Chemical non-equilibrium in stellar atmospheres

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1 Introduction

- **Layers of the sun**
  The inner layers are the core, radiative zone and the convection zone, the outer layers are the photosphre, chromosphere, transition region and the corona.

  Photosphere
  It is the outer shell from where most of the visible light is radiated. Star's does not have any solid/liquid surface so the photosphere is used to describe its surface. The temperature decreases as we go away from the sun in photosphere , it varies from about 4000 K to 6500 K. It is about 200-300 KM thick. It is composed mainly of Hydrogen and helium.

  Chromosphere
  It is the 2nd layer of the solar atmosphere , above photosphere and upto transition region, It's red in color, extends upto about 2000 km from sun's surface. Temperature increases with height in this layer, its around 4500 K at the start and around 25000 K at the top of chromosphere.

  Corona
  It's the Outermost layer of the sun's atmosphere and starts around 2000 KM above the photosphere and extends upto millions of km , the temperature here is much hotter than that of the photosphere and it is in excess of 1 million K, corona is much less denser than photosphere (the surface), it is made up of plasma (plasma is a state of matter made up of ions). It is much hotter than the surface and other layers. The composition is almost the same as of sun's interior with mainly Hydrogen but degree of ionization is much greater.

- **Coronal Heating problem**
  The corona is much much hotter than the surface (photosphere), what causes this?, and as we move away from the sun in this region the temperature increases opposite to what common sense says.

- **Stellar magnitude** Brightness of a star can be measured. It can be measured by inter comparison or quantitatively by measuring the energy received. For relative brightness we need a standard star. if F is irradiance (energy received per unit are per unit time), then the apparent magnitude is given by

$$m_1 - m_2 = -2.5log_{10}\frac{F_1}{F_2}$$

where 1 is for star1 and 2 is for star2 if we assign 0 by definition for a standard star it simplifies to

$$m_1 = -2.5log_{10}\frac{F_1}{F_s}$$

where F_s is the irradience of the standard star.
If we consider it for a given wavelength λ we can write it as

$$m_\lambda = -2.5log_{10}\frac{F_\lambda}{F_{s\lambda}}$$

- **Absolute magnitude**
Quantitative measurement of amount of radiation leaving the star. This is equal to the apparent magnitude of the star if it were viewed from 10 pc. i. e Absolute magnitude which is denoted by

$$M = m(10pc)$$

$$M - m = 5 - 5log_{10}(d)$$

where M is the absolute magnitude and m is the apparent magnitude and d is the distance of the star is pc.
The ratio of two stars can be described using magnitudes. The magnitudes which which measure the luminosity are called as **Bolometric magnitudes**. These measure all the radiation leaving the star.

- **Stellar color**
Brightness of a star depends on the wavelength at which it is observed. For a given star the magnitudes in different wavelengths maybe different, we may observe a star fainter than the standard star in blue region but brighter or same brightness in the visual region. The difference in these magnitudes tells the relative energy distribution in a star.

$$m_b - m_v = B - V$$

$$m_b - m_v > 0$$

for solar like stars, i.e for stars redder than vega (the ref star) we have

$$B - V > 0$$

, such stars are generally cooler than vega.

If stars have relatively more energy in the blue have smaller magnitudes in the blue, hence $B - V < 0$ for them. These stars are expected to be hotter than vega.

Luminosity of stars
Luminosity-Amount of radiation leaving the stellar surface per sec. If F is the surface flux then L is given by

$$L = 4\pi R^2 F$$

also since the same amount of radiation passes through any concentric sphere with radius r , it can also be given by

$$L = 4\pi r^2 f$$

equation we get

$$f = F\frac{R^2}{r^2}$$

- **Spectral lines**
  It is a line in a continuous spectrum reason being either absorption or emission of light in that frequency range by atoms or molecules. This happens due to the interaction between atom and photon of correct energy which is equal to the energy difference of 2 states, for example when a photon with energy E strikes an e- in E_1 energy state and if there is a energy state E_2 such that $E_2 = E_1 + E$, then the photon is absorbed and the electron jumps to E_2 state.

