11.72a

TABLE 11.10a FACTORS FOR OBTAINING VARIANCES IN CHAIN BLOCK DESIGNS

Treatments	f <sub>22</sub>	$f_{11}$	f <sub>12</sub>
In same group	0	0	(b-1)/3
1 group apart	(b - 2)	(b - 1)	(b-1)/3
2 groups apart	(3b - 8)	(2b - 4)	(5b - 9)/3
3 groups apart		(3b - 9)	(9b - 25)/3

In finding the shortest distance apart, number the groups as follows.

Block

1	. 2	3	 b
G	<sub>b</sub> G <sub>1</sub>	$G_2$	$G_{b-1}$
G	$G_2$	$G_3$	$G_b$
g	L 92	$g_3$	$g_b$

where  $g_i$  stands for the groups of treatments that are replicated once. In table 11.7a, for instance, A and I are one group apart, since A is in  $G_5$  and I in  $G_4$ . Similarly A (in  $G_5$ ) and q (in  $g_2$ ) are 2 groups apart while C (in  $G_1$ ) and k (in  $g_1$ ) are considered in the same group.

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#### TABLE 11.3 INDEX TO PLANS

t	k	<b>r</b>	$\boldsymbol{b}$	λ†	$\boldsymbol{\mathit{E}}$	Plan	Туре
4	2	3	6	1	.67	11.1	v
	3	3	4	2	.89	*	v
5	2	4	10	1	.62	11.2	v
	3	6	10	3	.83	11.1a	$\mathbf{v}$
	4	4	5	3	.94	*	v
6	2	5	15	1	.60	11.3	I
	3	5	10	<b>2</b>	.80	11.4	III
	3	10	20	4	.80	11.5	I
	4	10	15	6	.90	11.6	$\Pi$
	5	5	6	4	.96	*	V
7	2	6	21	1	.58	11.2a	II
	3	3	7	1	.78	11.7	v
	4	4	7	<b>2</b>	.88	11.8	v
	6	6	7	5	.97	*	v
8	<b>2</b>	7	28	1	.57	11.9	I
	4	7	14	3	.86	11.10	I
	7	7	8	6	.98	*	v
9	2	8	36	1	.56	11.3a	II
	4	8	18	3	.84	11.11	$\mathbf{II}$

TABLE 11.3 INDEX TO PLANS (Continued)

							_
t	$\boldsymbol{k}$	r	ь	λ†	$\boldsymbol{E}$	Plan	Type
9	5	10	18	5	.90	11.12	$\mathbf{II}$
	6	8	12	5	.94	11.13	$\mathbf{II}$
	8	8	9	7	.98	*	IV
10	2	9	45	1	.56	11.14	I
	3	9	30	<b>2</b>	.74	11.15	11
	4	6	15	2	.83	11.16	III
	5	9	18	4	.89	11.17	III
	6	9	15	5	.93	11.18	III
	9	9	10	8	.99	*	IV
11	<b>2</b>	10	55	1	.55	11.4a	$\mathbf{II}$
	5	5	11	2	.88	11.19	IV
	6	6	11	3	.92	11.20	IV
	10	10	11	9	.99	*	$\mathbf{IV}$
13	3	6	26	1	.72	11.21	II
	4	4	13	1	.81	11.22	IV
	9	9	13	6	.96	11.23	IV
15	3	7	35	1	.71	11.24	I
	7	7	15	3	.92	11.25	IV
	8	8	15	4	.94	11.26	$\mathbf{IV}$
16	6	6	16	2	.89	11.27	IV
	6	9	24	3	.89	11.28	II
	10	10	16	6	.96	11.29	IV
19	3	9	57	1	.70	11.30	II
	9	9	19	4	.94	11.31	IV
	10	10	19	5	.95	11.32	IV
21	3	10	70	1	.70	11.33	Ι
	5	5	21	1	.84	11.34	IV
	7	10	30	3	.90	11.35	III
25	4	8	50	1	.78	11.36	II
	9	9	25	3	.93	11.37	IV
28	4	9	63	1	.78	11.38	I
	7	9	36	2	.89	11.39	III
31	6	6	31	1	.86	11.40	IV
0.77	10	10	31	3	.93	11.41	IV
37	9	9 '	37	2	.91	11.42	IV
41	5	10	82 57	1 1	.82	11.43	II
5 <b>7</b>	8	8			.89	11.44	IV
73	9	9	73	1	.90	11.45	IV
91	10	10	91	1	.91	11.46	IV

<sup>†</sup> Number of times that two treatments appear together in the same block.

Plan 11.1  $t = 4, k = 2, r = 3, b = 6, \lambda = 1, E = .67, Type V$ 

Block	Block Rep. I			Rep	o. II		Rep	. III
(1)	1	2	(3)	1	3	(5)	1	4
(2)	3	4	(4)	2	4	(6)	2	3

Plan 11.2  $t = 5, k = 2, r = 4, b = 10, \lambda = 1, E = .62, Type V$ 

Block	_	os. I d II			s. III l IV
(1)	1	2	(6)	1	4
(2)	3	4	(7)	2	3
(3)	2	5	(8)	3	5
(4)	1	3	(9)	1	5
(5)	4	5	(10)	2	4

Plan 11.1a 
$$t = 5, k = 3, r = 6, b = 10, \lambda = 3, E = .83, Type V$$

]	Rep	s. I	, II,	F	leps	3. I	v, v,
Block	21	ıd I	II			id Y	
(1)	1	2	3	(6)	1	2	4
(2)	1	2	5	(7)	1	3	4
(3)	1	4	5	(8)	1	3	5
(4)	2	3	4	(9)	2	3	5
(5)	3	4	5	(10)	2	4	5

