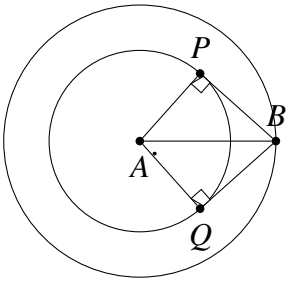
GEOMETRY

1 Construct a tangent to a circle of radius 4 units from a point on the concentric circle of radius 6 units.

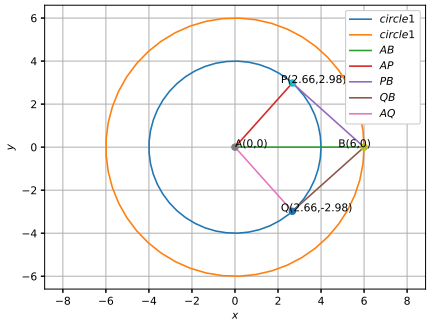
- **Solution** :



PB and QB are the tangents

- **Given** : r1=4 and r2 = 6

$$a = \sqrt{r2^2 - r1^2}$$
$$a = 4.47$$
$$c = r1, b = r2$$
$$p = \frac{b^2 + c^2 - a^2}{2b}$$
$$p = 2.66$$
$$q = \sqrt{c^2 - p^2}$$
$$q = 2.98$$
$$AB = r2$$
$$P = (2.66, 2.98)$$
$$Q = (2.66, -2.98)$$



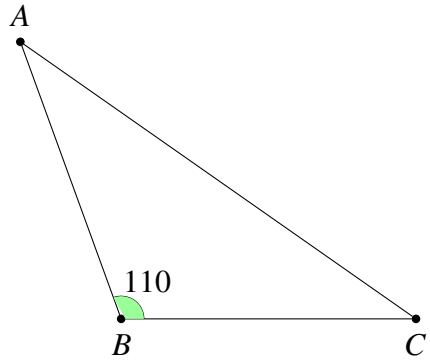
https://github.com/pratibha444/GEOMETRY/blob/master/figs/CIRCLE_CON.tex

https://github.com/pratibha444/GEOMETRY/blob/master/CODES/circle/circon.py

Triangle construction

2 Construct an isosceles triangle in which the lengths of the equal sides is 6.5 and the angle between them is 110°

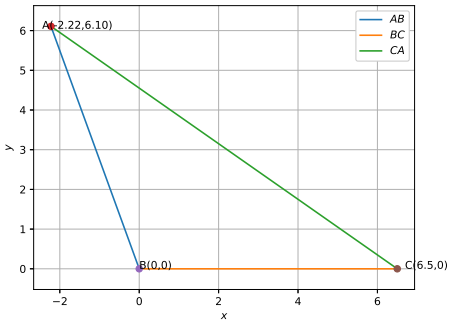
- **Solution**



- BC = 6.5
- AC = 10.64
- AB = 6.5
- $\angle B = 110$

- **Given**: BC = 6.5 and AB = 6.5 $\angle$ABC = 110

$$a = 6.5 and c = 6.5$$
$$b = \sqrt{a^2 + c^2 - 2accos(A)}$$
$$b = 10.64$$
$$p = \frac{a^2 + c^2 - b^2}{2a}$$
$$p = -2.22$$
$$q = \sqrt{c^2 - p^2}$$
$$q = 6.10$$



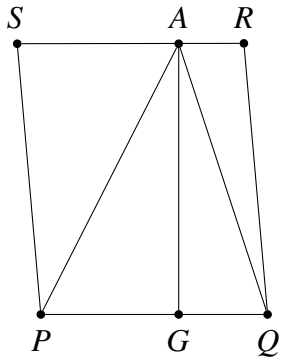
https://github.com/pratibha444/GEOMETRY/blob/master/figs/tri_iso.tex

https://github.com/pratibha444/GEOMETRY/blob/master/CODES/triangle/TRI_CON.py

Quadrilateral excercise

3 A farmer was having a field in the form of a parallelogram PQRS . She took any point A on RS and joined it to points P and Q. In how many parts the fields is divided? What are the shapes of these parts? The farmer wants to sow wheat and pulses in equal portions of the field separately. How should she do it?

