VECTORS

1 12^{th} Maths - EXERCISE-10.3

1. Find $|\overrightarrow{a}|$ and $\left|\overrightarrow{b}\right|$, if $(\overrightarrow{a}+\overrightarrow{b}).(\overrightarrow{a}-\overrightarrow{b})=8$ and $\overrightarrow{a}=8\left|\overrightarrow{b}\right|$.

Solution: Given points are

$$(\mathbf{a}+\mathbf{b})^\top(\mathbf{a}-\mathbf{b})=8 \tag{1}$$

$$|\mathbf{a}|=8\,|\mathbf{b}| \tag{2}$$

$$(\mathbf{a}+\mathbf{b})^\top(\mathbf{a}-\mathbf{b})=8 \tag{3}$$

$$\implies \mathbf{a}^\top\mathbf{a}+\mathbf{b}^\top\mathbf{a}-\mathbf{a}^\top\mathbf{b}-\mathbf{b}^\top\mathbf{b}=8 \tag{4}$$

$$\implies \|\mathbf{a}\|^2-\|\mathbf{b}\|^2=8 \tag{5}$$

$$\implies (\|8\mathbf{b}\|)^2-(\|\mathbf{b}\|)^2=8 \tag{6}$$

$$\implies 64\|\mathbf{b}\|^2-\|\mathbf{b}\|^2=8 \tag{7}$$

$$\implies 63\|\mathbf{b}\|^2=8 \tag{8}$$

$$\implies \|\mathbf{b}\|^2=\frac{8}{63} \tag{9}$$

$$\|\mathbf{b}\|=\sqrt{\frac{8}{63}} \tag{10}$$

$$\|\mathbf{b}\|=\frac{2\sqrt{2}}{3\sqrt{7}} \tag{11}$$

$$\|\mathbf{a}\| = 8\,\|\mathbf{b}\| \tag{12}$$

$$\implies \|\mathbf{a}\| = 8.\frac{2\sqrt{2}}{3\sqrt{7}} \tag{13}$$

$$\|\mathbf{a}\| = \frac{16\sqrt{2}}{3\sqrt{7}} \tag{14}$$