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Modeling stock prices in a portfolio using multidimensional geometric brownian motion

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Abstract. Modeling and forecasting stock prices of public corporates are important studies in financial analysis, due to their stock price characteristics. Stocks investments give a wide variety of risks. Taking a portfolio of several stocks is one way to minimize risk. Stochastic process of single stock price movements model can be formulated in Geometric Brownian Motion (GBM) model. But for a portfolio that consist more than one corporate stock, we need an expansion of GBM Model. In this paper, we use multidimensional Geometric Brownian Motion model. This paper aims to model and forecast two stock prices in a portfolio. These are PT. Matahari Department Store Tbk and PT. Telekomunikasi Indonesia Tbk on period January 4, 2016 until April 21, 2017. The goodness of stock price forecast value is based on Mean Absolute Percentage Error (MAPE). As the results, we conclude that forecast two stock prices in a portfolio using multidimensional GBM give less MAPE than using GBM for single stock price respectively. We conclude that multidimensional GBM is more appropriate for modeling stock prices, because the price of each stock affects each other.

Keywords: Stochastic Differential Equation, Multidimensional Geometric Brownian Motion, Two Dimensional Ito's Lemma, Mean Absolute Percentage Error

1. Introduction

Modeling and forecasting of stock prices are important topics in financial studies. Stock price prediction is one of most difficult task to solve in financial studies due to complex characteristic of stock market. Many investors want any forecasting method that could guarantee precise forecast and minimize investment risk from the stock market. This remains a motivating factor for researchers to expand and develop new models.

Financial researchers are interested in expanding stock price's forecast theory, in order to make important investment and financing decisions. Modeling stock price is one of the interesting works in financial studies. Modeling stock prices means generating price parts that a stock may follow in the future. Modeling stock prices is needed because future stock prices are uncertain (called stochastic), and they follow a set of characteristic that we can derive from historical data of stock prices. A prediction will be fit only if the underlying model is realistic. The model must reflect the behavior of stock prices and conform to historical data.

In this research, we expand the Geometric Brownian Motion (GBM) to multidimensional GBM method to simulate portfolio that consist of two stock prices in Indonesian Exchange. We focus on simulating and test the model using Indonesian stocks to assess how well the forecasted stock prices align with actual stock returns. For evaluating the forecast method, we calculate Mean Absolute Percentage Error (MAPE) between the actual and forecasted values.

This research paper is set out as follows: section 2 describes the literature related to multidimensional GBM. Section 3 details the data and research method used to model and forecast the multidimensional GBM. Section 4 presents result and discussion, and section 5 the conclusion.

2. Multidimensional Geometric Brownian Motion

There are two main studies in the financial analysis, technical analysis and fundamental analysis [1]. Fundamental analysis intends to determine a stock's value by focusing on underlying factors that affect a company's actual business and its future prospects. In the other side, technical analysis studies the price movement of a stock and predicts its future price movements.

Brownian motion was discovered by the biologist Robert Brown in 1827. The motion was fully captured by mathematician Norbert Wiener. Brownian motion is often used to explain the movement of time series variables. In 1900, Louis Bachelier first applied Brownian motion to the movements of the stock prices. Many literatures say that the stock market prices exhibit random walk. The random walk theory is the idea that stocks take a random and unpredictable path, making it near impossible to outperform the market without assuming additional risk.. The Geometric Brownian Motion (GBM) model incorporates this idea of random walks in stock prices through its uncertain component, along with the idea that stocks maintain price trends over time as the certain component [2].

GBM process is growing on some literatures that focus on testing the validity of the model and accuracy of forecast using Brownian motion. Abidin and Jaffar [3] use GBM to forecast future closing prices of small sized companies in Kuala Lumpur Stock Exchange. According to them, GBM can be used to forecast a maximum of two week closing prices. It was also found that one week's data was enough to forecast the share prices using GBM. Marathe and Ryan [4] discuss the process for checking whether a given time series follows the GBM process. They found that of the four industries they studied, the time series for usage of established services met the criteria for a GBM process; while the data form growth of emergent services did not. Reddy and Clinton (2016) simulate Australian Companies' stock prices using GBM. The results show that over all time horizons the chances of a stock price simulated using GBM moving in the same direction as real stock prices did was just a little greater than 50 percent. GBM Model to forecast and calculate VaR value on some Indonesian corporate stocks has been studied refers to [5], [6], and [7]. The results show that GBM model is suitable for modeling stock prices, but it can be expanded to multidimensional GBM model.