Plan 11.3 
$$t = 6, k = 2, r = 5, b = 15, \lambda = 1, E = .60, Type I$$

Block	k Rej			Rep	o. II		Rep	o. III		Rep	. IV		Rep	. v
(1)	1	2	(4)	1	3	(7)	ī	4	(10)	1	5	(13)	1	6
(2)	3	4	(5)	2	5	(8)	2	6	(11)	2	4	(14)	2	3
(3)	5	6	(6)	4	6	(9)	3	5	(12)	3	6	(15)	4	5

Plan 11.4 
$$t = 6$$
,  $k = 3$ ,  $r = 5$ ,  $b = 10$ ,  $\lambda = 2$ ,  $E = .80$ , Type III

Block	ζ.						
(1)	1	2	5	(6)	$\overline{2}$	3	4
(2)	1	2	6	(7)	2	3	
(3)	1	3	4	(8)	2	4	6
(4)	1	3	6	(9)	3	5	6
(5)	1	4	5	(10)	4	5	6

<sup>\*</sup>These plans are constructed by forming all possible combinations of the t numbers in groups of size k. The number of blocks b serves as a check that no group has been missed.

 $t = 7, k = 4, r = 4, b = 7, \lambda = 2, E = .88, Type V$ 

Plan 11.8

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Plan 11.5  $t = 6, k = 3, r = 10, b = 20, \lambda = 4, E = .80, Type I$ 

. ,	,	,	,					
Rep. I		Rep. II		Rep. III		Re	р. 🌃	,
1 2 3	(3)	1 2 4	(5)	1 2 5	(7)	1	2 6	
4 5 6	(4)	3 5 6	(6)	3 4 6	(8)	3	4 5	
Rep. V		Rep. VI		Rep. VII		Rep	. VII	I
1 3 4	(11)	1 3 5	(13)	1 3 6	(15)	1	4 5	
2 5 6	(12)	2 4 6	(14)	2 4 5	(16)	2	3 6	
Rep. IX		Rep. X						
1 4 6	(19)	1 5 6						
2 3 5	(20)	2 3 4						
	Rep. V 1 3 4 2 5 6 Rep. IX 1 4 6	Rep. I 1 2 3 4 5 6 (4) Rep. V 1 3 4 (11) 2 5 6 (12) Rep. IX 1 4 6 (19)	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$					

Plan 11.6  $t = 6, k = 4, r = 10, b = 15, \lambda = 6, E = .90, Type II$ 

loc	k R	leps. I	and l	II		Re	ps. II	I and	IV		$\mathbf{R}$	eps. V	and	VI
(1)	1	2	3	4	(4)	1	2	3	5	(7)	1	2	3	6
(2)	1	4	5	6	(5)	1	2	4	6	(8)	1	3	4	5
(0)	~	_			(0)	2	4	5	6	(0)	2	4	5	6
(3)	<b>2</b>	3	5	6	(6)	3	4	o	U	(9)	-	**		
3)	-		and		(0)	_	eps. D			(9)		- 4		
o) O)	-				(13)	_				(9)				
	-	s. VII		VIII		_	eps. I	X and	X	(9)		*		

Plan 11.2a  $t = 7, k = 2, r = 6, b = 21, \lambda = 1, E = .58$ , Type II

	Rej	ps, I		Reps	. III -		Rep	s. V		
Block	and	lI f		and	IV		and	and VI		
(1)	1	2	(8)	1	3	(15)	1	4		
(2)	2	6	(9)	2	4	(16)	2	3		
(3)	3	4	(10)	3	5	(17)	3	6		
(4)	4	7	(11)	4	6	(18)	4	5		
(5)	1	5	(12)	5	7	(19)	2	5		
(6)	5	6	(13)	1	6	(20)	6	7		
(7)	3	7	(14)	2	7	(21)	1	7		

