

Excercise

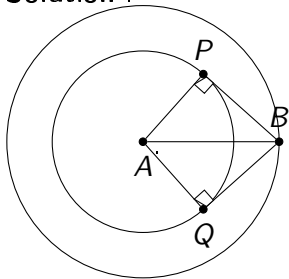
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8 Jan 2020

Circle construction

Construct a tangent to a circle of radius 4 units from a point on the concentric circle of radius 6 units.

• **Solution :**



PB and QB are the tangents

- Given : $r_1=4$ and $r_2 = 6$

$$a = \sqrt{r_2^2 - r_1^2}$$

$$a = 4.47$$

$$c = r_1, b = r_2$$

$$p = \frac{b^2 + c^2 - a^2}{2b}$$

$$p = 2.66$$

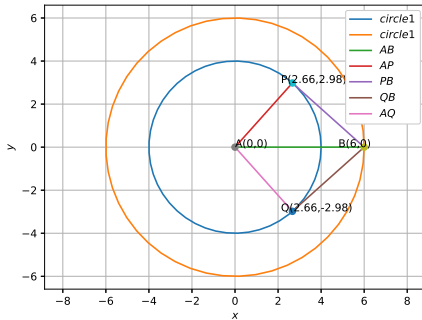
$$q = \sqrt{c^2 - p^2}$$

$$q = 2.98$$

$$AB = r_2$$

$$P = (2.66, 2.98)$$

$$Q = (2.66, -2.98)$$

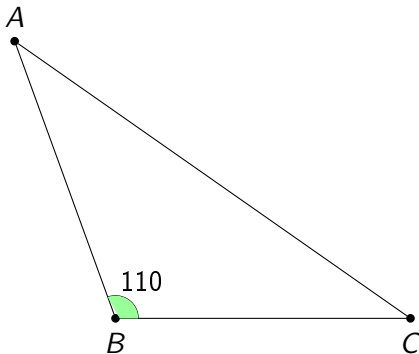


- https://github.com/pratibha444/GEOMETRY/blob/master/figs/CIRCLE_CON.tex
- <https://github.com/pratibha444/GEOMETRY/blob/master/CODES/circle/circon.py>

Triangle construction

- 1 Construct an isosceles triangle in which the lengths of the equal sides is 6.5 and the angle between them is 110°

- **Solution**



- $BC = 6.5$
- $AC = 10.64$
- $AB = 6.5$
- $\angle B = 110$

- **Given:** $BC = 6.5$ and $AB = 6.5$
 $\angle ABC = 110$

$$a = 6.5 \text{ and } c = 6.5$$

$$b = \sqrt{a^2 + c^2 - 2ac \cos(A)}$$

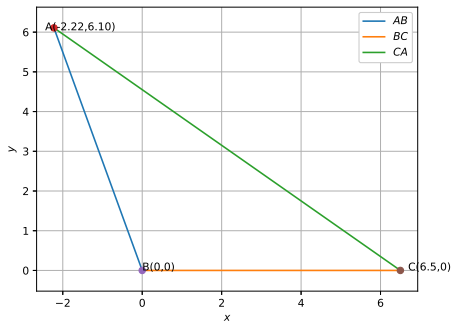
$$b = 10.64$$

$$p = \frac{a^2 + c^2 - b^2}{2a}$$

$$p = -2.22$$

$$q = \sqrt{c^2 - p^2}$$

$$q = 6.10$$

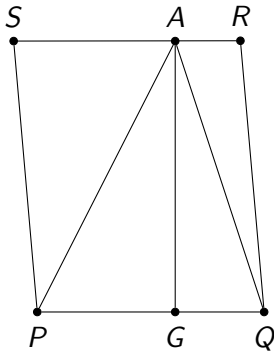


- https://github.com/pratibha444/GEOMETRY/blob/master/figs/tri_iso.tex
- https://github.com/pratibha444/GEOMETRY/blob/master/CODES/triangle/TRI_CON.py

Quadrilateral exercise

- 2 A farmer was having a field in the form of a parallelogram PQRS . She took any point A on RS and joined it to points P and Q. In how many parts the fields is divided? What are the shapes of these parts? The farmer wants to sow wheat and pulses in equal portions of the field separately. How should she do it?