The current practice of stocks analysis is based on the assumption that the time series of closing price of stock could represent the behavior of the each stock. Recently, there is an attempt where researchers represent all stocks in a portfolio moves dependently and correlated each other. This assumptions lead to consider a new methodology to construct multidimensional GBM where each stock is represented as GBM model and all stocks move as multidimensional GBM. Multidimensional GBM model that represents stock prices in the future is affected by three parameters, there are expectation of stock return, risk of stock, and correlation between stock return.

A stochastic differential equation (SDE) has the form

$$X(t) = X(0) + \int_0^t \mu\big(s, X(s)\big)ds + \int_0^t \sigma\mu\big(s, X(s)\big)dW(s)$$
$$0 \leq t \leq T \qquad (1)$$

In this equation, $\mu(t,x)$ and $\sigma(t,x)$ are two continuous deterministic functions. $\{X(t)\}$ is the solution of the stochastic differential equation (1) with initial value $X(0)$ and for convenience, we also call $\{X(t)\}$ an Ito process although the latter is more general. $\mu\big(t, X(t)\big)$ is referred to as the drift and

$\sigma\big(t, X(t)\big)$ is often referred to as the infinitesimal deviation of the SDE or the volatility of the stochastic process $\{X(t)\}$. Equation (1) is often written in a differential form as follow [7]

$$dX(t) = \mu\big(t, X(t)\big)dt + \sigma\big(t, X(t)\big)dW(t) \tag{2}$$

with initial condition $X(0)$, or simply

$$dX = \mu(t, X)dt + \sigma(t, X)dW \tag{3}$$

The expression (2) and (3) looks very similar to an ordinary differential equation (ODE) $dx = f(t, x)dt$.

Theorem 1 One Dimensional Ito's Lemma
Let $\{X(t)\}$ be a solution of the stochastic differential equation (3) and $g(t, x)$a deterministic function which is continuously differential in t and continuously twice differentiable in x. Then the stochastic process $\big\{g\big(t, X(t)\big)\big\}$ is a solution of the following SDE

$$dg(t, X) = \left[\frac{\partial g(t,X)}{\partial t} + \mu(t, X)\frac{\partial g(t,X)}{\partial t} + \frac{1}{2}\sigma^2(t, X)\frac{\partial^2 g(t,X)}{\partial t^2}\right]dt + \sigma(t, X)\frac{\partial g(t,X)}{\partial t}dW \tag{4}$$

Suppose the function $S(X) = \log X$ and $X(0)$ is initial value of X. By using One Dimensional Ito's Lemma we get

$$\ln X(t) = \left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma\big(W(t) - W(0)\big) \tag{5}$$

then

$$X(t) = X(0)\exp\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma\big(W(t) - W(0)\big) \tag{6}$$

Two or higher-dimensional stochastic processes and stochastic differential equations are necessary when modeling more than one risky asset or modeling jointly risky assets and stochastic interest rates [9]. A pair of standard Brownian motion processes $\{W_1(t), W_2(t)\}$ is said to be correlated two-dimensional standard Brownian motion if

1. Increments $W_1(t) - W_1(s)$ and $W_2(t) - W_2(s)$, $t > s$, are independent of $W_1(y)$ and $W_2(y)$ for any $0 \le y \le s$. In other words, the pair of processes as a vector has independent increments.
2. The covariance
$$cov\big(W_1(t), W_2(t)\big) = E\{W_1(t), W_2(t)\} = \rho t \quad -1 \le \rho \le 1$$

A pair of stochastic processes $\{X_1(t), X_2(t)\}$ is a solution of a two dimensional SDE

$$\begin{aligned} dX_1 &= \mu_1(t, X_1, X_2)dt + \sigma_{11}(t, X_1, X_2)dW_1 + \sigma_{12}(t, X_1, X_2)dW_2 \\ dX_2 &= \mu_2(t, X_1, X_2)dt + \sigma_{21}(t, X_1, X_2)dW_1 + \sigma_{22}(t, X_1, X_2)dW_2 \end{aligned} \tag{7}$$

