State Space Models for Exponential

State Space Approach

- Each model is represented as consisting of a measurement equation that describes the observed data and some transition equations that describe how the unobserved components or states (level, trend, seasonal) change over time.
- Hence these are referred to as “state space models”.
- For each forecasting method there exist two models:
  - one with additive errors
  - one with multiplicative errors.

Types of Errors

- There can be two possible errors in estimation of time series values
  - Additive Error
  - Multiplicative Error
- In model option of ets() of R, we specify "A" for additive and "M" for multiplicative

Additive Error

- The model equation with additive error (e_t):

$$y_{t+1} = L_t + T_t + e_t$$

- In additive error type, the errors are assumed to have fixed magnitude, irrespective of the current (level + trend) of the series

Multiplicative Error

- The model equation with multiplicative error (e_t):

$$y_{t+1} = (L_t + T_t) \times (1 + e_t)$$

- In multiplicative error type, error is percentage increase in the current (level + trend) of the series

State Space Models in R

- In R, each state space model is recognized as ETS for **E**rror, **T**rend, **S**easonal.

Syntax : ets(ts, model="ZZZ", damped, alpha, beta, gamma, phi,…)

Where

ts : a numeric vector or time series object

model : The first letter denotes the error type ("A", "M" or "Z"); the second letter denotes the trend type ("N","A","M" or "Z"); and the third letter denotes the season type ("N","A","M" or "Z"). In all cases, "N"=none, "A"=additive, "M"=multiplicative and "Z"=automatically selected. So, for example, "ANN" is simple exponential smoothing with additive errors, "MAM" is multiplicative Holt-Winters' method with multiplicative errors, and so on.

alpha : smoothing constant for level, if NULL then it is estimated

beta : smoothing constant for trend , if NULL then it is estimated

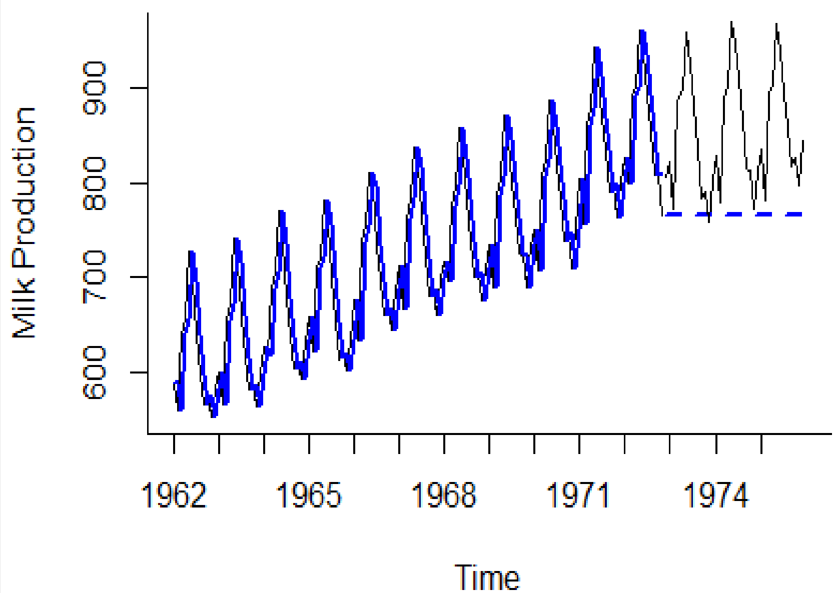
gamma : smoothing constant for seasonal component, if NULL then it is estimated

