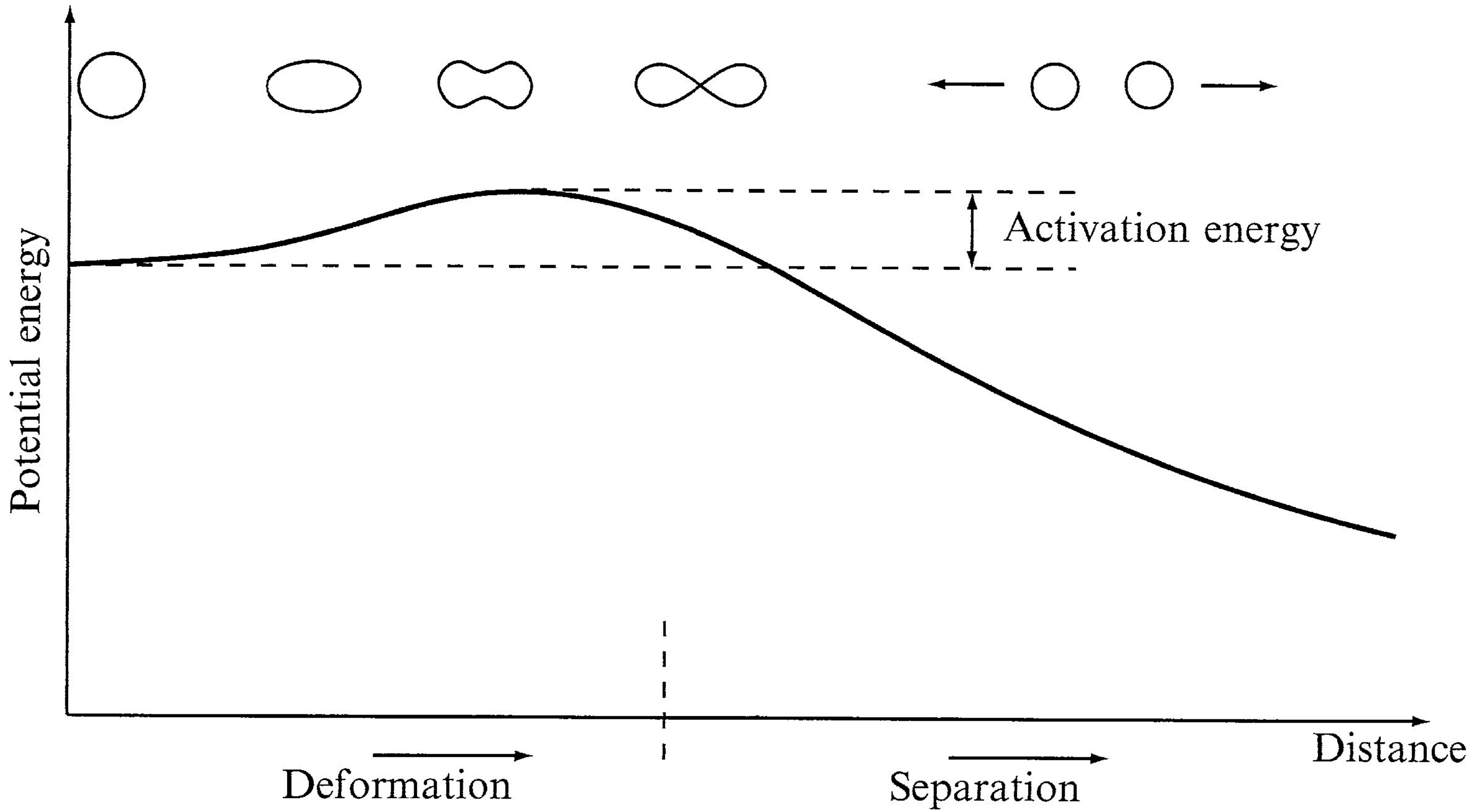
Induced fission will occur only if the absorbed neutron provides an **excitation energy** E_{ex} great enough to overcome the fission barrier (or **critical energy**) E_f.

As an example, let us consider the capture of a thermal neutron by a ^{235}U nucleus to form the compound state $^{236}U^*$.

The excitation energy is

$$E_{ex} = \left[m(^{236}U^*) - m(^{236}U)\right]c^2$$

and can be found directly from the mass energies of ^{235}U and n, if we assume that the neutron's kinetic energy is negligible.