Where $\big\{\big(W_1(t), W_2(t)\big)\big\}$ is uncorrelated standard Brownian motion, if

$$X_1(t) = X_1(0) + \int_0^t \mu_1\big(s, X_1(s), X_2(s)\big)ds + \sum_{j=1}^2 \int_0^t \mu_{1j}\big(s, X_1(s), X_2(s)\big)dW_j(s)$$

$$X_2(t) = X_2(0) + \int_0^t \mu_2\big(s, X_1(s), X_2(s)\big)ds + \sum_{j=1}^2 \int_0^t \mu_{2j}\big(s, X_1(s), X_2(s)\big)dW_j(s)$$

Theorem 2 Two Dimensional Ito's Lemma
Let $\{(X_1(t), X_2(t))\}$ *be a solution to SDE (5) and* $g(t, x_1, x_2)$ *be a function which is continuously differentiable in t and continuously twice differentiable jointly with respect to* x_1 *and* x_2*. Then* $g(t, X_1(t), X_2(t))$ *is a solution of the following SDE*

$$dg(t, X_1, X_2) = \frac{\partial g(t,X_1,X_2)}{\partial t} dt + \frac{\partial g(t,X_1,X_2)}{\partial x_1} dX_1 + \frac{\partial g(t,X_1,X_2)}{\partial x_2} dX_2$$
$$+\frac{1}{2}\left[\left((\sigma_{11}^2 + \sigma_{12}^2)\frac{\partial^2 g(t,X_1,X_2)}{\partial {x_1}^2} + (\sigma_{21}^2 + \sigma_{22}^2)\frac{\partial^2 g(t,X_1,X_2)}{\partial {x_2}^2}\right) + 2(\sigma_{11}\sigma_{21} + \sigma_{12}\sigma_{22})\frac{\partial^2 g(t,X_1,X_2)}{\partial x_1 \partial x_2}\right] dt \quad (8)$$

Suppose the function $S(X) = \log X$ and $X(0)$ is initial value of *X*. By using Two Dimensional Ito's Lemma refer to [10] and [11] we get

$$X_1(t) = X_1(0) \exp\left(\mu_1 - \frac{1}{2}\sum_{j=1}^{2} {\sigma_{1j}}^2\right) t + \sum_{j=1}^{2} \sigma_{1j} \left(W(t) - W(0)\right) \quad (9)$$

$$X_2(t) = X_2(0) \exp\left(\mu_2 - \frac{1}{2}\sum_{j=1}^{2} {\sigma_{2j}}^2\right) t + \sum_{j=1}^{2} \sigma_{2j} \left(W(t) - W(0)\right) \quad (10)$$

The stochastic differential equation is widely applicable in stochastic analysis and its area of application, for example in finance. Equation (6) is the asset price model that is able to predict an asset price at specific time, *t*, when the assets move as one dimensional Geometric Brownian Motion. And equation (9) and (10) is the asset price model that is able to predict an asset price at specific time, *t*, when the assets move together as two dimensional Geometric Brownian Motion model. According to Abidin and Jaffar (2014) there are three measurement of forecasting model which involve time period *t*. The measurements are the number of period forecast, *n*, actual value in time period at time *t*, X_t, and forecast value in time period *t*, F_t.

The widely used method to evaluate accuracy measure of forecasting that considers the effect of the magnitude of the actual values is the Mean Absolute Percentage Error (MAPE). The MAPE has the form

$$MAPE = \frac{\sum_{t=i}^{n} X(t) - F(t)}{n} \quad (11)$$

Table 1 shows a scale of judgement of forecasting accuracy using MAPE.

Table 1. A Scale Judgement of Forecast Accuracy

MAPE	Forecast Accuracy
< 10%	Highly accurate
11% - 20%	Good forecast
21% - 50%	Reasonable forecast
> 51%	Inaccurate forcast

Source: Abidin and Jaffar (2014)

The smaller of the MAPE value, the more accurate the forecasting model is.