phi : smoothing constant for damping, if NULL then it is estimated

Simple Smoothing model=ANN

```
ses <- ets(train.ts, model = "ANN")
```

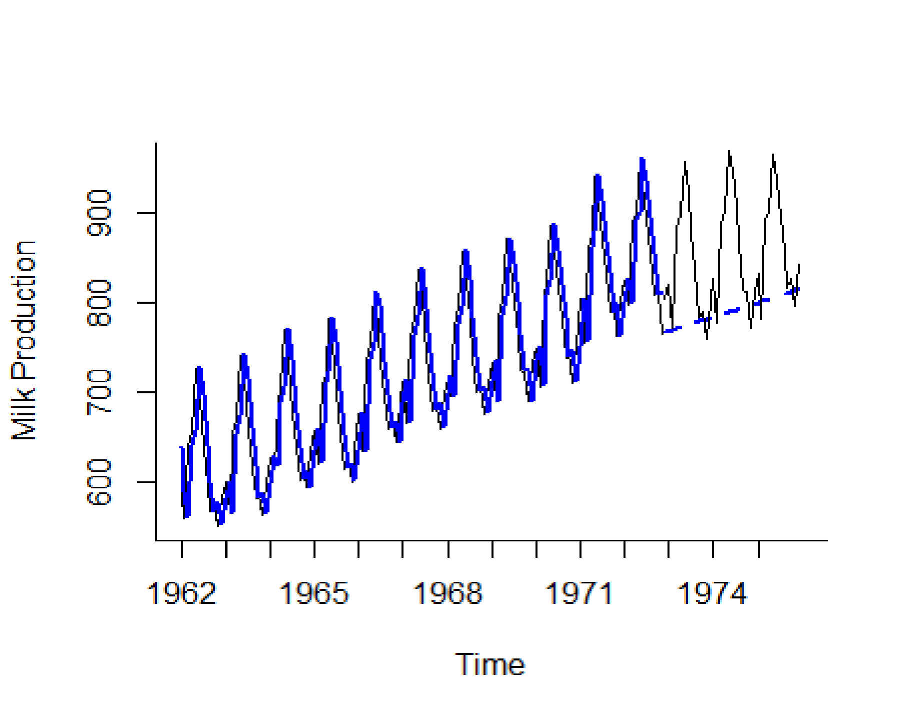
```
> ses
ETS(A,N,N)

Call:
 ets(y = train.ts, model = "ANN")

  Smoothing parameters:
    alpha = 0.9999

  Initial states:
    l = 589.2891

  sigma:  44.1128

     AIC     AICc      BIC
1634.780 1634.873 1640.530
```



Holt's Linear Trend model=AAN

```
hesAA <- ets(train.ts, model = "AAN")
```

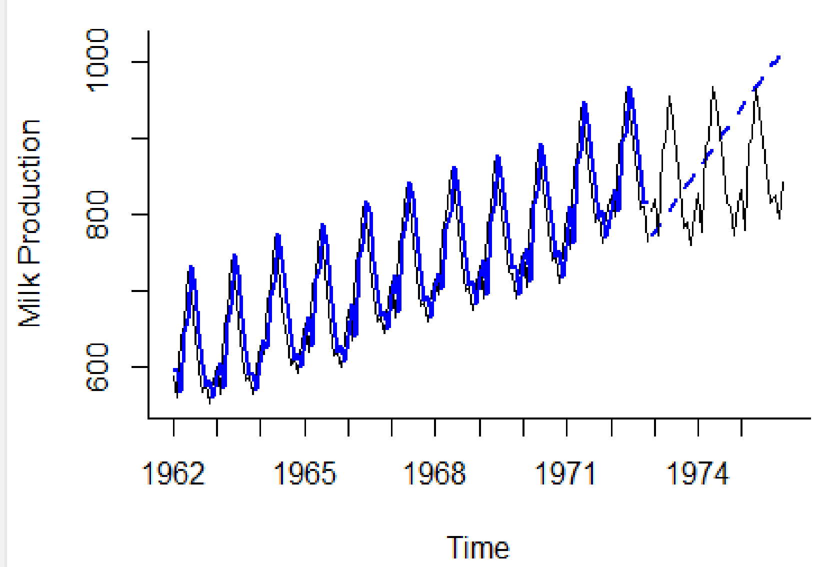
```
> hesAA
ETS(A,A,N)

Call:
 ets(y = train.ts, model = "AAN")

  Smoothing parameters:
    alpha = 0.9998
    beta  = 1e-04

  Initial states:
    l = 637.5476
    b = 1.3407

  sigma:  44.3094

     AIC     AICc      BIC
1639.945 1640.262 1651.445
```