Plan 11.7  $t = 7, k = 3, r = 3, b = 7, \lambda = 1, E = .78, Type V$ 

Block

(1)	1	2	4	(3)		4		(5)	5	6	1
(2)		3		(4)	4	5	7	(6)	6	7	2

(7) 7 1 3

```
Block
       (1) 3 5 6 7
                               (4) 1 2 3 6
                                                       (7) 2 4 5 6
       (2) \ \overline{1} \ 4 \ 6 \ 7
                               (5) 2 3 4 7
            1 2 5 7
                               (6) 1 3 4 5
Plan 11.9
              t = 8, k = 2, r = 7, b = 28, \lambda = 1, E = .57, Type I
    Block Rep. I
                                                                  Rep. IV
                             Rep. II
                                               Rep. III
      (1) 1
                2
                                          (9) 1
                                                     4
                                                            (13) 1
                        (5) 1
      (6) 2
                                         (10) 2
                                                            (14) 2
                                         (11) \overline{3}
      (3) 5
                        (7)
                            4
                                                            (15) 4
                        (8) 6
                                         (12) 5
                                                     8
                                                            (16) 6
      (4) 7
                             Rep. VI
                                               Rep. VII
           Rep. V
                       (21) \overline{1}
                                         (25) 1
     (17) 1
                                         (26) 2
     (18) 2
                       (22) 2
                       (23) 3
                                         (27) 3
     (19) 3
                                               4
     (20) 5
                       (24)
                            4
                                          (28)
Plan 11.10 t = 8, k = 4, r = 7, b = 14, \lambda = 3, E = .86, Type I
                                                Rep. III
                                                                   Rep. IV
 Block Rep. I
                            Rep. II
   (1) \overline{1} \overline{2} \overline{3} \overline{4}
                      (3) 1 2 7 8
                                                              (7) \ \overline{1 \ 4 \ 6 \ 7}
                                          (5) 1 3 6 8
   (2) 5 6 7 8
                      (4) 3 4 5 6
                                          (6) 2 4 5 7
                                                              (8) 2 3 5 8
         Rep. V
                            Rep. VI
                                               Rep. VII
   (9) 1 2 5 6
                     (11) 1 3 5 7
                                         (13) 1 4 5 8
  (10) 3 4 7 8
                     (12) 2 4 6 8
                                         (14) 2 3 6 7
Plan 11.3a t = 9, k = 2, r = 8, b = 36, \lambda = 1, E = .56, Type II
         Reps. I
                           Reps. III
                                               Reps. V
                                                                  Reps. VII
  Block and II
                            and IV
                                                and VI
                                                                  and VIII
   (1) 1
                      (10) 1
                                          (19)
                                                             (28) 1
                                                                         5
    (2) 2
                      (11) 2
                                          (20)
                                               2
                                                             (29) \ \ 2
    (3)
        3
                      (12) 3
                                          (21) \ \ 2
                                                             (30) \ \ \overline{3}
        4
    (4)
                      (13) 4
                                          (22)
                                                             (31) 4
    (5) 5
                      (14) \ 5
                                          (23) 5
                                                             (32) \ \ 3
    (6) 1
                      (15) 6
                                          (24) 6
                                                      8
                                                             (33)
                                                                   6
    (7) 3
                      (16) 1
                                          (25)
                                                             (34)
                                                                   2
    (8) 8
                      (26) 1
                                                             (35) 7
    (9) \ \ 5
                      (18) \ \ 2
                                          (27) \ \ \overline{3}
                                                             (36)
```

Plan 11.11 $t = 9, k = 4, r = 8, b = 18, \lambda = 3, E$	= 84, Type II	[
----------------------------------------------------------	---------------	---

Block		eps. I, a					-	V, VI, nd VII		
(1)	ī	4	6	7	(10)	1	2	5	7	
(2)	$\overline{2}$	6	8	9	(11)	2	3	5	6	
(3)	1	3	8	9	(12)	3	4	7	9	
(4)	1	2	3	4	(13)	1	2	4	9	
(5)	1	5	7	8	(14)	1	5	6	9	
(6)	4	5	6	9	(15)	1	3	6	8	
(7)	2	3	6	7	(16)	4	6	7	8	
(8)	2	4	5	8	(17)	3	4	5	8	
(9)	3	5	7	9	(18)	2	7	8	9	

Plan 11.12  $t = 9, k = 5, r = 10, b = 18, \lambda = 5, E = .90, Type II$ 

		Rep	s. I	, II	Ι,	]	Reps	. VI	, V	Η,
Block	( I)	II, I	ν,	and	V	V	ΙΙΪ,	IX,	an	ďΧ
(1)	1	2	3	7	8	(10) 1	2	3	5	9
(2)	1	2	4	6	8	(11) 1	2	5	6	8
(3)	2	3	5	8	9	(12) 1	3	4	5	6
(4)	2	3	4	6	9	$(13)$ $\bar{2}$	3	4	7	8
(5)	1	3	4	5	7	(14) 2	4	5	7	9
(6)	2	4	5	6	7	(15) 3	5	6	7	8
(7)	1	3	6	7	9	$(16)$ $\bar{1}$	4	7	8	9
(8)	1	4	5	8	9	$(17) \ \bar{3}$	4	6	8	9
(9)	5	6	7	8	9	$(18) \ \bar{1}$	2	6	7	9

Plan 11.13  $t = 9, k = 6, r = 8, b = 12, \lambda = 5, E = .94$ , Type II

אטטנע		rect	JS. J	L ELLI	u ı	1		к	eps	. II	Laı	nd.	ĮV
(1)	1	2	4	5	7	8	(4)	1	2	5	6	7	9
<b>(2)</b>	2	3	5	6	8	9	(5)	1	3	4	5	8	9
(3)	1	3	4	6	7	9	(6)	$\overline{2}$	3	4	6	7	8
	_1	Зер	s. V	an	d V	Ι	]	Rej	ps.	VII	an	d V	 'III
(7)	1	3	5	- 6	7	8	(10)	4,	5	6	7	8	9
(8)	1	2	4	6	8	9	(11)	1	2	3	4	5	6
(9)	2	3	4	- 5	7	9	(12)	1	2	3	7	8	9