3. Data and Method
Data is derived from publicly available databases obtained from two companies' stock price included in The Indonesian Exchange (IDX) Top Ten Blue 2016. The two companies are PT Matahari

Department Store Tbk (LPPF) and PT Telekomunikasi Indonesia Tbk (TLKM). Daily stock price data was obtained from the Yahoo! Finance database over the period 4 January 2016 to 21 April 2017 having a total number of 317 observations. We divide those data by 239 observations as in sample data and 78 observations as out sample data. The data composed of four elements, namely: open price, low price, high price and close price respectively. In this research the closing price is chosen to represent the price of the index to be predicted. Closing price is chosen because it reflects all the activities of the index in a trading day.

The stochastic differential equation will be particularly important in modeling many asset classes because GBM deals with randomness, volatility, drift and return on investment [12]. According to Wilmott (2000), investors' main concern will be on the return on investment which refers to the percentage growth in the value of an asset. If X_t is asset value on the day, then the return from day *i* to day *t*+1 is given by

$$R_t = \ln \frac{X_t}{X_{t-1}} \quad (12)$$

If n is the number of returns in the sample, then the drift μ can be presented by the mean of the returns distribution and volatility s can be represented by the sample standard deviation. We are modeling simultaneous stock prices model with two dimensional GBM as (9) and (10). Then compute the MAPE with equation (11).

4. Results and Discussion

Figure 1 and 2 shows the line plot for LPPF and TLKM historical stock prices.

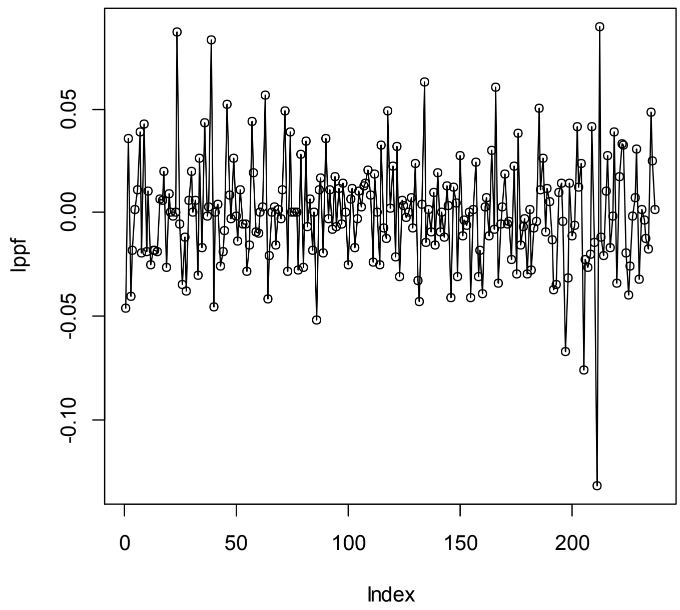

Figure 1. Graphical Representation of LPPF Stock Closing Prices

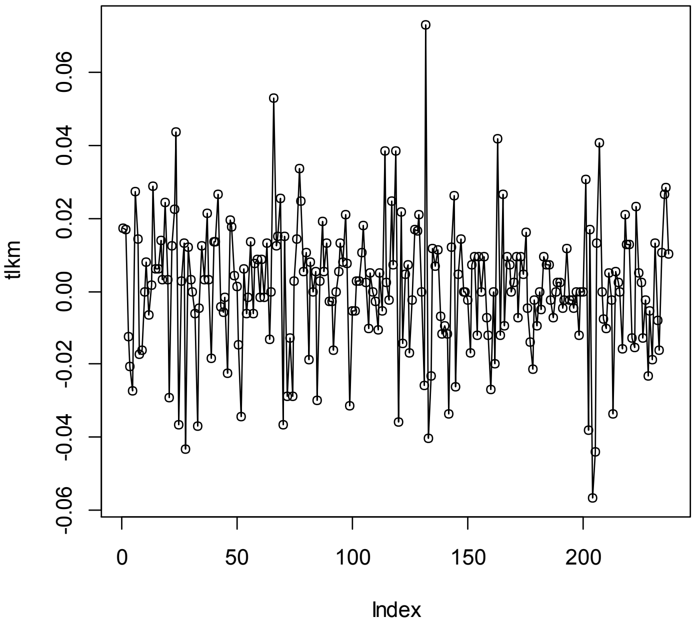

Figure 2. Graphical Representation of TLKM Stock Closing Prices

Table 2 reports the in sample descriptive statistics for each stock, expected annual return, and expected annual volatility. A total of 239 stocks were analysed from two companies as defined on the IDX website.