Holt's Linear Trend model=MAN

```
hesMA <- ets(train.ts, model = "MAN")
```

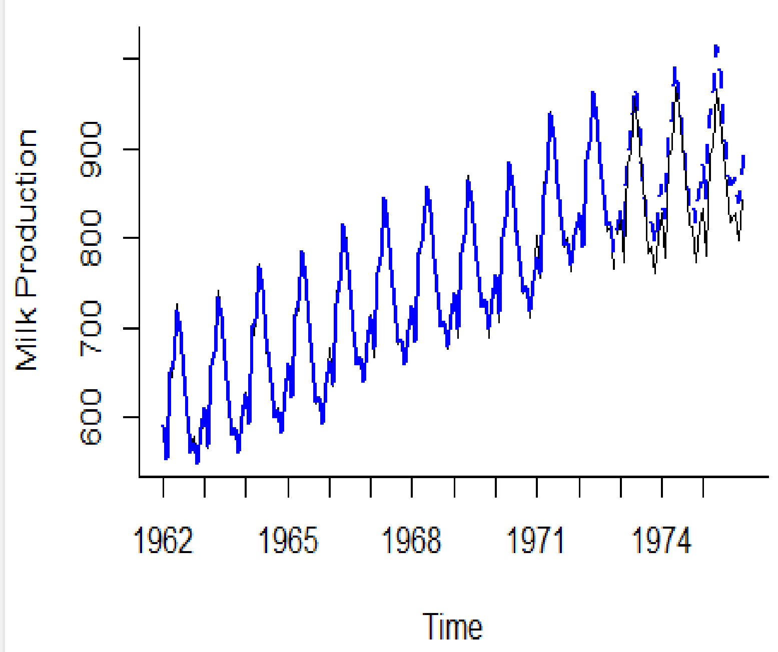
```
> hesMA
ETS(M,A,N)

Call:
 ets(y = train.ts, model = "MAN")

  Smoothing parameters:
    alpha = 0.9969
    beta  = 1e-04

  Initial states:
    l = 590.2998
    b = 6.8014

  sigma:  0.0617

     AIC     AICc      BIC
1643.063 1643.381 1654.564
```



Holt-Winters Method model=AAA

```
hwAA <- ets(train.ts, model = "AAA")
```

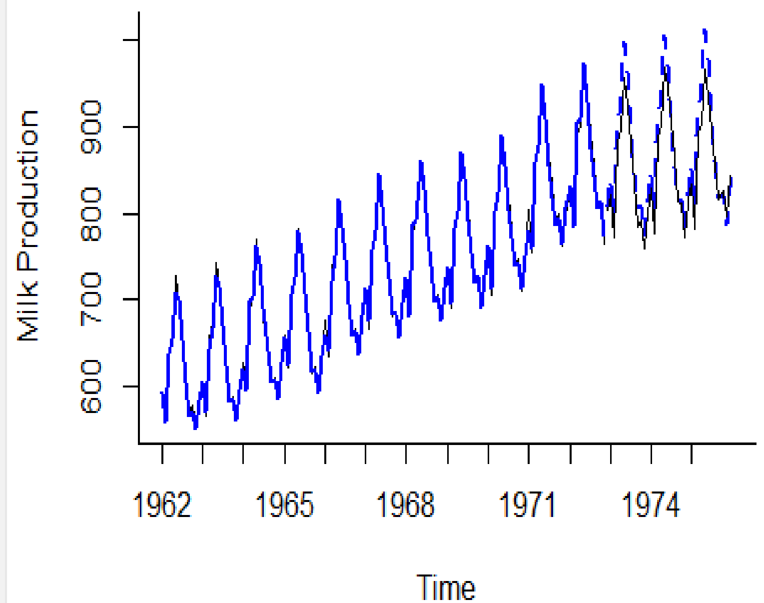
```
> hwAA
ETS(A,A,A)

Call:
 ets(y = train.ts, model = "AAA")

  Smoothing parameters:
    alpha = 0.6799
    beta  = 1e-04
    gamma = 1e-04

  Initial states:
    l = 605.2517
    b = 1.8674
    s=-42.4299 -78.1707 -49.0858 -52.9431 -12.6427 30.1153
           81.8793 110.4519 50.4173 34.5289 -54.7018 -17.4186

  sigma:  6.7803

     AIC     AICc      BIC
1172.125 1176.897 1218.128
```



