

VECTORS

1 12th Maths - EXERCISE-10.3

1. Find $|\vec{a}|$ and $|\vec{b}|$, if $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 8$ and $|\vec{a}| = 8|\vec{b}|$.

Solution: Given points are

$$(\mathbf{a} + \mathbf{b})^\top (\mathbf{a} - \mathbf{b}) = 8 \quad (1)$$

$$|\mathbf{a}| = 8|\mathbf{b}| \quad (2)$$

$$(\mathbf{a} + \mathbf{b})^\top (\mathbf{a} - \mathbf{b}) = 8 \quad (3)$$

$$\Rightarrow \mathbf{a}^\top \mathbf{a} + \mathbf{b}^\top \mathbf{a} - \mathbf{a}^\top \mathbf{b} - \mathbf{b}^\top \mathbf{b} = 8 \quad (4)$$

$$\Rightarrow \|\mathbf{a}\|^2 - \|\mathbf{b}\|^2 = 8 \quad (5)$$

$$\Rightarrow \|8\mathbf{b}\|^2 - \|\mathbf{b}\|^2 = 8 \quad (6)$$

$$\Rightarrow 64\|\mathbf{b}\|^2 - \|\mathbf{b}\|^2 = 8 \quad (7)$$

$$\Rightarrow 63\|\mathbf{b}\|^2 = 8 \quad (8)$$

$$\Rightarrow \|\mathbf{b}\|^2 = \frac{8}{63} \quad (9)$$

$$\|\mathbf{b}\| = \sqrt{\frac{8}{63}} \quad (10)$$

$$\|\mathbf{b}\| = \frac{2\sqrt{2}}{3\sqrt{7}} \quad (11)$$

$$\|\mathbf{a}\| = 8\|\mathbf{b}\| \quad (12)$$

$$\Rightarrow \|\mathbf{a}\| = 8 \cdot \frac{2\sqrt{2}}{3\sqrt{7}} \quad (13)$$

$$\|\mathbf{a}\| = \frac{16\sqrt{2}}{3\sqrt{7}} \quad (14)$$