Potential energy
Activation energy
Distance
Deformation
Separation

$$m(^{236}U^*) = m(^{235}U) + m_n$$
$$= (235.043924 \text{ u} + 1.008665 \text{ u})$$
$$= 236.052589 \text{ u}$$

$m(^{236}\text{U}) = 236.045563$ u

and so

$$E_{ex} = (236.052589 - 236.045563 \text{ u}) \times 931.502 \text{ MeV/u}$$
$$= 0.007026 \times 931.502 \text{ MeV/u} = 6.5 \text{ MeV} .$$

The critical energy for ^{236}U is calculated to be 6.2 MeV. Thus, the energy needed to excite ^{236}U into a fissionable state is exceeded by the energy we get by adding a thermal neutron to ^{235}U.

This means that ^{235}U can be fissioned with low-energy neutrons, consistent with its large cross-section in the thermal region (*fissile material*).

The following table shows the results of similar calculations (fissionability test) for other high-mass nuclides.

Target nuclide	Fissioning nuclide	E_{ex} (MeV)	E_f (MeV) A + 1	Fission by thermal neutrons
^{235}U	^{236}U	6.5	6.2	Yes
^{238}U	^{239}U	4.8	6.6	No
^{239}Pu	^{240}Pu	6.4	6.0	Yes
^{243}Am	^{244}Am	5.5	6.3	No

For ^{239}Pu, as was the case for ^{235}U, we see that $E_{ex} > E_f$; this means that fission by thermal neutrons is predicted to occur for these nuclides.

For ^{238}U and ^{243}Am, we have $E_{ex} < E_f$; thus, there is not enough energy to surmount the barrier, or to tunnel through it effectively.

What about the energy released by the fission process and where does it go?

To see this, consider another typical fission process for ^{235}U

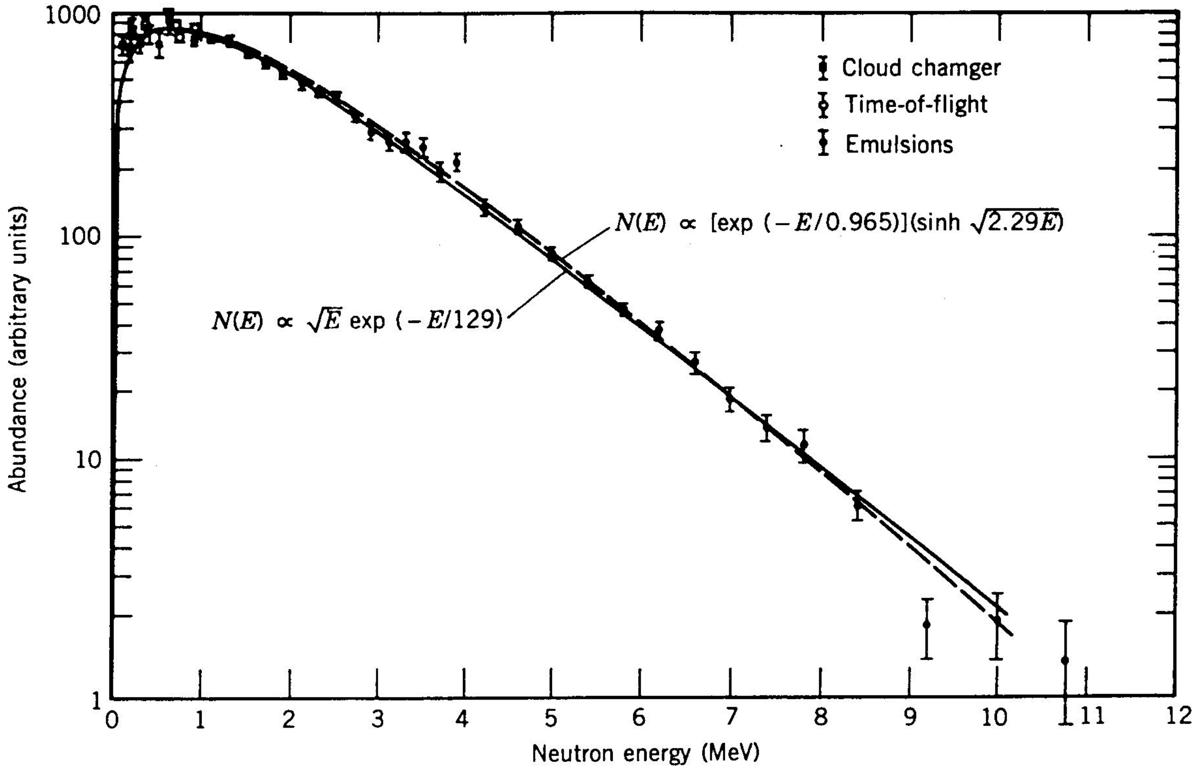
$$^{235}\mathrm{U} + \mathrm{n} \rightarrow {}^{236}\mathrm{U}^{*} \rightarrow {}^{141}\mathrm{Cs} + {}^{93}\mathrm{Rb} + 2\mathrm{n} \ .$$