Table 2. Descriptive Statistics

Parameter	LPPF	TLKM
Mean	-0.00056	0.00100
Variance	0.00069	0.00032
Standard Deviation	0.02622	0.01776
Covariance	0.00013	

Before we analyse with GBM model, we have to fulfill the assumptions, that is test of normality and independencies. Table 3 shows the Kolmogorov-Smirnov test for normality of LPPF and LTKM stock prices return.

Table 3. Kolomogorov-Smirnov Test for Normality Assumptions

Stock	D-Stats	p-Value
LPPF	0,076323	0,1264
TLKM	0,083778	0,07181

The result shows that we have to receive the null hyphotesis and conclude that stock prices return of LPPF and TLKM come from Normal Distribution. Based on equation (9) and (10), we can forecast and compare the forecast value with the out sample data. Table 4 provides the forecast and actual prices for LPPF and TLKM, counter by using Two Dimensional Geometric Brownian Motion. It illustrates that on January 2, 2017 until April 2. 2017. Figure 3 provides the graphs of the stock market by comparing the actual prices and the forecast prices.

Table 4. Forecast and Actual Value for LPPF Using GBM

`Date	LPPF		TLKM	
	Forecast	Actual	Forecast	Actual
Jan 02, 2017	15125	15070	3980	3974
Jan 03, 2017	15050	15222	3950	3945
Jan 04, 2017	15150	15327	3950	3946
Jan 05, 2017	15525	15469	3950	4086
Jan 06, 2017	15800	14867	4000	4011
Jan 09, 2017	15575	14940	4020	4039
Jan 10, 2017	15675	14656	4000	4090
Jan 11, 2017	15325	14688	3960	4033
Jan 12, 2017	15250	15199	3960	3959
Jan 13, 2017	15150	15375	3950	4070
Jan 16, 2017	14925	15011	3950	3975
Jan 17, 2017	14925	15446	3970	3919
Jan 18, 2017	14700	14771	3960	3962
Jan 19, 2017	14900	14672	3970	3986
Jan 20, 2017	14875	14384	3830	3945
Jan 23, 2017	14875	14849	3840	3851
Jan 24, 2017	15025	15210	3910	3663
Jan 25, 2017	15150	15322	3900	3629
Jan 26, 2017	15100	15776	3940	3631
Jan 27, 2017	14950	15699	3890	3670
Jan 30, 2017	14975	15547	3860	3613
Jan 31, 2017	14775	15125	3870	3494
Feb 01, 2017	14775	15551	3940	3610
Feb 02, 2017	14950	16366	3950	3640
Feb 03, 2017	15125	16535	3950	3587
Feb 06, 2017	15150	16246	3960	3629
Feb 07, 2017	15250	16188	3920	3613
Feb 08, 2017	15325	15445	3870	3609
Feb 09, 2017	15325	15155	3870	3681
Feb 10, 2017	15325	14835	3890	3661
Feb 13, 2017	15150	14970	3920	3705
Feb 14, 2017	15150	15152	3860	3699

