MCL 601

Applied Probability & Statistics

Odd Semester 2021

MANIPAL SCHOOL OF INFORMATION SCIENCES

MANIPAL

*(A constituent unit of MAHE, Manipal)*

Problem Set 1 — October 24, 2021

1. From a group of 5 assistant professors, 6 associate professors, and 4 professors, a group of 2 assistant professors, 2 associate professors, and 3 professors need to be chosen to form a thesis committee of 7 members; How many different such thesis committees are possible?

2. 9 computers are brought in for servicing (and machines are serviced one at a time). Of the 9 computers, 3 are PCs, 4 are Macs, and 2 are Linux machines. Assume that all computers of the same type are indistinguishable (i.e., all the PCs are indistinguishable, all the Macs are indistinguishable, etc.).

   (a) In how many distinguishable ways can the computers be ordered for servicing?

   (b) In how many distinguishable ways can the computers be ordered if the first 5 machines serviced must include all 4 Macs?

   (c) In how many distinguishable ways can the computers be ordered if 2 PCs must be in the first three and 1 PC must be in the last three computers serviced?

3. Say you have 20 lakh rupees that must be invested among 4 possible companies. Each investment must be in integral units of 1 lakh rupees, and there are minimal investments that need to be made if one is to invest in these companies. The minimal investments are 1, 2, 3, and 4 lakh rupees, respectively for company 1, 2, 3, and 4. How many different investment strategies are available if:

   (a) an investment must be made in each company?

   (b) investments must be made in at least 3 of the 4 companies?

4. Eleven soccer players are to be divided into 4 functional groups: 3 forwards, 3 midfields, 4 defenses, and 1 goalie. There are only 2 people who can play goalie. Both of these two players can play any other position. Of the remaining 9, 4 can play only forward or midfield; the other 5 can play only defense or midfield. We want to calculate the number of possible ways to divide the team into the 4 functional groups. Follow the hint below and get to the answer:

$$\text{Select 1 goalie out of 2 in } \binom{?}{?} \text{ ways } \textbf{AND} \begin{cases} \text{Remaining goalie plays defense} \\ \textbf{OR} \\ \text{Remaining goalie plays midfield} \\ \textbf{OR} \\ \text{Remaining goalie plays forward} \end{cases}$$

5. In how many ways can r identical server requests be distributed among n servers so that the ith server receives *at least* m_i requests, for each $i = 1, 2, \ldots, n$? You can assume that $r \geq (m_1 + m_2 + \cdots + m_n)$.

6. Suppose a particle starting from the origin can move *only* up or down; the *binomial option pricing model* addresses stock price movements using such an idea.
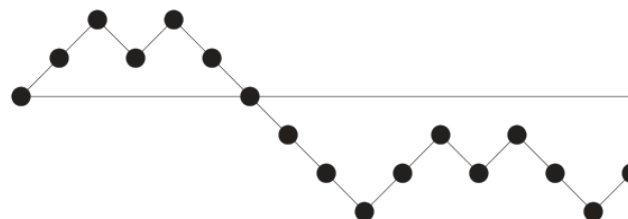
Show that the number of ways the particle can move from the origin to position k in n steps is $\binom{n}{\frac{n+k}{2}}$. Assume that $n + k$ is even.