The energy released can be calculated from the masses to be 181 MeV.

This result, although deduced for a specific set of final products, is typical of the average energy (~200 MeV) released in the fission of ^{235}U by thermal neutrons.

The bulk of the disintegration energy (~80%) appears as kinetic energy of the fission fragments. They are easily stopped and, in a solid medium, travel only a fraction of a mm from their point of origin.

The energy distribution of the prompt neutrons following fission of ^{235}U is shown below. The mean energy is ≈2 MeV and, with an average of about 2.5 neutrons per fission, the average energy carried by the neutrons is ≈5 MeV (2.5%).



Energy spectrum of neutrons emitted in the thermal neutron fission of ^{235}U (Leachman, 1956)

Other energy releases in fission appear as follows:

• prompt gamma rays:	8 MeV
• β decays from radioactive fragments:	19 MeV
• gamma decays from radioactive fragments:	7 MeV

These are only estimates and will depend on the exact nature of the fission products and their decays. However, within 1–2 MeV, they are characteristic of most decays.

The prompt gamma rays are emitted essentially at the instant of fission (later than the prompt neutrons, but still within 10^{-14} s).

The β and gamma radiations are emitted according to the decay chains associated with the heavy and light fragments.

The chain reaction

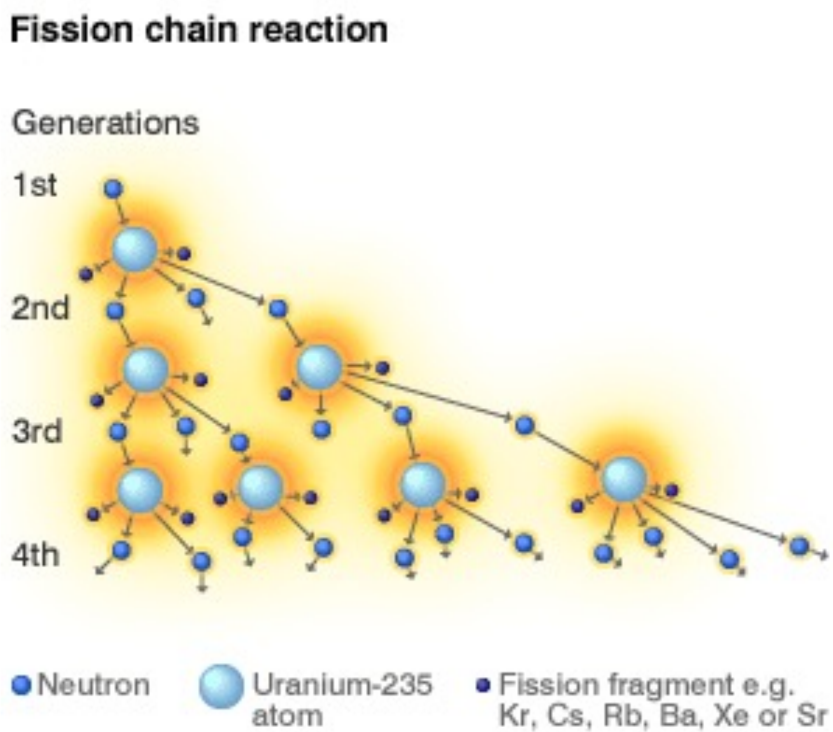
Since more than one neutron is produced per fission event, a **chain reaction** where these extra neutrons induce further fissions to produce another generation is, in principle, possible following a neutron induced fission in bulk uranium.

