TRIANGLES

9^{th}Math - Chapter 7

Problem

In the given Figure 1, AC=AE,AB=AD and $\angle BAD = \angle EAC$.Show that BC=DE.

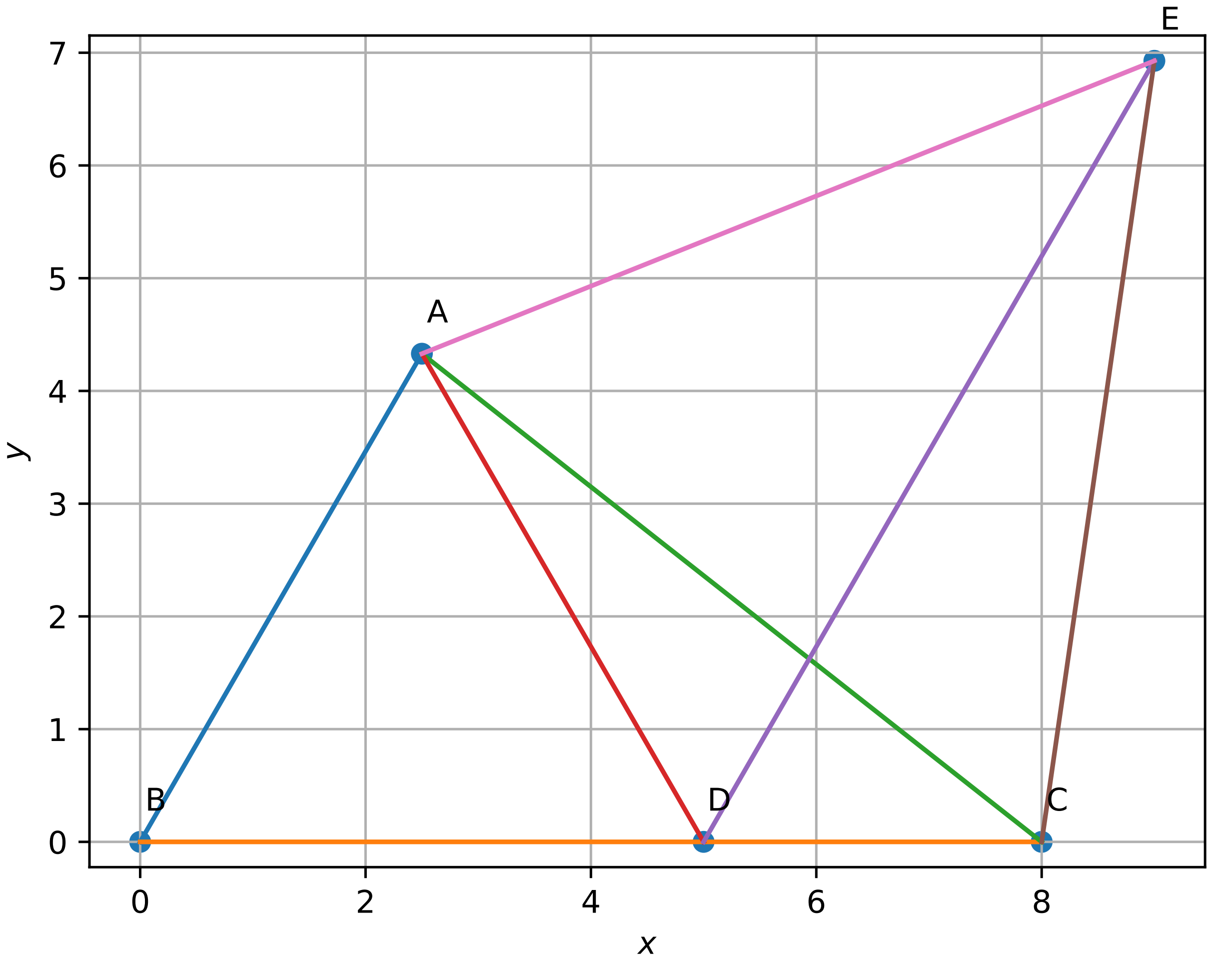

Figure 1

Construction

The input parameters for construction.

