VECTORS

$1 \quad 12^{th} \text{ Maths}$ - EXERCISE-10.3

1. Find $|\overrightarrow{a}|$ and $|\overrightarrow{b}|$, if $(\overrightarrow{a}+\overrightarrow{b}).(\overrightarrow{a}-\overrightarrow{b})=8$ and $\overrightarrow{a}=8|\overrightarrow{b}|$.

Solution: Given points are

$$(\mathbf{a} + \mathbf{b})^{\top} (\mathbf{a} - \mathbf{b}) = 8 \tag{1}$$

$$|\mathbf{a}| = 8|\mathbf{b}|\tag{2}$$

$$(\mathbf{a} + \mathbf{b})^{\top} (\mathbf{a} - \mathbf{b}) = 8 \tag{3}$$

$$\mathbf{a}^{\top}\mathbf{a} + \mathbf{b}^{\top}\mathbf{a} - \mathbf{a}^{\top}\mathbf{b} - \mathbf{b}^{\top}\mathbf{b} = 8 \tag{4}$$

$$\|\mathbf{a}\|^2 - \|\mathbf{b}\|^2 = 8$$
 (5)

$$(\|8\mathbf{b}\|)^2 - (\|\mathbf{b}\|)^2 = 8 \tag{6}$$

$$64\|\mathbf{b}\|^2 - \|\mathbf{b}\|^2 = 8\tag{7}$$

$$63\|\mathbf{b}\|^2 = 8\tag{8}$$

$$\left\|\mathbf{b}\right\|^2 = \frac{8}{63} \tag{9}$$

$$\|\mathbf{b}\| = \sqrt{\frac{8}{63}} \tag{10}$$

$$\|\mathbf{b}\| = \frac{2\sqrt{2}}{3\sqrt{7}}\tag{11}$$

$$|\mathbf{a}| = 8|\mathbf{b}|\tag{12}$$

$$|\mathbf{a}| = 8.\frac{2\sqrt{2}}{3\sqrt{7}}\tag{13}$$

$$|\mathbf{a}| = \frac{16\sqrt{2}}{3\sqrt{7}}\tag{14}$$