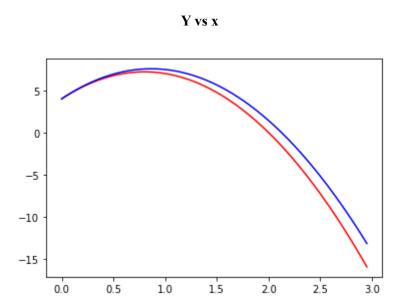
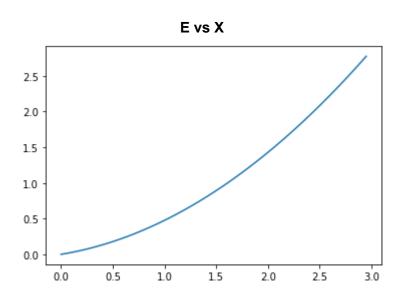
ODE 1





# **Coupled ODE 1**

While solving a set of M coupled ODEs the function 'Eul' may be extended to return an array  $Y = Eul(F(x, y), x_0, y_0, h, N, M)$ , where Y is the M-dimensional array of solutions and F is a M-dimensional array of RHS functions for coupled ODEs. Using this modified function find the phase space trajectory of a particle of unit mass in a potential V(x). You need to solve the Hamilton's equations of the system, given by:

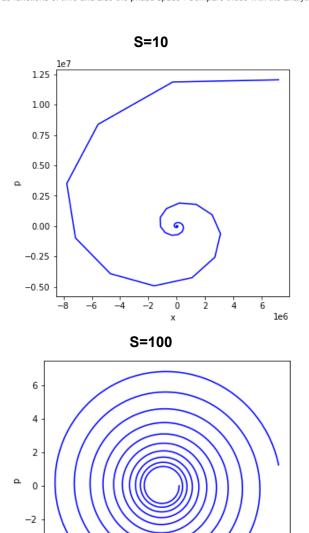
$$\frac{dp}{dt} = -\frac{\partial H}{\partial x}$$

$$\frac{dx}{dt} = \frac{\partial H}{\partial p}$$

a)

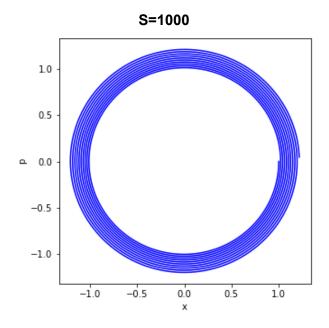
Start by taking simple harmonic oscillator potential  $V(x)=\frac{1}{2}kx^2$ ,  $k=1\frac{N}{m}$  What is the period T of this oscillator? Use h=T/S with S=10, 100 and 1000 for the step size and follow the trajectory over a time 10 T considering the initial condition (x,px)=(1.0,0.0). For all the cases, compare the following The time period  $T=2\pi$ 

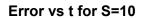
i) Show the trajectory i.e. x and p as functions of time and also the phase space . Compare these with the analytical solutions.

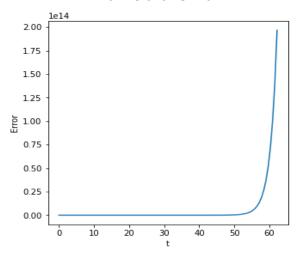


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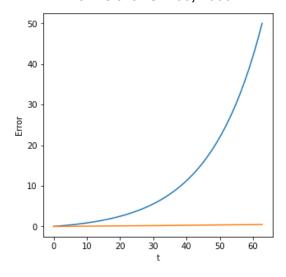
-6

