CIRCLES

9^{th}Math - Chapter 10

Problem

If the non-parallel sides of a trapezium are equal,prove that it is cyclic.

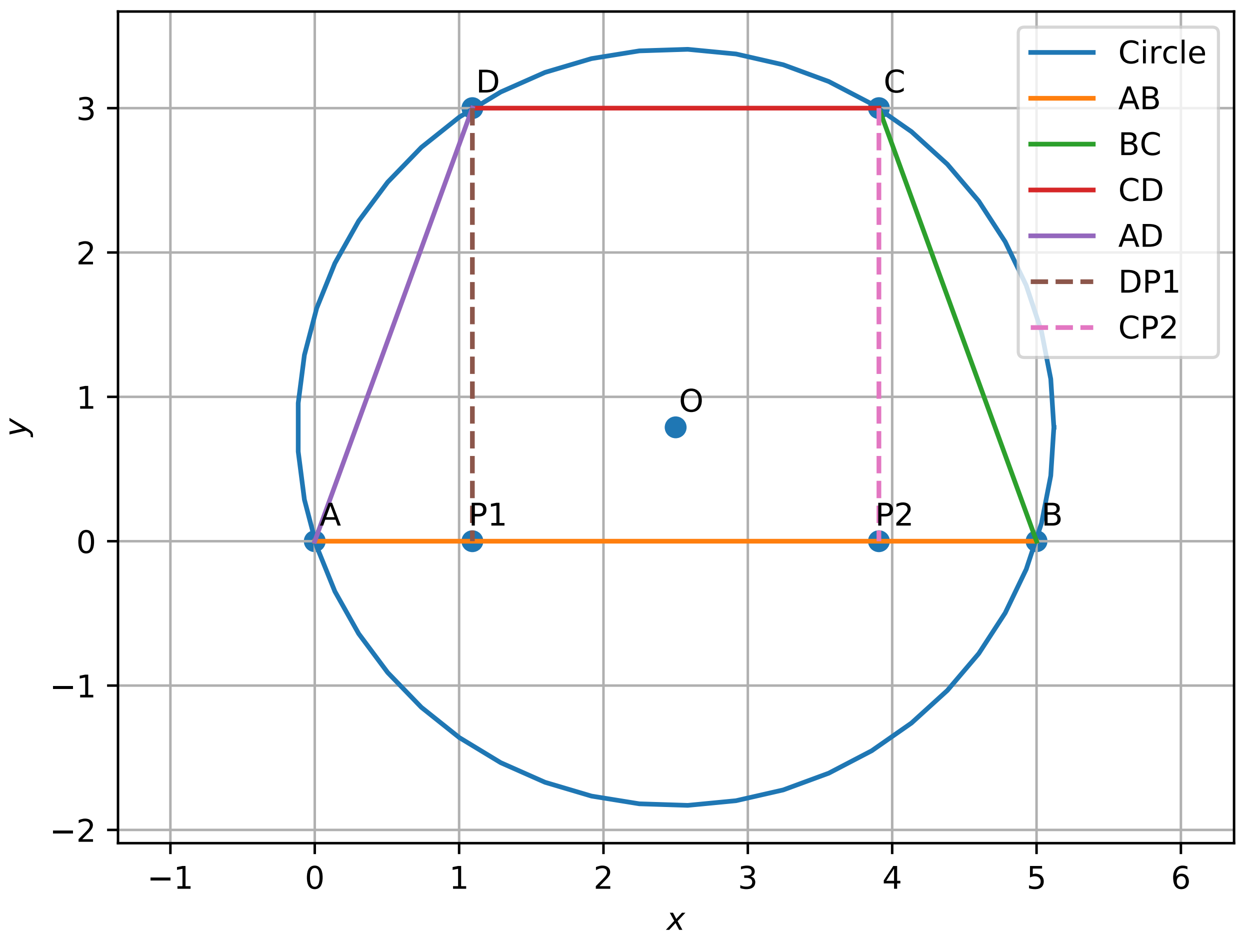

Figure 1

Construction

The input parameters for construction.

Symbol	Values	Description
θ	$70°$	$\angle A$
b	5	AB
h	3	Altitude
r	2.62	OB

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} b \\ 0 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} b - h\cot\theta \\ h \end{pmatrix}, \mathbf{D} = h \begin{pmatrix} \cot\theta \\ 1 \end{pmatrix} \tag{1}$$

$$\mathbf{P_1} = \mathbf{A} + \begin{pmatrix} \cot\theta \\ 0 \end{pmatrix}, \mathbf{P_2} = \mathbf{B} - \begin{pmatrix} \cot\theta \\ 0 \end{pmatrix} \tag{2}$$

Solution:
Theorm: If the sum of a pair of opposite angles of a quadrilateral is 180°, the quadrilateral is cyclic.

Proof:

$$\angle DAP_1 = \cos^{-1} \frac{(\mathbf{A} - \mathbf{D})^\top (\mathbf{A} - \mathbf{P_1})}{\|\mathbf{A} - \mathbf{D}\| \|\mathbf{A} - \mathbf{P_1}\|} = 70° \tag{3}$$

$$\angle DCP_2 = \cos^{-1} \frac{(\mathbf{C} - \mathbf{D})^\top (\mathbf{P_2} - \mathbf{C})}{\|\mathbf{C} - \mathbf{D}\| \|\mathbf{P_2} - \mathbf{C}\|} = 90° \tag{4}$$

$$\angle BCP_2 = \cos^{-1} \frac{(\mathbf{B} - \mathbf{C})^\top (\mathbf{P_2} - \mathbf{C})}{\|\mathbf{B} - \mathbf{C}\| \|\mathbf{P_2} - \mathbf{C}\|} = 20° \tag{5}$$

from (3),(4) and (5)

$$\angle DAP_1 + \angle DCP_2 + \angle BCP_2 = 180° \tag{6}$$

from (6) given quadrilateral is cyclic.