Computational Physics 05

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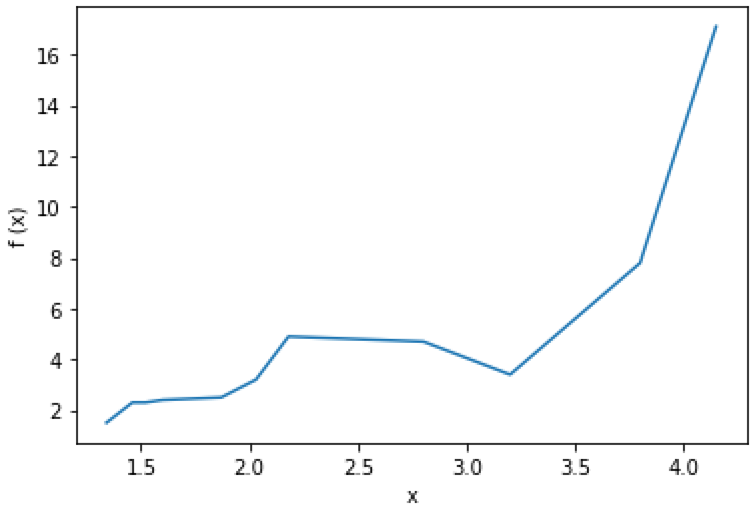
Numerical Integration 1

Plot and integrate the tabulated data given below by a suitable method. Take the endpoints of the table as integration limits

x	1.34	1.46	1.52	1.6	1.87	2.03	2.18	2.8	3.2	3.8	4.15
f(x)	1.5	2.3	2.3	2.4	2.5	3.2	4.9	4.7	3.4	7.8	17.1

The plot of the given points



The Value of integration is 14.59

Numerical Integration 2

1).