• **Solution :**



- $S = (-0.52, 5.97)$
- $A = (3.04, 5.97)$
- $R = (4.47, 5.57)$
- Distance between S and A

$$z = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$
$$z = \sqrt{(3.04 + 0.52)^2 + (5.97 - 5.97)^2}$$
$$z = 3.55$$

- Distance between A and R

$$z = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$
$$z = \sqrt{(4.47 - 3.04)^2 + (5.97 - 5.97)^2}$$
$$z = 1.43$$

- $PQ = 5$
- $SP = 6$
- $AG = 6$
- $\triangle APQ = \triangle PAS + \triangle QAR$
- $\triangle APQ = \frac{1}{2} \times l \times b$
 $\triangle APQ = \frac{1}{2} \times 6 \times 5$
 $\triangle APQ = 15cm^2$
- $\triangle PAS = \frac{1}{2} \times l \times b$
 $\triangle PAS = \frac{1}{2} \times 6 \times 3.55$
 $\triangle PAS = 10.65cm^2$
- $\triangle QRA = \frac{1}{2} \times l \times b$
 $\triangle QRA = \frac{1}{2} \times 6 \times 1.43$
 $\triangle QRA = 4.38cm^2$
- $10.65 + 4.38 = 15$
- Hence Area of $\triangle APQ = \text{Area of } \triangle PAS + \text{Area of } \triangle QAR$

- After joining the point A to p and Q, the feild is divided into 3 parts.
- All the three parts are in triangle shape.
As PQRS is a parallelogram so

$$\text{Area of } PQRS = \text{Area of } \triangle APS + \triangle ARQ + \triangle PAQ \quad \text{---(1)}$$

*Area of triangle is half of parallelogram
if they have same base and lie between same parallel lines.*

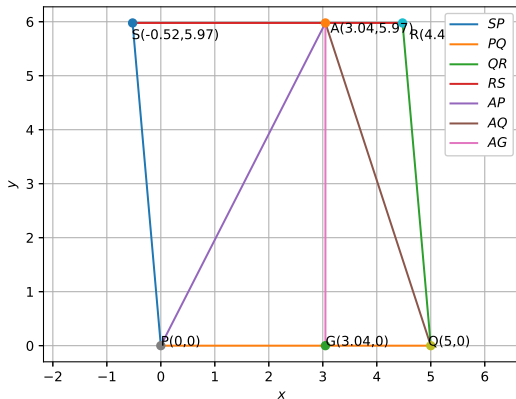
$$\text{Area of } \triangle PAQ = \frac{1}{2} \text{Area of } PQRS$$

$$\triangle PAQ = \frac{1}{2} \text{area}(\triangle APS + \triangle ARQ + \triangle APQ)$$

$$2\triangle PAQ - \triangle PAQ = \text{area}(\triangle APS + \triangle ARQ)$$

$$\triangle PAQ = \triangle APS + \triangle ARQ$$

Hence the farmer can sow wheat in $\triangle PAQ$ and pulses in $\triangle APS$ and $\triangle ARQ$

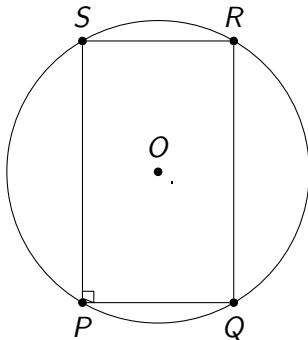


- <https://github.com/pratibha444/GEOMETRY/blob/master/figs/FARM.tex>
- https://github.com/pratibha444/GEOMETRY/blob/master/CODES/quad/QUAD_EXCERCISE.py

Circle exercise

- 8 Prove that a cyclic parallelogram is a rectangle.