  Emission spectrum
  When an electron makes transition from an higher energy state to a lower energy state it emits a photon with an energy $E_1 - E_2$, by conservation of energy $E_1 - E_2 = hf$ where f is the frequency of the emitted photon. Substituting $f = \frac{c}{\lambda}$ we get the wavelength of the line we observe, so we have

  $$\frac{1}{\lambda} = \frac{1}{hc}(E_1 - E_2)$$

  since there are many possible transitions each giving out different wavelengths, the collection of all these different lines of different wavelengths make the emission spectrum.

  Absorption spectrum
  When white light is passed through a sample, some of the light is absorbed by the sample which results is black lines in the continuous spectrum of white light. In this case the transition of the electron is from lower energy state to a higher energy state by absorbing photon of that particular energy hence the black line due to the absence of photon of that wavelength.

  These spectral lines can be used to identify a particular molecule/atom present in a sample. It is used to identify components of stars. The strength of the line indicates the abundance of that particular component.

- **Temperature estimates for stars**
  If we want to determine the temperature of a stellar gas, we must know how the radiation of gas changes with temperature. To analyse the radiation we must have some source to compare it to, and it's radiation properties should be independent of the material it is made of. It is called the black-body.

  Black Body
  It does not reflect any light falling on it. A black body looks black if it doesn't emit radiation of it's own. But it can emit radiation of it's own. A black body must be in thermodynamic equilibrium. It does not have temperature difference. The amount of energy (radiation) emitted by the blackbody per unit are per unit sec depends on the temperature of the blackbody and is given by

  $$F = \sigma T^4$$

  where $\sigma = 5.67 * 10\hat{}-8Wm^{-2}K^{-4}$ is called the stefan-boltzman constant.

  All black bodies at the same temperature emit the same kind of radiation irrespective of what they are made of.
  The intensity distribution for a black body is given by the planck's formula as

  $$I_\lambda = \frac{2hc^2}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1} = B_\lambda$$

Intensity increases for all wavelengths for increasing temperatures. The maxima shifts towards shorter wavelengths with increase in temperature. This is given by wien's displacement law

$$\lambda_{max} T = constant$$

Effective temperature of stars If the energy received by earth from sun is f per cm^2, then the total energy leaving the sun's surface must be $L = 4\pi d^2 f$

the amount of energy leaving sun's surface per cm^2 is $F = \frac{L}{4\pi R^2}$ where R is the radius of the sun. which gives

$$F = f(\frac{d}{R})^2$$

now by equating this in stefan-boltzman law, we have get the temperature which a blackbody should have to radiate the same amount of energy per cm^2 as the star, this temperature is called the effective temperature of the star.

Wien temperature Relative energy distribution of star can be used to determine temperature by using wein's displacement law.

- **Basic about radiative transfer**

  Radiative intensity is given by $I_\lambda = \frac{E_\lambda}{d\sigma cos\theta d\omega d\lambda}$.

  Now when when a beam of radiation passes through a volume of gas, there will be 2 things happening,
  i) there will be some amount of absorption of the radiation by the gas.
  ii)there will be emission of some radiation by the gas.

  Absorption
  The amount of energy that's absorbed when it travels through distance of ds can be given as

  $$dE_\lambda = -k_\lambda ds I_\lambda ds d\omega d\sigma$$

  Emission The amount of energy that's emitted when it travels through distance of ds can be given as

  $$dE_\lambda = \epsilon_\lambda d\lambda ds d\omega d\sigma$$

  combining these two we get

  $$dE_\lambda = I_\lambda d\sigma d\omega d\lambda = -k_\lambda ds I_\lambda ds d\omega d\sigma + \epsilon_\lambda d\lambda ds d\omega d\sigma$$

  simplifying we get

  $$\frac{dI_\lambda}{ds} = -k_\lambda I_\lambda + \epsilon_\lambda$$

  writing $\frac{\epsilon_\lambda}{k_\lambda} = S_\lambda$
  we get

  $$\frac{dI_\lambda}{d\tau} = -I_\lambda + S_\lambda$$

  This is called as the radiative transfer equation.
  S is called the source function. τ is called the optical depth.