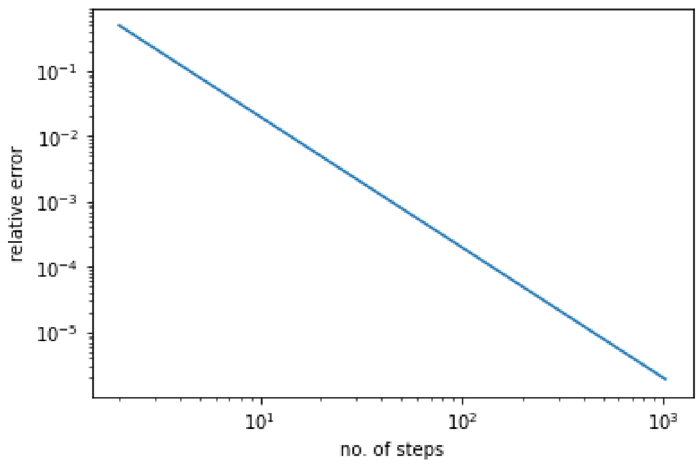
$f(x) = x^2$ from x= -1 to x= +1

Integral values

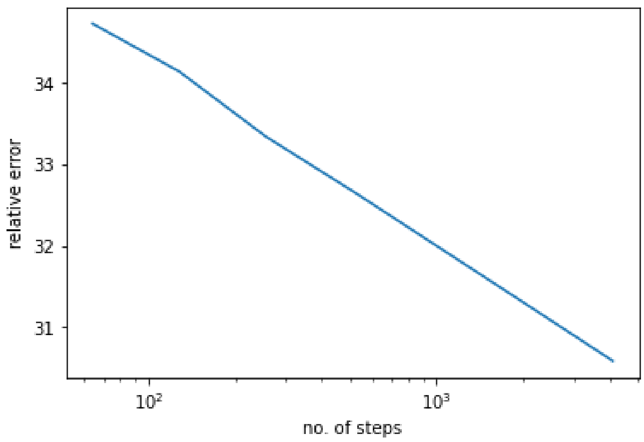
1). By Trapezoidal rule =0.666

2). By Simpson's rule =0.666

Error vs. n graph

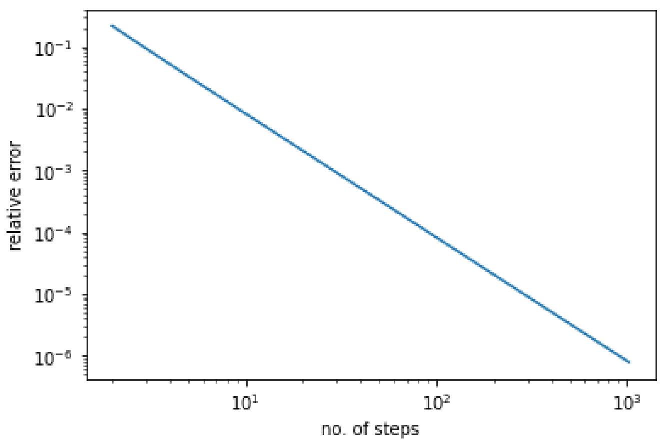


For Trapezoidal rule

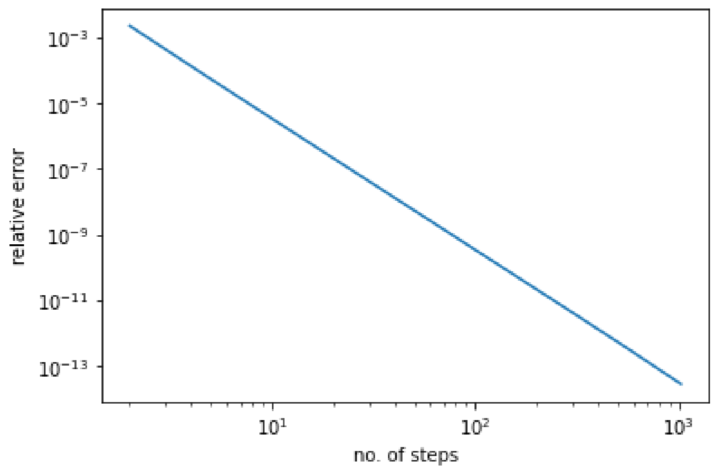


For Simpson’s method

2). $f(x) = sin(x)$**,** $x = 0$ **to** $x = \pi$
By Simpson's method= 2.0000
By Trapezoidal rule=1.9835
Error vs n graphs

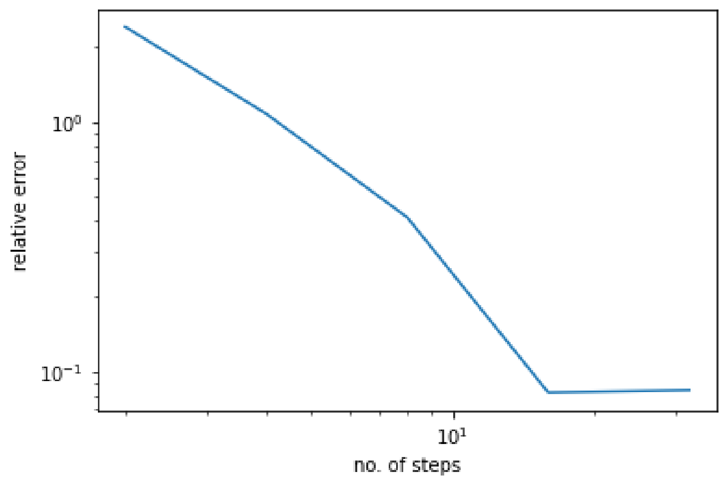


Simpson’s method

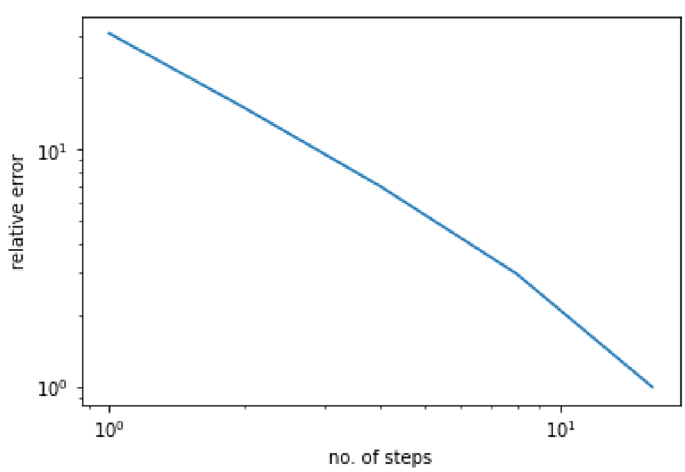


Trapezoidal method

3). $f(x) = (sin(x)/x)^2$ **from** $x = 0$ **to** $x = \infty$ **analytical ans is** $\pi/2$
Simpson's=1.5658
Trapezoidal=1.5658



