# TRIANGLES

## $9^{th}$Math - Chapter 7

## 1 problem

In the given figure,AC=AE,AB=AD and $\angle BAD = \angle EAC$.Show that BC=DE.

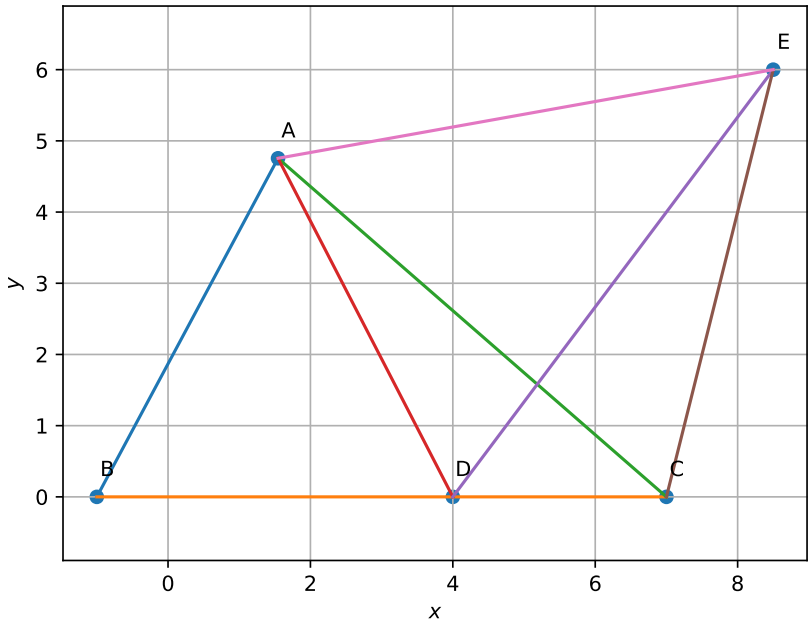


Figure 1

## 2 construction

The input parameters for construction.

| Symbol | Values | Description |
|---|---|---|
| $d$ | 7 | length of side BC |
| $\theta 1$ | 72° | $\angle BAD$=$\angle EAC$ |

Table 2

$$\mathbf{A} = 5\begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix}$$
$$\mathbf{B} = \begin{pmatrix}-1\\ 0\end{pmatrix}$$

$$\mathbf{C} = \begin{pmatrix} d \\ 0 \end{pmatrix}$$
$$\mathbf{D} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$$
$$\mathbf{E} = \begin{pmatrix} 8.5 \\ 6 \end{pmatrix}$$

# 3 solution

**Given:**

$$AC = AE \tag{1}$$
$$AB = AD \tag{2}$$
$$\angle BAD = \angle EAC \tag{3}$$

**To prove :**

$$BC = DE \tag{4}$$

**Proof**
In $\triangle ABC$ and in $\triangle ADE$

$$\|\mathbf{A} - \mathbf{B}\| = \left\| \begin{pmatrix} 2.54 \\ 4.75 \end{pmatrix} \right\| = 5.3 \tag{5}$$
$$\|\mathbf{A} - \mathbf{D}\| = \left\| \begin{pmatrix} -2.46 \\ 4.75 \end{pmatrix} \right\| = 5.3 \tag{6}$$
$$\implies \|\mathbf{A} - \mathbf{B}\| = \|\mathbf{A} - \mathbf{D}\| \tag{7}$$
$$\text{or, } AB = AD \tag{8}$$
$$\|\mathbf{A} - \mathbf{C}\| = \left\| \begin{pmatrix} -5.46 \\ 4.75 \end{pmatrix} \right\| = 7.2 \tag{9}$$
$$\|\mathbf{A} - \mathbf{E}\| = \left\| \begin{pmatrix} -6.96 \\ -1.25 \end{pmatrix} \right\| = 7.1 \tag{10}$$
$$\implies \|\mathbf{A} - \mathbf{C}\| = \|\mathbf{A} - \mathbf{E}\| \tag{11}$$
$$\text{or, } AC = AE \tag{12}$$
$$\mathbf{m_1} = \mathbf{A} - \mathbf{B} \tag{13}$$
$$\mathbf{m_2} = \mathbf{A} - \mathbf{C} \tag{14}$$
$$\mathbf{n_1} = \mathbf{A} - \mathbf{D} \tag{15}$$
$$\mathbf{n_2} = \mathbf{A} - \mathbf{E} \tag{16}$$
$$\theta_1 = \cos^{-1} \frac{\mathbf{m_1}^\top \mathbf{m_2}}{\|\mathbf{m_1}\|\|\mathbf{m_2}\|} \tag{17}$$
$$\implies \theta_1 = \cos^{-1} \frac{\begin{pmatrix} 2.54 & -4.75 \end{pmatrix} \begin{pmatrix} -5.46 \\ 4.75 \end{pmatrix}}{(5.3)(7.2)} = 102^\circ \tag{18}$$
$$\theta_2 = \cos^{-1} \frac{\mathbf{n_1}^\top \mathbf{n_2}}{\|\mathbf{n_1}\|\|\mathbf{n_2}\|} \tag{19}$$
$$\implies \theta_2 = \cos^{-1} \frac{\begin{pmatrix} -2.46 & -4.75 \end{pmatrix} \begin{pmatrix} -6.96 \\ -1.25 \end{pmatrix}}{(5.3)(7.1)} = 102^\circ \tag{20}$$

from (18) and (20)

$$\angle BAC = \angle DAE \tag{21}$$

from (8),(12) and (21)

$$\triangle ABC \cong \triangle ADE \tag{22}$$

from (22)

$$BC = DE \tag{23}$$