

Master's Theorem.

If  $f(n) \in O(n^k)$  or  $f(n) = c \cdot n^d$  where  $d \geq 0$  in recurrence  
 $T(n) = aT(n/b) + f(n)$  then.

$$T(n) \in \begin{cases} O(n^k) & \text{if } (a < b^d) \\ O(n^k \log n) & \text{if } (a = b^d) \\ O(n \log_b a) & \text{if } (a > b^d) \end{cases}$$

1)  $T(n) = 8T(n/2) + 1000n^2$ .

$a=8$     $b=2$     $f(n) = 1000n^2 = c \cdot n^d$   $\therefore d=2$ .

$b^d = 2^2 = 4$

Hence  $a > b^d$

$T(n) \in O(n \log_b a)$ .

$\therefore T(n) \in O(n^3)$ .

$\log_b a = \log_2 8 = 3$

2)  $T(n) = 2T(n/2) + n^2$ .

$a=2$     $b=2$     $d=2$ .

$b^d = 2^2 = 4$ .

Hence  $a < b^d$ .  $T(n) \in O(n^d)$

$\Rightarrow T(n) \in O(n^2)$

3)  $T(n) = 2T(n/2) + 10n$ .

$a=2$     $b=2$     $d=1$

$b^d = 2^1 = 2$ .

Hence,  $a = b^d$ .

$T(n) \in O(n^d \log n)$ .

$\Rightarrow T(n) \in O(n \log n)$