CIRCLES

9^{th}Math - Chapter 10

Problem

If the non-parallel sides of a trapezium are equal,prove that it is cyclic.

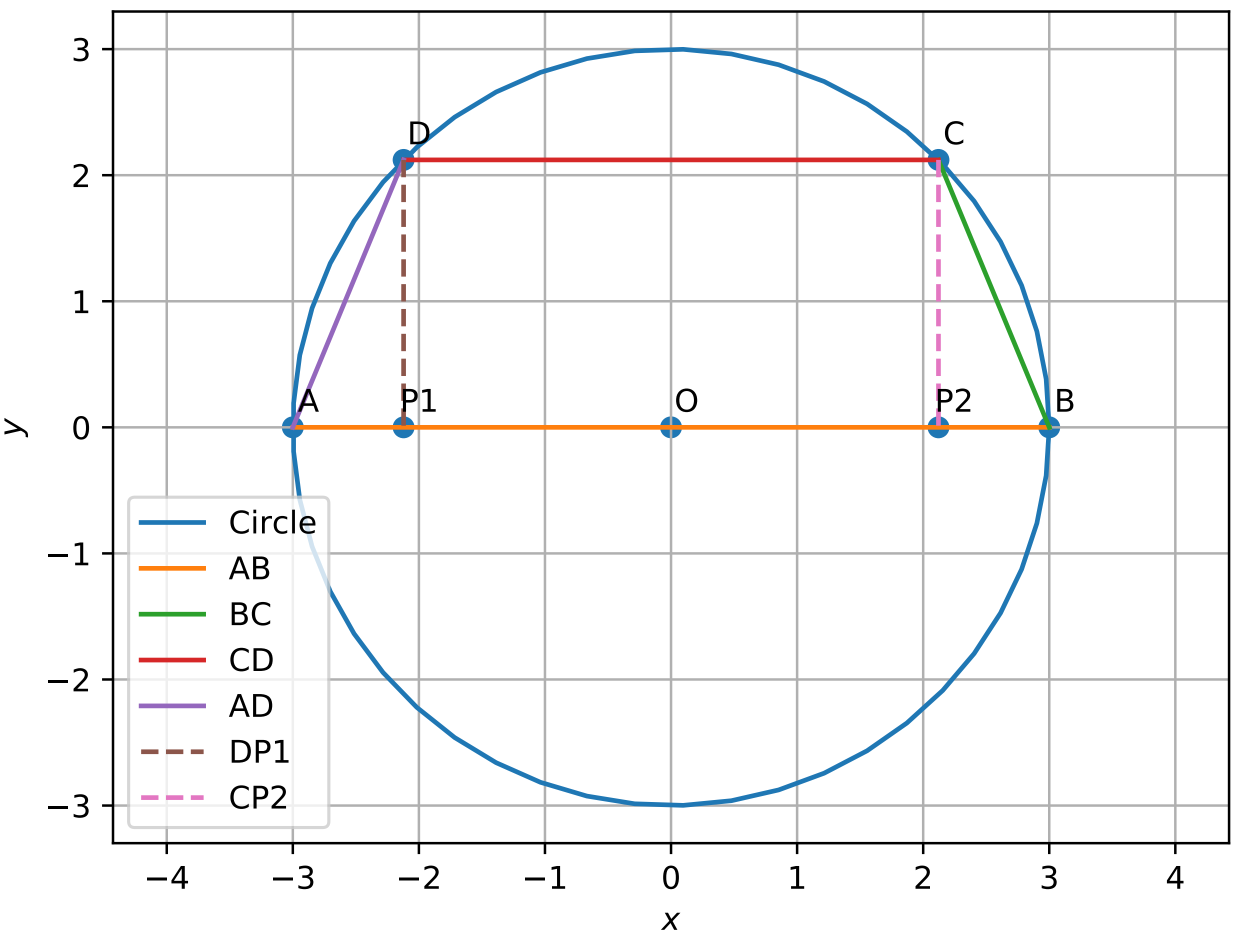

Figure 1

Construction

The input parameters for construction.

Symbol	Values	Description
r	3	Radius of the circle
b	2.1	Altitude
θ_1	45°	Assumed angle
θ_2	180°	Assumed angle
θ_3	0°	Assumed angle

$$\mathbf{A} = r\begin{pmatrix}\cos\theta_2\\ \sin\theta_2\end{pmatrix}, \mathbf{B} = r\begin{pmatrix}\cos\theta_3\\ \sin\theta_3\end{pmatrix}, \mathbf{C} = r\begin{pmatrix}\cos\theta_1\\ \sin\theta_1\end{pmatrix}, \mathbf{D} = r\begin{pmatrix}-\cos\theta_1\\ \sin\theta_1\end{pmatrix} \tag{1}$$

$$\mathbf{P_1} = b\begin{pmatrix}\cos\theta_2\\ \sin\theta_2\end{pmatrix}, \mathbf{P_2} = b\begin{pmatrix}\cos\theta_3\\ \sin\theta_3\end{pmatrix} \tag{2}$$

Solution:
Theorm: If the sum of a pair of opposite angles of a quadrilateral is 180°, the quadrilateral is cyclic.

Proof:

$$\angle DAP_1 = \cos^{-1}\frac{(\mathbf{A}-\mathbf{D})^\top(\mathbf{A}-\mathbf{P_1})}{\|\mathbf{A}-\mathbf{D}\|\|\mathbf{A}-\mathbf{P_1}\|} = 67.5^\circ \tag{3}$$

$$\angle DCP_2 = \cos^{-1}\frac{(\mathbf{C}-\mathbf{D})^\top(\mathbf{P_2}-\mathbf{C})}{\|\mathbf{C}-\mathbf{D}\|\|\mathbf{P_2}-\mathbf{C}\|} = 90^\circ \tag{4}$$

$$\angle BCP_2 = \cos^{-1}\frac{(\mathbf{B}-\mathbf{C})^\top(\mathbf{P_2}-\mathbf{C})}{\|\mathbf{B}-\mathbf{C}\|\|\mathbf{P_2}-\mathbf{C}\|} = 22.5^\circ \tag{5}$$

from (3),(4) and (5)

$$\angle DAP_1 + \angle DCP_2 + \angle BCP_2 = 180^\circ \tag{6}$$

from (6) given quadrilateral is cyclic.