Plan 11.14 t	= 10, k = 2,	$r=9, b=45, \lambda=$	1, E = .56, T	ype I
Block Rep. I			Rep. I	
$(1)  \overline{1}  2$		3 (11) 1 4	$(16) \ \overline{1} \ \ \xi$	5 (21) 1 6
(2) 3 4	(7) 2	7 (12) 2 10	(17) 2 8	$\overline{3}$ (22) $\overline{2}$ $\overline{9}$
(3) 5 6	(8) 4	8 (13) 3 7	(18) 3 10	$\overline{0}$ (23) $\overline{3}$ 8
(4) 7 8	(9) 5	9 (14) 5 8	(19) 4 9	(24) 4 10
(5) $\overline{9}$ 10		$\overline{0}$ (15) $\overline{6}$ 9	(20) 6 7	7 (25) 5 7
Rep. V	Rep. V	 /II Rep. VIII	Rep. I	x
(26) $\frac{1}{1}$ $\frac{7}{7}$		8 (36) 1 9	(41) 1 10	-
$(27)$ $\frac{1}{2}$ $\frac{1}{6}$		$\frac{3}{3}$ (37) $\frac{1}{2}$ $\frac{3}{4}$	(42) 2	_
(28) 3 9	• • • —	$\frac{3}{6}$ (38) $\frac{2}{3}$ $\frac{1}{5}$	(43) 3 6	-
(29) 4 5		$\frac{0}{0}$ (39) $\frac{0}{6}$ $\frac{0}{8}$	(44) 4 7	
$(30)$ $\frac{1}{8}$ $\frac{10}{10}$	` ′	$\frac{1}{9}$ (40) $\frac{3}{7}$ $\frac{3}{10}$	(45) 8 9	-
Di 11 15 . 4	10 7 2 -	0 } 20 \		·
Plan 11.15 t	Reps. I,	$b = 9$ , $b = 30$ , $\lambda = 2$ Reps. IV,		Reps. VII,
Block	II, and III	V, and VI		VIII, and IX
(1)	1 2 3	$(11) \frac{7}{1 \ 2 \ 4}$	(21)	$\phantom{00000000000000000000000000000000000$
(2)	2 5 8	$(12)  \overline{2}  \overline{3}  \overline{6}$	• •	$\frac{2}{6}$ $\frac{6}{7}$
(3)	3 4 7	(13) 3 4 8	` '	3 8 9
(4)	1 4 6	$(14) \ \overline{4} \ 5 \ 9$		2 4 10
(5)	5 7 8	$(15) \ \overline{1} \ \overline{5} \ \overline{7}$	(25)	3 5 6
(6)	4 6 9	(16) 6 8 9	• •	1 6 8
(7)	1 7 9	(17) 3 7 10	, ,	2 7 9
(8)	2 8 10	$(18) \ \overline{1} \ 8 \ 10$		4 7 8
(9)	3 9 10	$(19)  \frac{2}{2}  \frac{5}{5}  9$	, ,	1 9 10
(10)	5 6 10	$(20) \frac{2}{6} \frac{1}{7} \frac{10}{10}$	, ,	4 5 10
	- 10 h - 4 m	$=6, b=15, \lambda=2$		
Block	— 10, n — <del>1</del> , r	0, 0 10, X 2	<i>i, E</i> = .00, 13	ype III
(1) 1	2 3 4	(6) 1 6 8 1	<u>0</u> (11)	3 5 9 10
(2) 1	2 5 6	(7) 2 3 6	9 (12)	3 6 7 10
(3) 1	3 7 8	(8) 2 4 7 1	<u>0</u> (13)	3 4 5 8
$(4)  \overline{1}$	4 9 10	(9) 2 5 8 1	0 (14)	4 5 6 7
(5) 1	5 7 9	(10) 2 7 8	9 (15)	4 6 8 9
	= 10, k = 5, i	$b=9$ , $b=18$ , $\lambda=4$	$E = .89, T_1$	ype III
Block	<del></del>			
(1) $\frac{1}{1}$ $\frac{2}{1}$	3 4 5	$\begin{array}{cccccccccccccccccccccccccccccccccccc$		2 5 6 8 10
$(2) \frac{1}{1} \frac{2}{1}$	3 6 7	(8) 1 4 8 9		2 6 7 9 10
(3) $\frac{1}{1}$ $\frac{2}{1}$	4 6 9	(9) 1 5 7 9		3 4 6 7 10
$(4)  \frac{1}{1}  \frac{2}{1}$	5 7 8	(10) 2 3 4 8		3 4 5 7 9
(5) 1 3		(11) 2 3 5 9		3 5 6 8 9
(6) 1 3	7 8 10	(12) 2 4 7 8	9 (18)	4 5 6 7 8

Plan 11.18  $t = 10, k = 6, r = 9, b = 15, \lambda = 5, E = .93$ , Type III

13100	JK.							_							_					
(1)	1	2	4	5	8	9	(6)	2	3	4	6	8	10	(11)	1	4	5	7	8	10
(2)	5	6	7	8	9	10	(7)	1	2	6	7	9	10	(12)	1	2	3	5	7	10
(3)	$\tilde{2}$	4	5	6	9	10	(8)	1	3	5	6	8	9	(13)	2	3	5	6	7	8
(4)	1	2	4	6	7	8	(9)	1	2	3	8	9	10	(14)	1	3	4	5	6	10
(5)	3	4	7	8	9	10	(10)	2	3	4	5	7	9	(15)	1	3	4	6	7	9

