CHAPTER-11
THREE DIMENSIONAL GEOMETRY

EXERCISE-11.2

1. Show that the three lines with direction cosines $\frac{12}{13}, \frac{-3}{13}, \frac{-4}{13}; \frac{4}{13}, \frac{12}{13}, \frac{3}{13}; \frac{3}{13}, \frac{-4}{13}, \frac{12}{13};$ are mutually perpendicular.

2. Show that the line through the points $(1, -1, 2), (3, 4, -2)$ is perpendicular to the line through the points$(0, 3, 2)$ and$(3, 5, 6)$.

3. Show that the line through the points $(4, 7, 8), (2, 3, 4)$ is parallel to the line through the points $(-1, -2, 1), (1, 2, 5)$.

4. Find the equation of the line which passes through the point $(1, 2, 3)$ and is parallel to the vector $3\hat{i} + 2\hat{j} - 2\hat{k}$

5. Find the equation of the line in vector and in cartesian form that passes through the point with position vector $2\hat{i} - \hat{j} + 4\hat{k}$ and is in direction $\hat{i} + 2\hat{j} - \hat{k}$.

6. Find the cartesian equation of the line which passes through the point $(-2, 4, -5)$ and parallel to the line given by$\frac{x+3}{3} = \frac{y-4}{5} = \frac{z+8}{6}$.

7. The cartesian equation of a line is $\frac{x-5}{3} = \frac{y+4}{7} = \frac{z-6}{2}$. Write its vector form.

8. Find the vector and the cartesian equations of the lines that passes through the origin and $(5, -2, 3)$.

9. Find the vector and the cartesian equations of the line that passes through the points $(3, -2, -5), (3, -2, 6)$.

10. Find the angle between the following pairs of lines:

    (a) $\overrightarrow{r} = 2\hat{i} - 5\hat{j} + \hat{k} + \lambda(3\hat{i} + 2\hat{j} + 6\hat{k})$ and
    $\overrightarrow{r} = 7\hat{i} - 6\hat{k} + \mu(\hat{i} + 2\hat{j} + 2\hat{k})$

    (b) $\overrightarrow{r} = 3\hat{i} + \hat{j} - 2\hat{k} + \lambda(\hat{i} - \hat{j} - 2\hat{k})$ and
    $\overrightarrow{r} = 2\hat{i} - \hat{j} - 56\hat{k} + \mu(3\hat{i} - 5\hat{j} - 4\hat{k})$

11. Find the angle between the following pairs of lines:

    (a) $\frac{x-2}{2} = \frac{y-1}{5} = \frac{z+3}{-3}$ and $\frac{x+2}{-1} = \frac{y-4}{8} = \frac{z-5}{4}$.

    (b) $\frac{x}{2} = \frac{y}{2} = \frac{z}{1}$ and $\frac{x-5}{4} = \frac{y-2}{1} = \frac{z-3}{8}$.

12. Find the values of p so that the lines $\frac{1-x}{3} = \frac{7y-14}{2p} = \frac{z-3}{2}$ and $\frac{7-7x}{3p} = \frac{y-5}{1} = \frac{6-z}{5}$ are at right angles.

13. Show that the lines $\frac{x-5}{7} = \frac{y+2}{-5} = \frac{z}{1}$ and $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ are perpendicular to each other.

14. Find the shortest distance between the lines
    $\overrightarrow{r} = (\hat{i} + 2\hat{j} + \hat{k}) + \lambda(\hat{i} - \hat{j} + \hat{k})$ and
    $\overrightarrow{r} = 2\hat{i} - \hat{j} - \hat{k} + \mu(2\hat{i} + \hat{j} + 2\hat{k})$

15. Find the shortest distance between the lines
    $\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}$ and $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$

16. Find the shortest distance between the lines whose vector equations are
    $\overrightarrow{r} = (\hat{i} + 2\hat{j} + 3\hat{k}) + \lambda(\hat{i} - 3\hat{j} + 2\hat{k})$ and
    $\overrightarrow{r} = 4\hat{i} + 5\hat{j} + 6\hat{k} + \mu(2\hat{i} + 3\hat{j} + \hat{k})$

17. Find the shortest distance between the lines whose vector equations are
    $\overrightarrow{r} = (1-t)\hat{i} + (t-2)\hat{j} + (3-2t)\hat{k}$ and
    $\overrightarrow{r} = (s+1)\hat{i} + (2s-1)\hat{j} - (2s+1)\hat{k}$