VECTORS

1 12^{th} Maths - EXERCISE-10.3

1. Find $|\overrightarrow{a}|$ and $\left|\overrightarrow{b}\right|$, if $(\overrightarrow{a}+\overrightarrow{b}).(\overrightarrow{a}-\overrightarrow{b})=8$ and $\overrightarrow{a}=8\left|\overrightarrow{b}\right|$.

Solution: Given points are

$$(\mathbf{a}+\mathbf{b})^{\top}(\mathbf{a}-\mathbf{b})=8 \tag{1}$$

$$|\mathbf{a}|=8\,|\mathbf{b}| \tag{2}$$

$$(\mathbf{a}+\mathbf{b})^{\top}(\mathbf{a}-\mathbf{b})=8 \tag{3}$$

$$\mathbf{a}^{\top}\mathbf{a}+\mathbf{b}^{\top}\mathbf{a}-\mathbf{a}^{\top}\mathbf{b}-\mathbf{b}^{\top}\mathbf{b}=8 \tag{4}$$

$$(|\mathbf{a}|)^2-(|\mathbf{b}|)^2=8 \tag{5}$$

$$(|8\mathbf{b}|)^2-(|\mathbf{b}|)^2=8 \tag{6}$$

$$64\mathbf{b}^2-\mathbf{b}^2=8 \tag{7}$$

$$63\mathbf{b}^2=8 \tag{8}$$

$$\mathbf{b}^2=\frac{8}{63} \tag{9}$$

$$|\mathbf{b}|=\sqrt{\frac{8}{63}} \tag{10}$$

$$|\mathbf{b}|=\frac{2\sqrt{2}}{3\sqrt{7}} \tag{11}$$

$$|\mathbf{a}|=8\,|\mathbf{b}| \tag{12}$$

$$|\mathbf{a}|=8.\frac{2\sqrt{2}}{3\sqrt{7}} \tag{13}$$

$$|\mathbf{a}|=\frac{16\sqrt{2}}{3\sqrt{7}} \tag{14}$$