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Problem 1:

Given $||X||' = ||SX||$,

1. If $X \ != 0$, $||X||' > 0$
2. Let k be a scalar, $||kX|| = ||SkX|| = |k|\ ||SX|| = |k|\ ||X||'$
3. $||\ X + Y||' = ||\ S(X+Y)|| = ||\ SX + SY|| <= ||SX|| + ||SY|| = ||X||' + ||Y||'$

   $||\ X + Y||' <= ||X||' + ||Y||'$

As $||\cdot||'$ satisfies all the above 3 conditions , it is a norm

Given $||\cdot||$ is a subordinate matrix, to show $||A||' = ||SAS^{-1}||$ is a matrix norm

1. If $A \ != 0$, $||A||' = ||SAS^{-1}|| > 0$
2. $||kA||' = ||SkAS^{-1}|| = |k|\ ||SAS^{-1}|| = |k|\ ||A||'$
3. $||A+B||' = ||S(A+B)S^{-1}|| = ||SAS^{-1} + SBS^{-1}||$

   $<= ||SAS^{-1}|| + ||SBS^{-1}|| = ||A||' + ||B||'$

   $||A+B||' <= ||A||' + ||B||'$

4. $||AB||' = ||SABS^{-1}|| = ||SAS^{-1}SBS^{-1}||$

   $<= ||SAS^{-1}||\ ||SBS^{-1}|| = ||A||'\ ||B||'$

   $||AB||' <= ||A||'\ ||B||'$

Since, $||A||' = ||SAS^{-1}||$ satisfies all the above 4 conditions it is a matrix norm

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Problem 2:

Given A is a positive definite matrix, i.e., $X^T A X > 0$ (X is a Vector, A is a matrix)

Let $(1/X X^T) = k$, some scalar

A^2: $X^T A^2 X = X^T A A X = (1/X X^T)(X^T A X X^T A X) =k (X^T A X)(X^T A X) > 0$

$X^T A^2 X > 0$

A^3 : $X^T A^3 X = X^T A^2 A X = (1/X X^T)(X^T A^2 X X^T A X) = k(X^T A^2 X)(X^T A X) > 0$

$X^T A^3 X > 0$

A^4: $X^T A^4 X = X^T A^3 A X = (1/X X^T)(X^T A^3 X X^T A X) = k(X^T A^3 X)(X^T A X) > 0$

$X^T A^4 X > 0$

Therefore A^n is a positive definite matrix for $n = \{1, 2, 3, 4, \ldots\ldots\}$

Consider $X^T A A^{-1} X = (1/X X^T)(X^T A X X^T A^{-1} X)$

$X^T I X = k(X^T A X)(X^T A^{-1} X)$

$X^T I X > 0$, $X^T A X > 0$, $k > 0 \Rightarrow X^T A^{-1} X > 0$

Therefore A^{-1} is a positive definite

A^{-2}: $X^T A^{-2} X = X^T A^{-1} A^{-1} X = (1/X X^T)(X^T A^{-1} X X^T A^{-1} X) =k (X^T A^{-1} X)(X^T A^{-1} X) > 0$
$X^T A^{-2} X > 0$

A^{-3} : $X^T A^{-3} X = X^T A^{-2} A^{-1} X = (1/X X^T)(X^T A^{-2} X X^T A^{-1}X) = k(X^T A^{-2} X)(X^T A^{-1}X) > 0$
$X^T A^{-3} X > 0$

A^{-4}: $X^T A^{-4} X = X^T A^{-3} A^{-1} X = (1/X X^T)(X^T A^{-3} X X^T A^{-1} X) = k(X^T A^{-3} X)(X^T A^{-1}X) > 0$
$X^T A^{-4} X > 0$

Therefore A^n is a positive definite matrix for all $n = \{\ldots\ldots, -3, -2, -1, 1, 2, 3, 4, \ldots\ldots\}$

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Problem 3:

When solving system of linear equations, Ax = B,

Guass siedel method converges only if matrix A is either diagonally dominant or symmetric and Positive definite

For the 1st set of linear equations Guass siedel method converges and the solution is x = 1, y = 1, z = 1 (A is a diagonally dominant matrix)

Output:

Select 1 or 2: 1

3x + y + z = 5

x + 3y -z = 3

3x + y - 5z = -1

No. of iterations:100

Solution matrix X

[[1.]

[1.]

[1.]]

For the 2nd set of linear equations, Guass siedel method diverges since matrix A is neither diagonally dominant nor symmetric and positive definite

Select 1 or 2: 2

3x + y + z = 5

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$3x + y - 5z = -1$

$x + 3y - z = 3$

Gauss Seidel method does not convergence (A is neither diagonally dominant nor positive definite)