Plan 11.4a 
$$t = 11, k = 2, r = 10, b = 55, \lambda = 1, E = .55, Type II$$

Block		ps. I d II		Reps			•	os. V i VI			s, VII VIII			в. IX d <u>X</u>
(1)	1	2	(12)	1	3	(23)	1	4	(34)	1	5	(45)	1	6
(2)	$\overline{2}$	11	(13)	2	6	(24)	2		(35)	2	9	(46)	2	5
(3)	3	10	(14)	3	5	(25)	3	7	(36)	3	6	(47)	3	4
(4)	4	5	(15)	4	10	<b>(26)</b>	4	6	(37)	2	4	(48)	4	7
(5)	5	6	(16)	5	9	(27)	5	10	(38)	5	7	(49)	5	8
(6)	6	7	(17)	6	8	(28)	6	9	(39)	6	10	(50)	6	11
(7)	1	7	(18)	2	7	(29)	7	11	(40)	7	8	(51)	7	10
(8)	3	8	(19)	1	8	(30)	2	8	(41)	4	8	(52)	8	9
(9)	4	9	(20)	7	9	(31)	1	9	(42)	9	11	(53)	3	9
(10)	9	10	(21)	10	11	(32)	8	10	(43)	1	10	(54)	2	10
(11)	8	11	(22)	4	11	(33)	5	11	(44)	3	11	(55)	1	11

Plan 11.19  $t = 11, k = 5, r = 5, b = 11, \lambda = 2, E = .88$ , Type IV Block

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DIOOL				
(1)	1	2	3	5
<b>(2)</b>	2	3	4	6
(3)	3	4	5	7

Plan 11.20 
$$t = 11, k = 6, \tau = 6, b = 11, \lambda = 3, E = .92, Type IV$$

### Block

(1)	4	6	7	9	10	11
(2)	1	5	7	8	10	11
(3)	1	2	6	8	9	11
(4)	1	2	3	7	9	10
(5)	2	3	4	8	10	11

Plan 11.21 
$$t = 13, k = 3, r = 6, b = 26, \lambda = 1, E = .72, Type II$$

	F	teps.	I,			eps. 🧎	
Block	II,	and	III		V,	and	VI
(1)	1	3	9	(14)	2	5	6
(2)	2	4	10	(15)	3	6	7
(3)	3	5	11	(16)	4	7	8
(4)	4	6	12	(17)	5	8	9
(5)	5	7	13	(18)	6	9	10
(6)	1	6	8	(19)	7	10	11
(7)	2	7	9	(20)	8	11	12
(8)	3	8	10	(21)	9	12	13
(9)	4	9	11	(22)	1	10	13
(10)	5	10	12	(23)	1	2	11
(11)	6	11	13	(24)	2	3	12
(12)	1	7	12	(25)	3	4	13
(13)	2	8	13	(26)	1	4	5
	_				_		

Plan 11.22  $t = 13, k = 4, r = 4, b = 13, \lambda = 1, E = .81, Type IV$ 

# Block

(1)	1	2	4	10	(6)	6	7	9	2
(2)	2	3	5	11	(7)	7	8	10	3
(3)	3	4	6	12	(8)	8	9	11	4
(4)	4	5	7	13	(9)	9	10	12	5
(5)	5	6	8	1	(10)	10	11	13	6

(11)	11	12	1	7
(12)	12	13	2	8
(13)	13	1	3	9

Plan 11.23	t=13,k	= 9, r = 9, b	$= 13, \lambda = 6, E :$	= .96, Type IV
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(8	13	12	11	9	8	7	6	5	3
(9	13	12	10	9	8	7	6	4	1
(10	13	11	10	9	8	7	5	2	1
(11	12	11	10	9	8	6	3	2	1
(12	13	12	11	10	9	7	4	3	$\overline{2}$
(13	13	12	11	10	8	5	4	3	1
	13	12	11	9	6	5	4	2	ī

(8)	1	2	3	5	6	7	10	12	13
(9)	1	2	3	4	6	7	8	11	13
)	1	2	3	4	5	7	8	9	12
1)	2	3	4	5	6	8	9	10	13
	1	3	4	5	6	7	9	10	11

2 4 5 6 7 8 10 11 12

Rep. V

Plan 11.24 
$$t = 15, k = 3, r = 7, b = 35, \lambda = 1, E = .71, Type I$$

Block Rep. I					Į	Rep.	II		R	ep. ]	F	IV			
(1)	1	2	3	(6)	1	4	5	(11)	1	6	7	(16)	1	8	9
(2)	4	8	12	(7)	2	8	10	(12)	2	9	11	(17)	2	13	15
(3)	5	10	15	(8)	3	13	14	(13)	3	12	15	(18)	3	4	7
(4)	6	11	13	(9)	6	9	15	(14)	4	10	14	(19)	5	11	14
(5)	7	9	14	(10)	7	11	12	(15)	5	8	13	(20)	6	10	12
	)	Rep.	v		F	lер.	vi		R	ep. ¹	VII.				
(21)	1	10	11	(26)	1	12	13	(31)	1	14	15				
(22)	2	12	14	(27)	2	5	7	(32)	2	4	6				
(23)	3	5	6	(28)	3	9	10	(33)	3	8	11				
(24)	4	9	13	(29)	4	11	15	(34)	5	9	12				
(25)	7	8	15	(30)	6	8	14	(35)	7	10	13				

Plan 11.25 
$$t = 15, k = 7, r = 7, b = 15, \lambda = 3, E = .92$$
, Type IV

See incomplete latin squares Plan 13.7; randomize units in blocks ignoring replications.

Plan 11.26 
$$t = 15, k = 8, r = 8, b = 15, \lambda = 4, E = .94$$
, Type IV

See incomplete latin squares Plan 13.8; randomize units in blocks ignoring replications.

Plan 11.27 
$$t = 16, k = 6, r = 6, b = 16, \lambda = 2, E = .89$$
, Type IV

See incomplete latin squares Plan 13.9; randomize units in blocks ignoring replications.

