VECTORS

$1 \quad 12^{th} \text{ Maths}$ - EXERCISE-10.3

1. Find $|\overrightarrow{a}|$ and $|\overrightarrow{b}|$, if $(\overrightarrow{a}+\overrightarrow{b}).(\overrightarrow{a}-\overrightarrow{b})=8$ and $\overrightarrow{a}=8|\overrightarrow{b}|$.

Solution: Given points are

$$(\mathbf{a} + \mathbf{b}).(\mathbf{a} - \mathbf{b}) = 8 \tag{1}$$

$$|\mathbf{a}| = 8|\mathbf{b}|\tag{2}$$

$$(\mathbf{a} + \mathbf{b}).(\mathbf{a} - \mathbf{b}) = 8 \tag{3}$$

$$\mathbf{a}.(\mathbf{a} - \mathbf{b}) + \mathbf{b}.(\mathbf{a} - \mathbf{b}) = 8 \tag{4}$$

$$\mathbf{a}.\mathbf{a} - \mathbf{a}.\mathbf{b} + \mathbf{b}.\mathbf{a} - \mathbf{b}.\mathbf{b} = 8 \tag{5}$$

$$\mathbf{a}.\mathbf{a} - \mathbf{a}.\mathbf{b} + \mathbf{a}.\mathbf{b} - \mathbf{b}.\mathbf{b} = 8 \tag{6}$$

$$\mathbf{a.a} - \mathbf{b.b} = 8 \tag{7}$$

$$(|\mathbf{a}|)^2 - (|\mathbf{b}|)^2 = 8$$
 (8)

$$(|8\mathbf{b}|)^2 - (|\mathbf{b}|)^2 = 8 \tag{9}$$

$$64\mathbf{b}^2 - \mathbf{b}^2 = 8 \tag{10}$$

$$63\mathbf{b}^2 = 8\tag{11}$$

$$\mathbf{b}^2 = \frac{8}{63} \tag{12}$$

$$|\mathbf{b}| = \sqrt{\frac{8}{63}} \tag{13}$$

$$|\mathbf{b}| = \frac{2\sqrt{2}}{3\sqrt{7}}\tag{14}$$

$$|\mathbf{a}| = 8 \, |\mathbf{b}| \tag{15}$$

$$|\mathbf{a}| = 8.\frac{2\sqrt{2}}{3\sqrt{7}}\tag{16}$$

$$|\mathbf{a}| = \frac{16\sqrt{2}}{3\sqrt{7}}\tag{17}$$