The chain reaction is characterised by the **neutron multiplication factor** *k*, which is defined as the ratio of the number of neutrons in one generation to that in the preceding generation:

$k < 1$ subcritical

$k = 1$ critical

$k > 1$ supercritical



Consider the abundances (by weight) of the various isotopes in natural uranium:

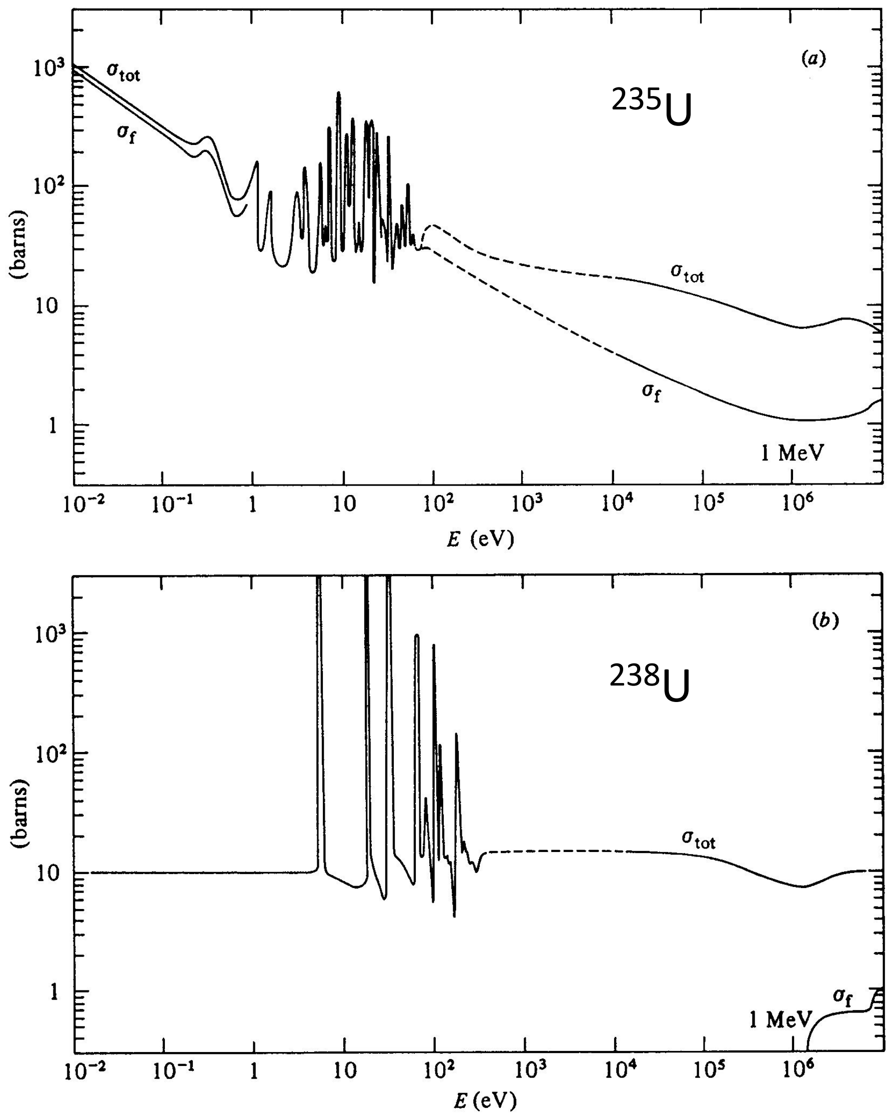
^{238}U: 99.28%

^{235}U: 0.72%

^{234}U: 0.0056% .

Given that natural uranium is over 99% ^{238}U, and that the threshold for neutron-induced fission for ^{238}U is >1 MeV, one might at first sight think that only a chain reaction maintained by fast neutrons is feasible in natural uranium. This is not, in fact, the case.

The very small fission cross-section σ_f of ~0.5 b for the fast fission of ^{238}U, and the relatively higher cross-section σ_s of ~4.5 b for inelastic (neutron) scattering (n, $n\gamma$), means that the energy is reduced without a chain reaction occurring.



Total and neutron-induced fission cross-sections as a function of energy for neutrons incident on (a) ^{235}U, and (b) ^{238}U. Note that both the horizontal and vertical scales are logarithmic (Garber & Kinsey, 1976)

Moderator

To utilise the favorable figures for thermal neutron induced fission in ^{235}U, it is necessary to slow down the fast neutrons which are produced in the fission process.