Feb 15, 2017	15150	14915	3860	3657
Feb 16, 2017	14800	15155	3870	3740
Feb 17, 2017	14275	15593	3870	3810
Feb 20, 2017	14275	15829	3870	3744
Feb 21, 2017	14475	16101	3880	3809
Feb 22, 2017	14750	15826	3880	3926
Feb 23, 2017	14975	15930	3840	3831
Feb 24, 2017	14400	16165	3840	3923
Feb 27, 2017	14000	15709	3870	3910
Feb 28, 2017	13650	14506	3850	3893
Mar 01, 2017	11725	14449	3850	3862
Mar 02, 2017	12925	14089	3830	3881
Mar 03, 2017	13100	14486	3850	3869
Mar 06, 2017	13075	14162	3920	3907
Mar 07, 2017	13200	14214	3950	3802
Mar 08, 2017	13050	13712	3880	3852
Mar 09, 2017	13225	13015	3960	3872
Mar 10, 2017	13200	13386	3950	3903
Mar 13, 2017	13050	12940	3950	3866
Mar 14, 2017	13150	12904	4050	3938
Mar 15, 2017	13225	12278	4040	3833
Mar 16, 2017	13525	12376	4140	3828
Mar 17, 2017	13850	12284	4110	3851
Mar 20, 2017	13750	12671	4100	3853
Mar 21, 2017	14075	12924	4090	3840
Mar 22, 2017	14350	13045	4070	3830
Mar 23, 2017	14250	13115	4090	3884
Mar 24, 2017	13900	12914	4080	3863
Mar 27, 2017	13550	12926	4080	3857
Mar 29, 2017	13325	13006	4150	3854
Mar 30, 2017	13125	13235	4140	3870
Mar 31, 2017	13175	13306	4130	3829
Apr 03, 2017	13500	13026	4170	3991
Apr 04, 2017	13125	13334	4250	3956
Apr 05, 2017	13275	13221	4250	3854
Apr 06, 2017	13400	13417	4170	3939
Apr 07, 2017	13250	12939	4130	3943
Apr 10, 2017	13475	12892	4100	3971
Apr 11, 2017	13300	12611	4150	4040

Apr 12, 2017	13875	12454	4150	3957
Apr 13, 2017	13450	12637	4090	4036
Apr 17, 2017	13350	12885	4010	4123
Apr 18, 2017	13375	12968	4070	4236
Apr 19, 2017	13375	12892	4070	4281
Apr 20, 2017	13250	13088	4110	4318
Apr 21, 2017	13300	13036	4420	4264

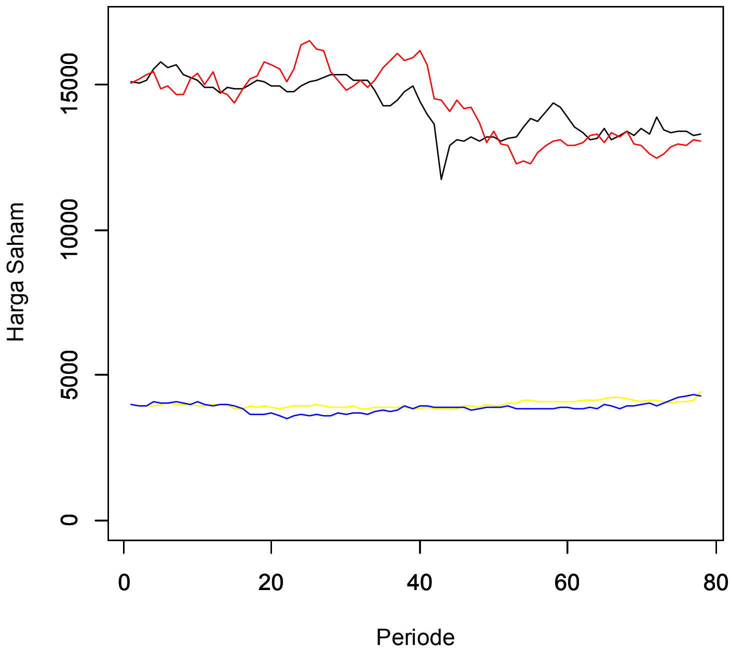

Figure 3. The Graph of Forecast Value vs. Actual Value for the LPPF and TLKM (black = actual value of LPPF ; red = forecast value of LPPF ; yellow = actual value of TLKM ; blue = forecast value of TLKM)

Then Table 5 shows the MAPE Value based on the data used in Multidimensional GBM model.

Table 5. MAPE Value for Multidimensional GBM Model

Stock	MAPE	Accuracy
LPPF	4,7335%	Highly accurate
TLKM	3,8726%	Highly accurate

According to Table 1, it shows that the forecast value is highly accurate for the model. Based on the results, the model was found capable of use on the data.

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5. Conclusion

This study explores the Multidimensional Geometric Brownian motion model for simulating stock price from The Indonesian Exchange (IDX) Top Ten Blue 2016. The two companies are PT Matahari Department Store Tbk (LPPF) and PT Telekomunikasi Indonesia Tbk (TLKM). The results show MAPE Value is less than 10%. It means that the method is very highly accurate for the model.

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