- Solution :



- $PQ = 2\text{cm}$ and $PS = 4\text{cm}$, radius = 4

- Calculation

$$a = 2$$

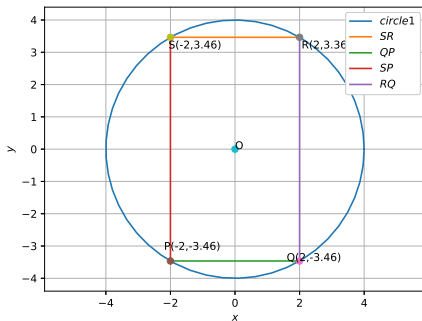
$$c = 4$$

$$b = \sqrt{c^2 - a^2}$$

$$b = 3.46$$

$$PQ = 2$$

$$PS = 4$$



- <https://github.com/pratibha444/GEOMETRY/blob/master/figs/CYCPA.tex>
- <https://github.com/pratibha444/GEOMETRY/blob/master/CODES/quad/QUADP.py>

To prove : PQRS is a rectangle

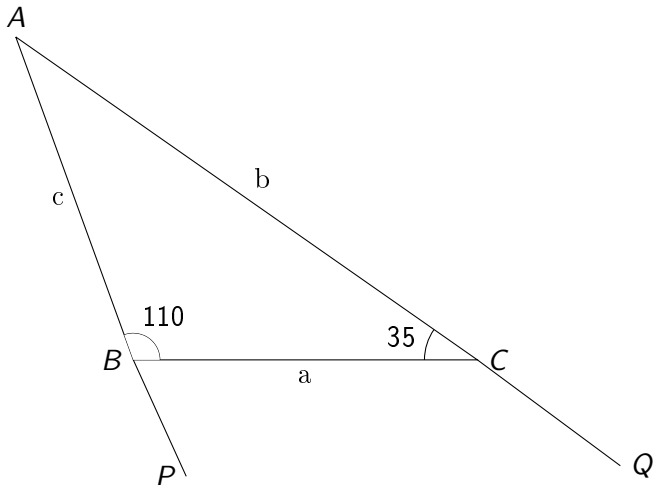
Proof:

- $\angle P = \angle R$ (opposite sides of parallelogram are equal)
- $\angle P + \angle R = 180^\circ$ (sum of opposite angles of a cyclic quadrilateral is 180°)
- $2\angle P = 180^\circ$
- $\angle P = 90^\circ$

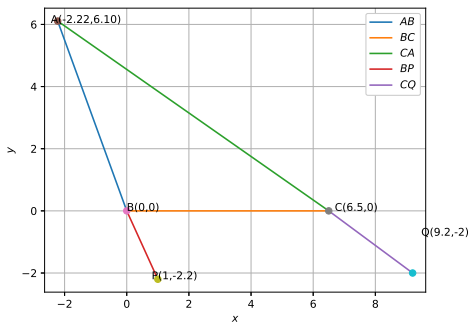
Hence PQRS is a rectangle as in rectangle one angle is 90°

Triangle exercise

- 1 Sides AB and AC of $\triangle ABC$ are extended to points P and Q respectively. Also, $\angle PBC < \angle QCB$. Show that $AC > AB$.



- $a = 6.5, b = 6.5, c = 10.64$
- $c^2 = b^2 + a^2 - 2ab\cos(B)$
- $\cos(C) = -\left(\frac{c^2 - b^2 - a^2}{2ab}\right)$
- $\cos(C) = 35^\circ$



- https://github.com/pratibha444/GEOMETRY/blob/master/figs/TRI_EX.tex
- https://github.com/pratibha444/GEOMETRY/blob/master/CODES/triangle/TRI_ISOP.py