Holt-Winters Method model=MAM

```
hwAM <- ets(train.ts, model = "MAM")
```

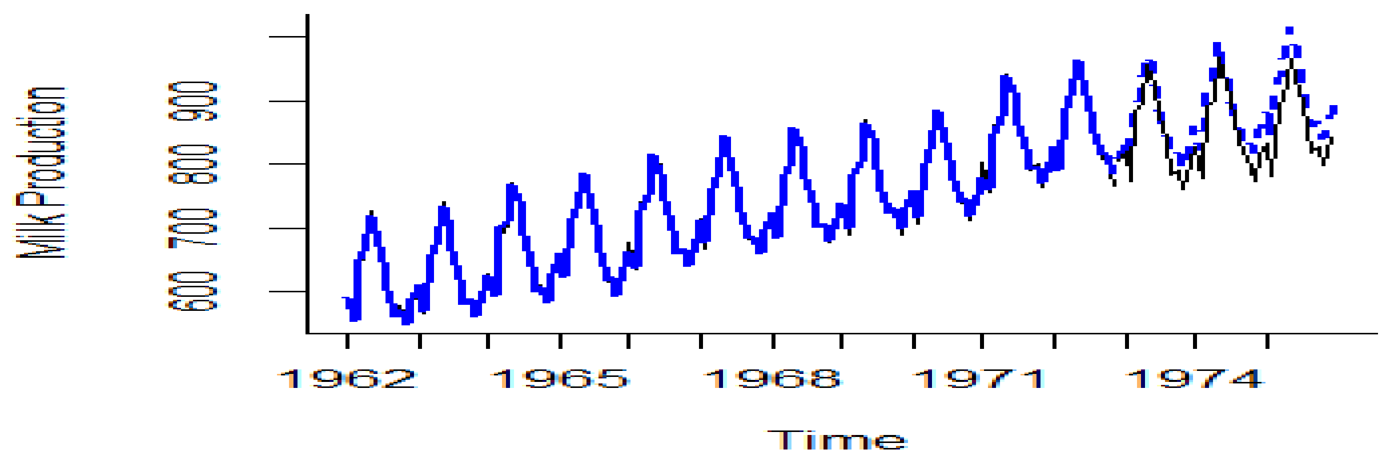
```
> hwAM
ETS(M,Ad,M)

Call:
 ets(y = train.ts, model = "MAM")

  Smoothing parameters:
    alpha = 0.9179
    beta  = 0.0131
    gamma = 3e-04
    phi   = 0.9796

  Initial states:
    l = 608.1447
    b = 0.7231
    s=0.9399 0.8916 0.9332 0.9292 0.984 1.0425
           1.1131 1.1537 1.0682 1.0462 0.9236 0.9749

  sigma:  0.0108

     AIC     AICc      BIC
1210.985 1216.401 1259.864
```



ets() without any model parameter

- Function ets() when no option specified for model, tries all the possible models and gives the best model by default using the AIC, AICc or BIC criterions.
- Minimizing the AIC gives the best model for prediction.

```
hwes <- ets(train.ts, allow.multiplicative.trend = TRUE,additive.only = FALSE)
```



Model and Accuracy

```
> hwes
ETS(A,A,A)

Call:
 ets(y = train.ts, additive.only = FALSE, allow.multiplicative.trend = TRUE)

  Smoothing parameters:
    alpha = 0.6799
    beta  = 1e-04
    gamma = 1e-04

  Initial states:
    l = 605.2517
    b = 1.8674
    s=-42.4299 -78.1707 -49.0858 -52.9431 -12.6427 30.1153
           81.8793 110.4519 50.4173 34.5289 -54.7018 -17.4186

  sigma:  6.7803

     AIC     AICc      BIC
1172.125 1176.897 1218.128
```

```
> accuracy(hwes.opt , valid.ts)
                       ME      RMSE       MAE          MPE      MAPE      MASE        ACF1 Theil's U
Training set  0.004482957  6.780315  5.113118 -0.004536466 0.7172879 0.2121552 -0.00143546        NA
Test set    -31.344975577 34.516856 31.344976 -3.714358918 3.7143589 1.3005760  0.75248253 0.6954029
```

References

- http://puterman.chcm.ubc.ca/babs502_11/statespace.pdf
- http://www.robjhyndman.com/papers/hksg.pdf