TRIANGLES

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Contents

1 Problem

In the given figure,$AC = AE$,$AB = AD$ and $\angle BAD = \angle EAC$.Show that $BC = DE$.

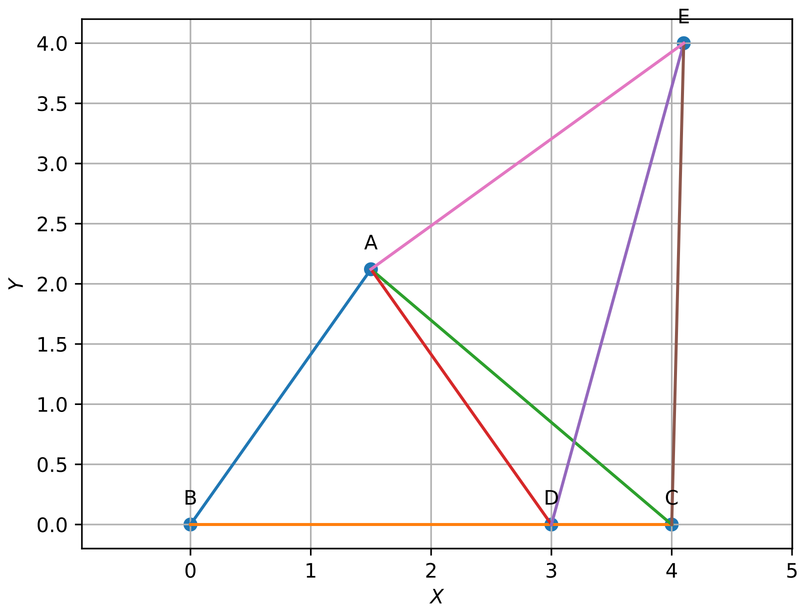

Figure 1:

2 Construction

The input parameters for this construction are

Symbol	Value
r	3
b	4
b1	3
$\theta 1$	$\frac{pi}{3}$
$\theta 2$	$\frac{pi}{4}$

$$\mathbf{A} = \begin{pmatrix} r\cos\theta \\ r\sin\theta \end{pmatrix}$$
$$\mathbf{B} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$
$$\mathbf{C} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$$
$$\mathbf{D} = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$$
$$\mathbf{E} = \begin{pmatrix} 4.1 \\ 4 \end{pmatrix}$$

3 Solution

We know that two triangles are said to be congruent if the sides and angles of one triangle are equal to the corresponding sides and angles of the other triangle.In congruent triangles corresponding parts are equal.

Given

$$\mathbf{A} - \mathbf{C} = \mathbf{A} - \mathbf{E} \tag{1}$$
$$\mathbf{A} - \mathbf{B} = \mathbf{A} - \mathbf{D} \tag{2}$$
$$\angle BAD = \angle EAC \tag{3}$$

Proof
Given that

$$\angle BAD = \angle EAC$$
$$\angle BAD + \angle DAC = \angle EAC + \angle DAC$$
$$\angle BAC = \angle DAE$$

In $\triangle ABC$ and in $\triangle ADE$

$$\mathbf{A}-\mathbf{B}=\mathbf{A}-\mathbf{D} \tag{4}$$

$$\angle BAC=\angle DAE \tag{5}$$

$$\implies cos\angle BAC=cos\angle DAE \tag{6}$$

$$\implies \frac{(\mathbf{A}-\mathbf{B})^T(\mathbf{A}-\mathbf{C})}{\|\mathbf{A}-\mathbf{B}\|\|\mathbf{A}-\mathbf{C}\|}=\frac{(\mathbf{A}-\mathbf{D})^T(\mathbf{A}-\mathbf{E})}{\|\mathbf{A}-\mathbf{D}\|\|\mathbf{A}-\mathbf{E}\|} \tag{7}$$

$$\mathbf{A}-\mathbf{C}=\mathbf{A}-\mathbf{E} \tag{8}$$

1. From (4), (7) and (8)

   taking the norms of the respective sides,

   $$\triangle ABC \cong \triangle ADE \tag{9}$$

2. From (4), (7) and (8) taking the norms of the respecttive sides

   $$BC=DE \tag{10}$$