- **Given** : P and Q are extended to AB and AC respectively.
And $\angle PBC < \angle QCB$ --(1)
So,

$$\angle PBC + \angle ABC = 180^\circ$$

$$\angle QCB + \angle ACB = 180^\circ$$

$$\angle PBC = 180 - \angle ABC \text{ --(2)}$$

$$\angle QCB = 180 - \angle ACB \text{ --(3)}$$

By substituting the value of $\angle PBC$ and $\angle QCB$ in equation 1

$$\begin{aligned} 180 - \angle ABC &< 180 - \angle ACB \\ -\angle ABC &< -\angle ACB \end{aligned}$$

multiplying both the sides by '-'

$$\begin{aligned} -(-\angle ABC) &> -(-\angle ACB) \\ \angle ABC &> \angle ACB \end{aligned}$$

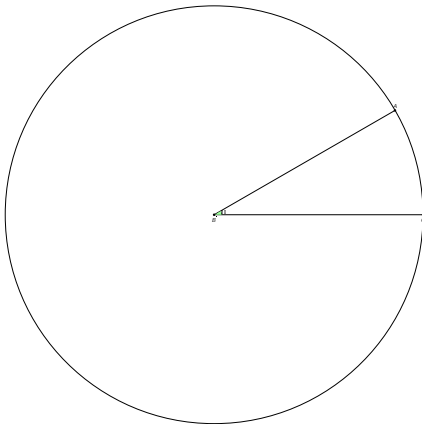
(sides opposite to greater angle is longer)

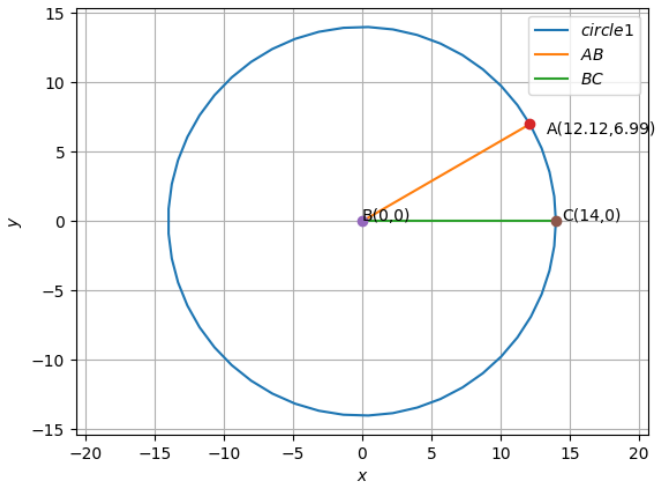
Miscellaneous Exercises

- 2 The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.

• **Solution :**

- Given $r = 14$





In 60 minutes minute hand covers 360°

For 5 minutes $6^\circ \times 5 = 30^\circ$

Here $\theta = 30^\circ$ and $r = 14\text{cm}$

$$\text{Area of sector} = \frac{\theta}{360} \times \pi r^2 = 51.31\text{cm}^2.$$

- <https://github.com/pratibha444/GEOMETRY/blob/master/figs/clock.tex>
- <https://github.com/pratibha444/GEOMETRY/blob/master/CODES/newclock.py>

Quadrilateral construction

- ③ Can you construct a quadrilateral PQRS with $PQ=3$, $RS=3$, $PS=7.5$, $PR=8$ and $SQ=4$?

- **Given:** Quadrilateral PQRS with $PQ = 3$ cm $RS = 3$ cm $PS = 7.5$ cm and $SQ = 4$ cm

We know from triangle inequality theorem that sum of any two sides is greater than the third side ,

But here we have $PQ = 3$ cm $PS = 7.5$ cm $SQ = 4$ cm (AS we get PQS triangle in quadrilateral PQRS where PQ and PS are sides and SQ is diagonal of quadrilateral .

And

$$PQ + SQ = 3 + 4 = 7$$

and

$$PS = 7.5 , \text{ So}$$

$$PQ + SQ < PS ,$$

But that equation is not true for another triangle.

So, we can't construct the given quadrilateral.