Symbol	Values	Description
θ	$60°$	$\angle BAD$=$\angle EAC$
a	8	BC
c	5	AB
$\mathbf{e_1}$	$\begin{pmatrix}1\\0\end{pmatrix}$	basis vector

Table 2

$$b = \sqrt{a^2 + c^2 - 2ac\cos\theta} \tag{1}$$

$$\mathbf{A} = c\begin{pmatrix}\cos\theta\\ \sin\theta\end{pmatrix}, \mathbf{B} = \begin{pmatrix}0\\0\end{pmatrix}, \mathbf{C} = \begin{pmatrix}a\\0\end{pmatrix}, \mathbf{D} = \left(2c\sin\frac{\theta}{2}\right)\mathbf{e_1} \tag{2}$$

$$\angle BCA = cos^{-1}\left(\frac{a^2+b^2-c^2}{2ab}\right) \tag{3}$$

$$\angle ACE = 90° - \frac{\theta}{2} \tag{4}$$

$$\phi = 180° - (\angle BCA + \angle ACE) \tag{5}$$

$$\mathbf{E} = \mathbf{C} + \left(2b\sin\frac{\theta}{2}\right)\begin{pmatrix}\cos\phi\\ \sin\phi\end{pmatrix} \tag{6}$$

$$\tag{7}$$

Solution: Given:

$$\mathbf{A} - \mathbf{C} = \mathbf{A} - \mathbf{E} \tag{8}$$

$$\mathbf{A} - \mathbf{B} = \mathbf{A} - \mathbf{D} \tag{9}$$

$$\angle BAD = \angle EAC \tag{10}$$

To prove :

$$\mathbf{B} - \mathbf{C} = \mathbf{D} - \mathbf{E} \tag{11}$$

Proof

In $\triangle ABC$ and in $\triangle ADE$

$$\|\mathbf{A} - \mathbf{B}\| = \left\|\begin{pmatrix}2.5\\4.33\end{pmatrix}\right\| = 5 \tag{12}$$

$$\|\mathbf{A} - \mathbf{D}\| = \left\|\begin{pmatrix}-2.6\\4.33\end{pmatrix}\right\| = 5 \tag{13}$$

$$\implies \|\mathbf{A} - \mathbf{B}\| = \|\mathbf{A} - \mathbf{D}\| \tag{14}$$

$$\|\mathbf{A} - \mathbf{C}\| = \left\|\begin{pmatrix}-5.5\\4.33\end{pmatrix}\right\| = 7 \tag{15}$$

$$\|\mathbf{A} - \mathbf{E}\| = \left\|\begin{pmatrix}-6.5\\-2.5\end{pmatrix}\right\| = 7 \tag{16}$$

$$\implies \|\mathbf{A} - \mathbf{C}\| = \|\mathbf{A} - \mathbf{E}\| \tag{17}$$

$$\angle BAC = \cos^{-1}\frac{(\mathbf{A}-\mathbf{B})^\top(\mathbf{A}-\mathbf{C})}{\|\mathbf{A}-\mathbf{B}\|\|\mathbf{A}-\mathbf{C}\|} = 82° \tag{18}$$

$$\angle DAE = \cos^{-1}\frac{(\mathbf{A}-\mathbf{D})^\top(\mathbf{A}-\mathbf{E})}{\|\mathbf{A}-\mathbf{D}\|\|\mathbf{A}-\mathbf{E}\|} = 82° \tag{19}$$

from (18) and (19)

$$\angle BAC = \angle DAE \tag{20}$$

from (14),(17) and (20)

$$\triangle ABC \cong \triangle ADE \tag{21}$$

from (21)

$$\mathbf{B} - \mathbf{C} = \mathbf{D} - \mathbf{E} \tag{22}$$