Plan 11.28  $t = 16, k = 6, r = 9, b = 24, \lambda = 3, E = .89$ , Type II

Reps. I, II, R					eps.	. IV	, V	,	R	ep	s. 1	Ή,	Π,			
Block and III				and VI								X				
6	11	12	(9)	1	3	6	8	13	15	(17)	1	4	5	8	10	11
′ 8	9	10	(10)	2	4	5	7	14	16	(18)	2	3	6	7	9	12
10	13	14	(11)	5	7	9	11	13	15	(19)	5	8	.9	12	13	16
12	15	16	(12)	6	8	10	12	14	16	(20)	1	4	6	7	13	16
10	15	16	(13)	2	4	6	8	9	11	(21)	1	4	9	12	14	15
12	13	14	(14)	$\overline{1}$	3	5	7	10	12	(22)	6	7	10	11	14	15
7 8	13	14	(15)	2	4	10	12	13	15	(23)	2	3	10	11	13	16
5 6	15	16	(16)	1	3	9	11	14	16	(24)	2	3	5	8	14	15
	nd I 5 6 7 8 9 10 1 12 9 10 1 12 7 8	nd III 5 6 11 7 8 9 9 10 13 1 12 15 9 10 15 1 12 13 7 8 13	nd III  5 6 11 12  7 8 9 10  9 10 13 14  1 12 15 16  9 10 15 16  1 12 13 14  7 8 13 14	nd III  5 6 11 12 (9)  7 8 9 10 (10)  9 10 13 14 (11)  1 12 15 16 (12)  9 10 15 16 (13)  1 12 13 14 (14)  7 8 13 14 (15)	nd III    5   6   11   12   (9)   1   7   8   9   10   (10)   2   9   10   13   14   (11)   5   1   12   15   16   (12)   6   9   10   15   16   (13)   2   1   12   13   14   (14)   1   7   8   13   14   (15)   2	nd III    5   6   11   12   (9)   1   3     7   8   9   10   (10)   2   4     9   10   13   14   (11)   5   7     1   12   15   16   (12)   6   8     9   10   15   16   (13)   2   4     1   12   13   14   (14)   1   3     7   8   13   14   (15)   2   4	and III and III and 5 6 11 12 (9) 1 3 6 7 8 9 10 (10) 2 4 5 9 10 13 14 (11) 5 7 9 1 12 15 16 (13) 2 4 6 1 12 13 14 (14) 1 3 5 7 8 13 14 (15) 2 4 10	and III     and V       5 6 11 12     (9) 1 3 6 8       7 8 9 10     (10) 2 4 5 7       9 10 13 14     (11) 5 7 9 11       1 12 15 16     (12) 6 8 10 12       9 10 15 16     (13) 2 4 6 8       1 12 13 14     (14) 1 3 5 7       7 8 13 14     (15) 2 4 10 12	and III     and VI       5 6 11 12     (9) 1 3 6 8 13       7 8 9 10     (10) 2 4 5 7 14       9 10 13 14     (11) 5 7 9 11 13       1 12 15 16     (12) 6 8 10 12 14       9 10 15 16     (13) 2 4 6 8 9       1 12 13 14     (14) 1 3 5 7 10       7 8 13 14     (15) 2 4 10 12 13	and III     and VI       5 6 11 12     (9) 1 3 6 8 13 15       7 8 9 10     (10) 2 4 5 7 14 16       9 10 13 14     (11) 5 7 9 11 13 15       1 12 15 16     (12) 6 8 10 12 14 16       9 10 15 16     (13) 2 4 6 8 9 11       1 12 13 14     (14) 1 3 5 7 10 12       7 8 13 14     (15) 2 4 10 12 13 15	and III     and VI       5 6 11 12     (9) 1 3 6 8 13 15     (17)       7 8 9 10     (10) 2 4 5 7 14 16     (18)       9 10 13 14     (11) 5 7 9 11 13 15     (19)       1 12 15 16     (12) 6 8 10 12 14 16     (20)       9 10 15 16     (13) 2 4 6 8 9 11     (21)       1 12 13 14     (14) 1 3 5 7 10 12     (22)       7 8 13 14     (15) 2 4 10 12 13 15     (23)	and III     and VI       5 6 11 12     (9) 1 3 6 8 13 15     (17) 1       7 8 9 10     (10) 2 4 5 7 14 16     (18) 2       9 10 13 14     (11) 5 7 9 11 13 15     (19) 5       1 12 15 16     (12) 6 8 10 12 14 16     (20) 1       9 10 15 16     (13) 2 4 6 8 9 11     (21) 1       1 12 13 14     (14) 1 3 5 7 10 12     (22) 6       7 8 13 14     (15) 2 4 10 12 13 15     (23) 2	and III     and VI       5 6 11 12     (9) 1 3 6 8 13 15     (17) 1 4       7 8 9 10     (10) 2 4 5 7 14 16     (18) 2 3       9 10 13 14     (11) 5 7 9 11 13 15     (19) 5 8       1 12 15 16     (12) 6 8 10 12 14 16     (20) 1 4       9 10 15 16     (13) 2 4 6 8 9 11     (21) 1 4       1 12 13 14     (14) 1 3 5 7 10 12     (22) 6 7       7 8 13 14     (15) 2 4 10 12 13 15     (23) 2 3	and III     and VI     and VI       5 6 11 12     (9) 1 3 6 8 13 15     (17) 1 4 5       7 8 9 10     (10) 2 4 5 7 14 16     (18) 2 3 6       9 10 13 14     (11) 5 7 9 11 13 15     (19) 5 8 9       1 12 15 16     (12) 6 8 10 12 14 16     (20) 1 4 6       9 10 15 16     (13) 2 4 6 8 9 11     (21) 1 4 9       1 12 13 14     (14) 1 3 5 7 10 12     (22) 6 7 10       7 8 13 14     (15) 2 4 10 12 13 15     (23) 2 3 10	and III     and VI     and I       5 6 11 12     (9) 1 3 6 8 13 15     (17) 1 4 5 8       7 8 9 10     (10) 2 4 5 7 14 16     (18) 2 3 6 7       9 10 13 14     (11) 5 7 9 11 13 15     (19) 5 8 9 12       1 12 15 16     (12) 6 8 10 12 14 16     (20) 1 4 6 7       9 10 15 16     (13) 2 4 6 8 9 11     (21) 1 4 9 12       1 12 13 14     (14) 1 3 5 7 10 12     (22) 6 7 10 11       7 8 13 14     (15) 2 4 10 12 13 15     (23) 2 3 10 11	and III       and VI       and IX         5 6 11 12       (9) 1 3 6 8 13 15       (17) 1 4 5 8 10         7 8 9 10       (10) 2 4 5 7 14 16       (18) 2 3 6 7 9         9 10 13 14       (11) 5 7 9 11 13 15       (19) 5 8 9 12 13         1 12 15 16       (12) 6 8 10 12 14 16       (20) 1 4 6 7 13         9 10 15 16       (13) 2 4 6 8 9 11       (21) 1 4 9 12 14         1 12 13 14       (14) 1 3 5 7 10 12       (22) 6 7 10 11 14         7 8 13 14       (15) 2 4 10 12 13 15       (23) 2 3 10 11 13