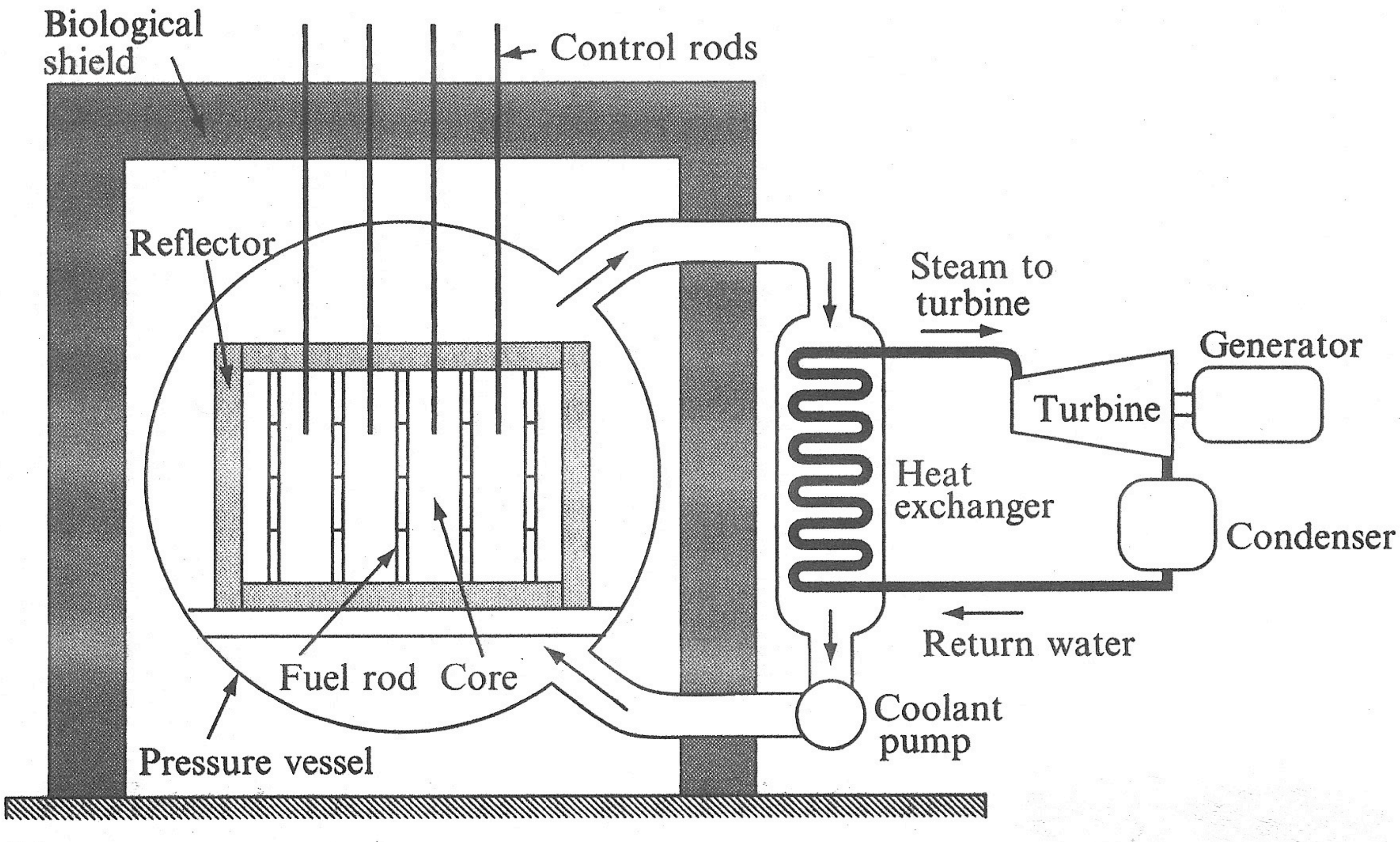
This is done by facilitating collisions with light nuclei in what we call a **moderator**. Materials used as moderators include light water (H_2O), graphite (i.e., C) and heavy water (D_2O).

Ordinary (light) water has a high capture cross-section for neutrons via the reaction $n + p \rightarrow d + \gamma$, and can only be used as a moderator if the fuel is enriched uranium.

Heavy water has a very small neutron capture cross-section, though when the reaction does occur it produces tritium (β^- emitter with a half-life of ~12 years), which is hazardous to biological systems.

Components of a nuclear power plant

A schematic layout of a typical power plant, based on a thermal fission reactor is shown in the next viewgraph.



Simplified schematic layout of a typical reactor power plant (PWR).