  Source function

  Considering radiative transfer in a black body, since in a black body the volume of gas is in thermodynamic equilibrium, A beam of light passing through that shouldn't change, so we have

  $$\frac{dI_\lambda}{ds} = -k_\lambda I_\lambda + \epsilon_\lambda = 0$$

this gives

$$S_\lambda = I_\lambda$$

If this assumption is made that $S_\lambda = I_\lambda$ then it is called as the local thermodynamic equlibrium.

we have I as the planck function (which is usually denoted by B), so in a complete thermodynamic equilibrium the source function equals the planck function.

Absorption and emission lines
In most stellar spectra, we see absorption line. In lab, a open hot gas will emission line spectrum while a cold gas in front of a hot light source will give absorption line spectrum. For stars, the deeper layers are hot emitting continuous spectrum, while outer layers are cooler and make the absorption line spectrum.

solving the diffrential equation we get

$$I_\lambda = I_{\lambda o} e^{-\tau_\lambda} + S_\lambda(1 - e^{-\tau_\lambda})$$

i) The first term - It gives the amount of radiation left after it travels an optical path of τ.
ii) The second term - It gives the contribution of the light from radiation emitted along the path.

Case $I_{\lambda o} = 0$
In this we have a volume of hot gas but no light is shining on it, the gas has some emission. The equation becomes,

$$I_\lambda = S_\lambda(1 - e^{-\tau_\lambda})$$

where $S_\lambda = B_\lambda$

Now there are two cases
a). When $\tau_\lambda << 1$ which means the medium is optically thin.

after expanding using taylor series and neglecting the higher order terms, we get

$$I_\lambda = \tau_\lambda S_\lambda$$

where $\tau_\lambda = k_\lambda s$, so the intensity leaving on the other side depends on k_λ, the frequencies at which we see the spectral lines are actually frequencies for which the absorption coefficient k_λ is very large. Therefore the wavelengths corresponding to large k_λ correspond to the wavelengths of the spectral lines. Intensity will be large at thosw wavelength where optical depth is large, i. e k_λ is large, so for $\tau_\lambda << 1$ we expect to see emission lines with large intensity at the wavelengths of large k

b). When $\tau_\lambda >> 1$ i. e $e^{\tau_\lambda} -> 0$, this is the optically very thick case. In this case we have

$$I_\lambda = S_\lambda$$

in LTE

$$I_\lambda = B_\lambda$$

here, the emitted intensity is independent k_λ. This is the case for a blackbody, as we can see the first term becomes 0 meaning that the body absorbs all the light going to it. The emitted intensity is only due to the emission within the box and is given by the source function.

The solar corona corresponds to the 1st case shows emission line spectrum. Stars do not show emission spectrum because they are optically very thick.

Case where $I_{\lambda o} \neq 0$

i)when $\tau_\lambda << 1$ we have

$$I_\lambda = I_{\lambda 0} - \tau_\lambda (I_{\lambda 0} - S_\lambda)$$

If $I_{\lambda 0} > S_\lambda$ then there is something subtracted which is proportional to optical depth, so there is more intensity missing at thosw wavelengths for which k_λ is larger. This is the case of absorption lines.

If $I_{\lambda 0} < S_\lambda$ the last term is positive which means there will be emission lines on top of background intensity of $I_{\lambda 0}$.

ii)when $\tau_\lambda >> 1$

For this otically thick case we have $I_\lambda = S_\lambda$ no matter what $I_{\lambda 0}$ is.

Observationally stars usually gives absorption spectrum which means for them $I_{\lambda 0} > S_\lambda$.i.e The intensity coming from the deepeer layers is larger than that producded at the top layers of the star.
Now assuming LTE (local thermodynamic equilibrium), we get $S_\lambda = B_\lambda$.

2 Molecules is solar atmosphere

$CO, SiH, CH, MgH, OH, HS, AlH, HCl, HF, NH, PH, SH, CO, NO, PO,$
$SO, SiO, CN, H_2, H^+, N_2^+, O_2, O_2^+, CN, HCl,$

References

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