Plan 11.29 
$$t = 16, k = 10, r = 10, b = 16, \lambda = 6, E = .96$$
, Type IV

See incomplete latin squares Plan 13.10; randomize units in blocks ignoring replications.

Plan 11.30 
$$t = 19, k = 3, r = 9, b = 57, \lambda = 1, E = .70, Type II$$

See extended incomplete latin squares Plan 13.15a; randomize units in blocks ignoring replications.

Plan 11.31 
$$t = 19, k = 9, r = 9, b = 19, \lambda = 4, E = .94$$
, Type IV

See incomplete latin squares Plan 13.11; randomize units in blocks ignoring replications.

Plan 11.32 
$$t = 19, k = 10, r = 10, b = 19, \lambda = 5, E = .95$$
, Type IV

See incomplete latin squares Plan 13.12; randomize units in blocks ignoring replications.

Plan 11.33  $t = 21, k = 3, r = 10, b = 70, \lambda = 1, E = .70, Type I$ 

Block Rep. I

(1)	1 2	2 3	(8)	_1	4	15	(15)	1	5	17	(22)	1	6	9	(2	9)	1	7	21
(2)	4 8	5 6	(9)	2	5	11	(16)	2	4	14	(23)	2	7	16	(3	(0)	2	13	17
(3)	7 8	3 9	(10)	3	9	16	(17)	3	7	11	(24)	3	8	21	(3	1)	3	10	18
(4)	10 11	12	(11)	6	17	20	(18)	6	10	19	(25)	4	17	19	(3	2)	4	8	11
(5)	13 14	15	(12)	7	12	19	(19)	8	16	20	(26)	5	10	13	(3	3)	5	16	19
(6)	16 17	7 18	(13)	8	13	18	(20)	9	15	18	(27)	11	15	20	(3	4)	6	12	15
(7)	19 20	21	(14)	10	14	21	(21)	12	13	21	(28)	12	14	18	(3	5)	9	14	20
																-			
	Rep.	VI		Re	p. V	/II		Re	p. V	Ш		$\mathbf{R}$	ер	IX			$\mathbf{R}\epsilon$	ep.	X
(36)		VI 3 10	(43)		p. V 11		(50)		p. V 12		(57)			$\frac{\mathrm{IX}}{19}$	(6	- i4)		ер. 14	
(36) (37)		3 10	()	1	11	18	(50) (51)		12		(57) (58)	1	13			i4) i5)	1		16
, ,	1 8	3 10 3 19	()	1	11 10	18		1 2	12	20 8	` '	1 2	13 9	19	(6		1	14 15	16
(37)	1 8 2 18 3 18	3 10 3 19	(44) (45)	$\frac{1}{2}$	11 10 5	18 20 12	(51)	1 2 3	12 6	20 8 19	(58)	1 2 3	13 9 4	19 12	(6 (6	5)	1 2	14 15 6	16 21
(37) (38)	1 8 2 18 3 18	3 10 3 19 5 17 2 16	(44) (45)	1 2 3 4	11 10 5	18 20 12 13	(51) (52)	1 2 3	12 6 14 18	20 8 19	(58) (59)	1 2 3 5	13 9 4 8	19 12 20	(6 (6	i5) i6)	1 2 3 4	14 15 6 7	16 21 13 10
(37) (38) (39)	1 8 2 18 3 18 4 19	3 10 3 19 5 17 2 16 9 21	(44) (45) (46) (47)	$ \begin{array}{c c} \hline 1\\ \hline 2\\ \hline 3\\ \hline 4\\ \hline 6 \end{array} $	11 10 5 9	18 20 12 13 21	(51) (52) (53)	$\frac{1}{2}$ $\frac{3}{4}$	12 6 14 18 7	20 8 19 21 15	(58) (59) (60)	$ \begin{array}{c c} 1\\ \hline 2\\ \hline 3\\ \hline 5\\ \hline 6 \end{array} $	13 9 4 8 7	19 12 20 14	(6 (6 (6	i5) i6) i7)	1 2 3 4 5	14 15 6 7	16 21 13 10 20
(37) (38) (39) (40)	1 8 2 18 3 18 4 12 5 9 6 1	3 10 3 19 5 17 2 16 9 21	(44) (45) (46) (47) (48)		11 10 5 9 16	18 20 12 13 21 17	(51) (52) (53) (54)	1 2 3 4 5 9	12 6 14 18 7 10	20 8 19 21 15 17	(58) (59) (60) (61)	$\frac{1}{2}$ $\frac{3}{5}$ $\frac{6}{10}$	13 9 4 8 7	19 12 20 14 18 16	(6 (6 (6 (6	i5) i6) i7) i8)	1 2 3 4 5 8	14 15 6 7 18	16 21 13 10 20 17