Simpson's method



Trapezoidal rule

Numerical Integration 4

We consider the bound 1-D motion of a particle of mass min a time independent potential $V(x)$. The fact that the energy Ewill be conserved allows us to integrate the equation of motion and obtain a solution in closed form. The time period of the oscillation T is given by

$$T = \int_a^b \frac{\sqrt{2m}}{\sqrt{E - V(x)}} dx$$

Where the limits a and b are obtained by solving $V(x) = E, a < x < b.$

a)

solving $E = V(x)$ we get a and b to be $\frac{+1}{\omega_o}\sqrt{\frac{2E}{m}}$ and $\frac{-1}{\omega_o}\sqrt{\frac{2E}{m}}$ respectively

given

$$V(x) = \frac{1}{2} m \omega_o^2 x^2$$

where $m = 1Kg, \omega_o = 2\pi sec^{-1}$

substituting $y = \frac{x}{\sqrt{\frac{2E}{m\omega_o^2}}}$, we replace the limit of integrals as follows, a by -1 and b by $+1$, the integral reduces to

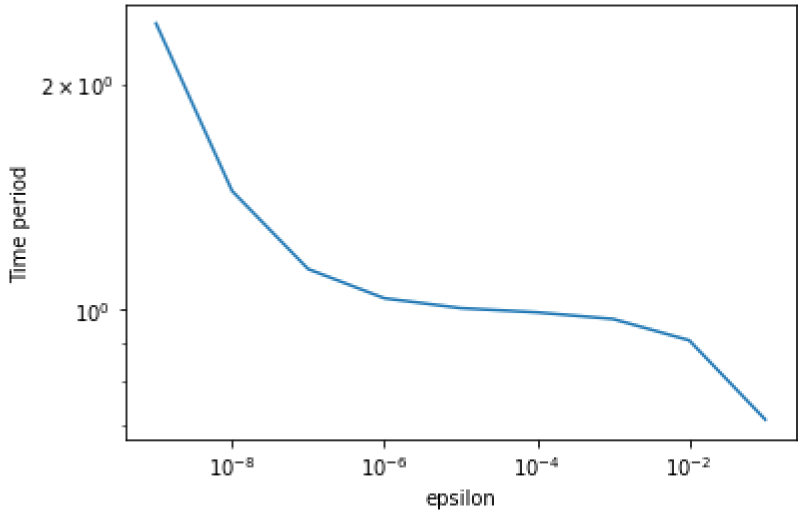
$$T\omega_o = \int_{-1}^{+1} \frac{2}{\sqrt{1 - y^2}} dy$$

Now solving this numerically from a value of $-1 + \epsilon$ to $1 - \epsilon$ we get the time period

The expected value is 1 sec, so the value we obtained is very close to the actual value

We get the Time period to be 1 sec

Epsilon vs no. of steps



b)

Given

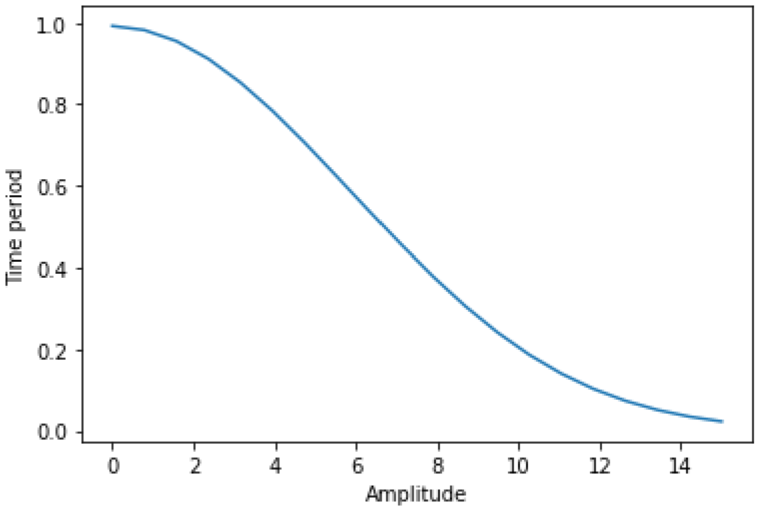
$$V(x) = \frac{m\omega_o^2 L^2}{2}[e^{\frac{x^2}{L^2}} - 1]$$

equating $E = V(a)$, and putting E in the equation, we get the final integral as

$$T = \frac{1}{5\pi}\int_{-A}^{A} \frac{dx}{\sqrt{e^{\frac{A^2}{L^2}} - e^{\frac{x^2}{L^2}}}}$$

where A is the amplitude

Time period vs Amplitude



We can see that for small amplitudes the Time period remains constant and does not depen on the Amplitude