

# VECTORS

## 1 12<sup>th</sup> Maths - EXERCISE-10.3

1. Find  $|\vec{a}|$  and  $|\vec{b}|$ , if  $(\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b}) = 8$  and  $|\vec{a}| = 8|\vec{b}|$ .

**Solution:** Given points are

$$(\mathbf{a} + \mathbf{b})^\top (\mathbf{a} - \mathbf{b}) = 8 \quad (1)$$

$$|\mathbf{a}| = 8|\mathbf{b}| \quad (2)$$

$$(\mathbf{a} + \mathbf{b})^\top (\mathbf{a} - \mathbf{b}) = 8 \quad (3)$$

$$\implies \mathbf{a}^\top \mathbf{a} + \mathbf{b}^\top \mathbf{a} - \mathbf{a}^\top \mathbf{b} - \mathbf{b}^\top \mathbf{b} = 8 \quad (4)$$

$$\implies \|\mathbf{a}\|^2 - \|\mathbf{b}\|^2 = 8 \quad (5)$$

$$\implies \|8\mathbf{b}\|^2 - \|\mathbf{b}\|^2 = 8 \quad (6)$$

$$\implies 64\|\mathbf{b}\|^2 - \|\mathbf{b}\|^2 = 8 \quad (7)$$

$$\implies 63\|\mathbf{b}\|^2 = 8 \quad (8)$$

$$\implies \|\mathbf{b}\|^2 = \frac{8}{63} \quad (9)$$

$$\|\mathbf{b}\| = \sqrt{\frac{8}{63}} \quad (10)$$

$$\|\mathbf{b}\| = \frac{2\sqrt{2}}{3\sqrt{7}} \quad (11)$$

$$\|\mathbf{a}\| = 8\|\mathbf{b}\| \quad (12)$$

$$\|\mathbf{a}\| = \frac{16\sqrt{2}}{3\sqrt{7}} \quad (13)$$