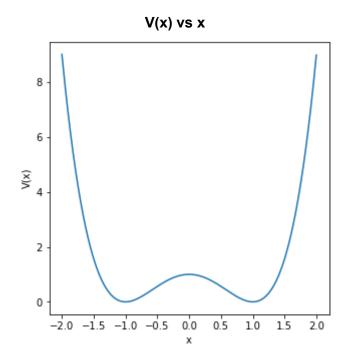


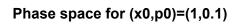


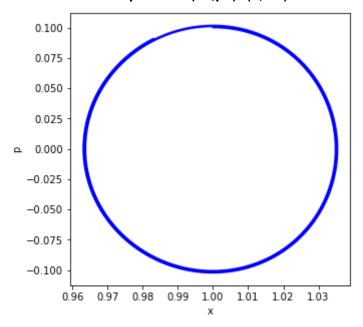


Error vs t for S=100, 1000

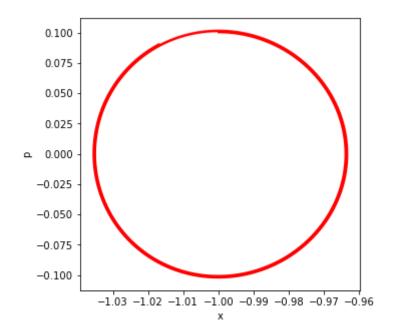




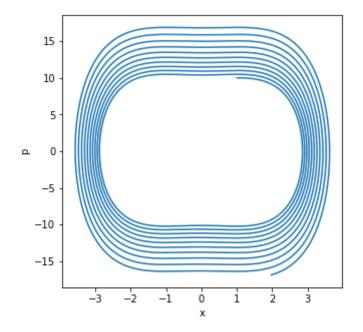




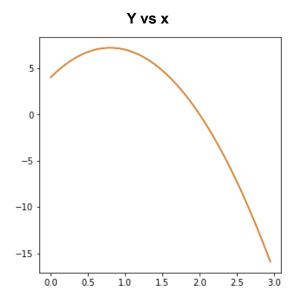
# Phase space for(x0,p0)=(-1,0.1)



### Phase space for (x0,p0)=(1,10)

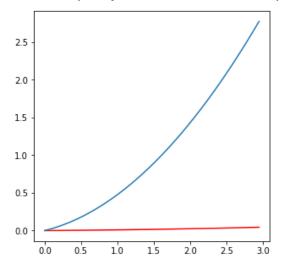


### RK 2nd order

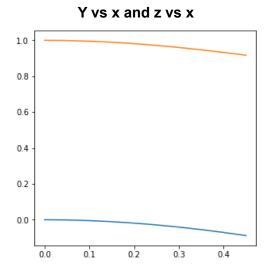


# 0.04 - 0.03 - 0.00 - 0.0 0.5 1.0 1.5 2.0 2.5 3.0

# Error vs x (comparision RK2 and Euiler)



### **Coupled ODE II**



### **Coupled ODE 4**

# Coupled equation 4

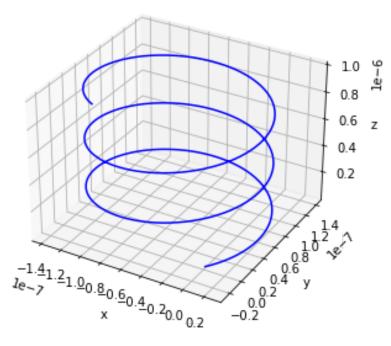
Write a program to follow the motion of an electron (e) in an electric field E(x,t) and a magnetic field B(x,t). Numerically determine the trajectory of an electron for 1 micro second with 1 nano second of time resolution by solving Lorentz force equation:

$$m\frac{d\vec{v}}{dt} = q(\vec{E} + \vec{v}\vec{B})$$

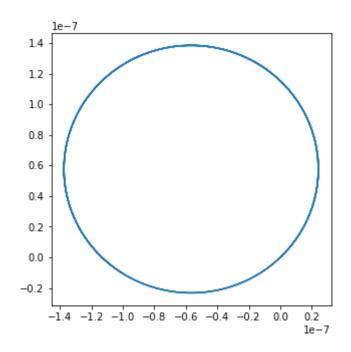
 $m\frac{d\vec{v}}{dt}=q(\vec{E}+\vec{v}\,\vec{B})$  Assume that the particle starts at the origin with velocity v=(1.0,1.0,1.0) m/sec for the following field configurations:

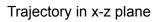
I) Uniform magnetic field  $10^{-4}\,$  Tesla along the z-axis.

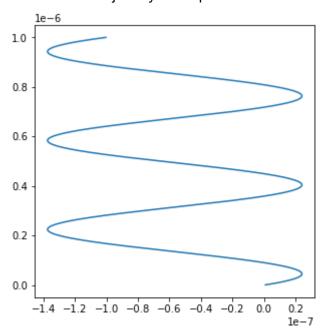
### The trajectory is a helix



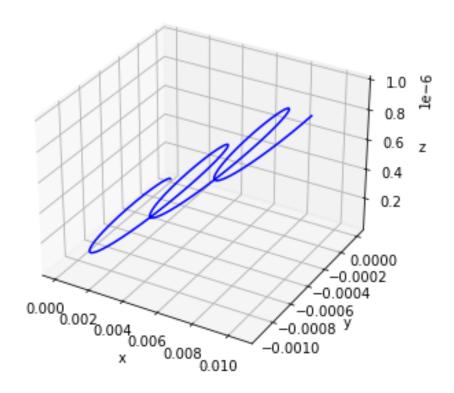
Trajectory in x-y plane



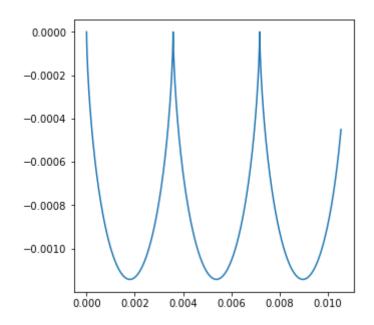




# II)Uniform magnetic field $10^{(-4)}\,Tesla$ along the z-axis and a uniform electric field 1V/m along the y-axis



Trajectory in x-y plane



Trajectory in x-z plane