Plan 11.34 
$$t = 21, k = 5, r = 5, b = 21, \lambda = 1, E = .84$$
, Type IV

See incomplete latin squares Plan 13.13; randomize units in blocks ignoring replications.

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Plan 11.35  $t = 21, k = 7, r = 10, b = 30, \lambda = 3, E = .90, Type III$ 

2	2	5	10	11	17	19	20	$(16) \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \$	7	7	10	13	18	20	21
5	}	6	11	12	18	20	21	$(17) \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \$	]	l	11	14	19	21	15
9	į	7	12	13	19	21	15	(18) 4		2	12	8	20	15	16
	;	1	13	14	20	15	16	$(19) \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \ \$	:	3	13	9	21	16	17
		2	14	8	21	16	17	(20) 6	4	4	14	10	15	17	18
•	7	3	8	9	15	17	18	$(21)$ $\bar{7}$	- 4	5	8	11	16	18	19
	L	4	9	10	16	18	19	(22) 1	:	2	4	8	9	11	21
		4	8	13	17	19	20	$(23) \ \ 2$	;	3	5	9	10	12	15
Ï	_	5	9	14	18	20	21	(24) 3	-	4	6	10	11	13	16
j		6	10	8	19	21	15	$(25)$ $\bar{4}$	- ;	5	7	11	12	14	17
	;	7	11	9	20	15	16	(26) 5	(	6	1	12	13	8	18
	7	1	12	10	21	16	17	(27) $6$	•	7	2	13	14	9	19
Į		2	13	11	15	17	18	(28) 7		1	3	14	8	10	20
2	_	3	14	12	16	18	19	(29) 1	-	2	3	4	5	6	7
1	i	6	9	12	17	19	20	(30) 8	. 1	9	10	11	12	13	14

Plan 11.36 
$$t = 25, k = 4, r = 8, b = 50, \lambda = 1, E = .78, Type II$$

See extended incomplete latin squares Plan 13.16a; randomize units in blocks ignoring replications.

Plan 11.37 
$$t = 25, k = 9, r = 9, b = 25, \lambda = 3, E = .93$$
, Type IV

See incomplete latin squares, Plan 13.1a; randomize units in blocks ignoring replications.

Plan	11.3	8	t ==	28,	k=4, r=9,	b =	63,	λ ==	1, E	= .78, Type	I				
Block	k	Rep	p. I				Rep	. II			Rep. II				
(1)	28	1	10	19	(8)	28	2	11	20	(15)	28	3	12	21	
<b>(2)</b>	2	9	13	16	(9)	3	1	14	17	(16)	4	2	15	18	
(3)	3	8	11	18	(10)	4	9	12	10	(17)	5	1	13	11	
<b>(4)</b>	4	7	23	24	(11)	5	8	24	25	(18)	6	9	25	26	
(5)	5	6	20	27	(12)	6	7	21	19	(19)	7	8	22	20	
(6)	12	17	22	25	(13)	13	18	23	26	(20)	14	10	24	27	
(7)	14	15	21	26	(14)	15	16	22	27	(21)	16	17	23	19	
		Rep	. IV				Rep	. V				Rep	. vi		
(22)	28	4	13	22	(29)	28	5	14	23	(36)	28	6	15	24	
(23)	5	3	16	10	(30)	6	4	17	11	(37)	7	5	18	12	
(24)	6	2	14	12	(31)	7	3	15	13	(38)	8	4	16	14	
(25)	7	1	26	27	(32)	8	2	27	19	(39)	9	3	19	20	
(26)	8	9	23	21	(33)	9	1	24	22	(40)	1	2	25	23	
(27)	15	11	25	19	(34)	16	12	26	20	(41)	17	13	27	21	
(28)	17	18	24	20	(35)	18	10	25	21	(42)	10	11	26	22	
		Rep	. VI	[			Rep.	VII	I			Rep	. IX		
(43)	28	7	16	25	(50)	28	8	17	26	(57)	28	9	18	27	
(44)	8	6	10	13	(51)	9	7	11	14	(58)	1	8	12	15	
(45)	9	5	17	15	(52)	1	6	18	16	(59)	2	7	10	17	
(46)	1	4	20	21	(53)	2	5	21	22	(60)	3	6	22	23	
(47)	2	3	26	24	(54)	3	4	27	25	(61)	4	5	19	26	
(48)	18	14	19	22	(55)	10	15	20	23	(62)	11	16	21	24	
(49)	11	12	27	23	(56)	12	13	19	24	(63)	13	14	20	25	