- **Solution** :



- S = (-0.52,5.97)
- A = (3.04,5.97)
- R = (4.47,5.57)
- Distance between S and A

$$z = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$
$$z = \sqrt{(3.04 + 0.52) + (5.97 - 5.97)}$$
$$z = 3.55$$

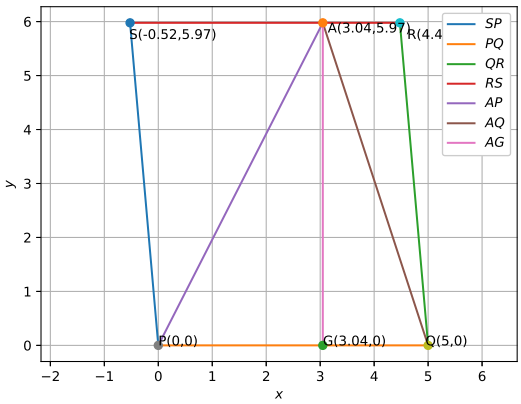
- Distance between A and R

$$z = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$
$$z = \sqrt{(4.47 - 3.04) + (5.97 - 5.97)}$$
$$z = 1.43$$

- PQ = 5
- SP = 6
- AG = 6
- △ APQ = △PAS + △ QAR
- △ APQ = $\frac{1}{2}$ X l X b
  △ APQ = $\frac{1}{2}$ X 6 X 5
  △ APQ = $15cm^2$
- △ PAS = $\frac{1}{2}$ X l X b
  △ PAS = $\frac{1}{2}$ X 6 X 3.55
  △ PAS = $10.65cm^2$
- △ QRA = $\frac{1}{2}$ X l X b
  △ QRA = $\frac{1}{2}$ X 6 X 1.43
  △ QRA = $4.38cm^2$
- 10.65 + 4.38 = 15
- Hence Area of △APQ = Area of △PAS + Area of △QAR
- After joining the point A to p and Q, the feild is divided into 3 parts.
- All the three parts are in triangle shape.
  As PQRS is a parallelogram so

  Area of PQRS = (Area of △APS + △ARQ + △PAQ) –(1)
  Area of triangle is half of parallelogram if they have same base and lie between same parallel lines.
  Area of △PAQ = $\frac{1}{2}$ Area of PQRS
  △PAQ = $\frac{1}{2}$area(△APS + △ARQ + △APQ
  2△PAQ - △PAQ = area(△APS + △ARQ)
  △PAQ = △APS + △ARQ

  Hence the farmer can sow wheat in △PAQ and pulses in △APS and △ARQ



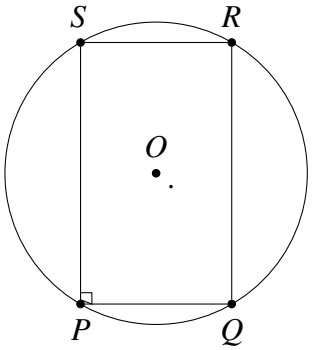
https://github.com/pratibha444/GEOMETRY/blob/master/figs/FARM.tex

https://github.com/pratibha444/GEOMETRY/blob/master/CODES/quad/QUAD_EXCERCISE.py

Circle excercise

4 Prove that a cyclic parallelogram is a rectangle.

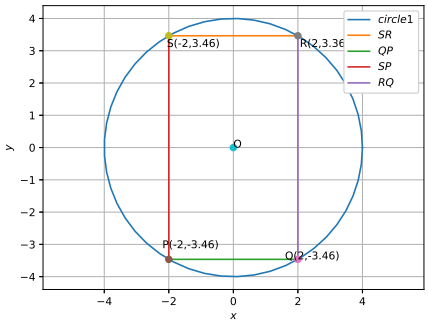
- Solution :



- PQ = 2cm and PS = 4cm, radius = 4
- **Calculation**

$$a = 2$$
$$c = 4$$
$$b = \sqrt{c^2 - a^2}$$
$$b = 3.46$$

PQ =2
PS = 4



https://github.com/pratibha444/GEOMETRY/blob/master/figs/CYCPA.tex

https://github.com/pratibha444/GEOMETRY/blob/master/CODES/quad/QUADP.py

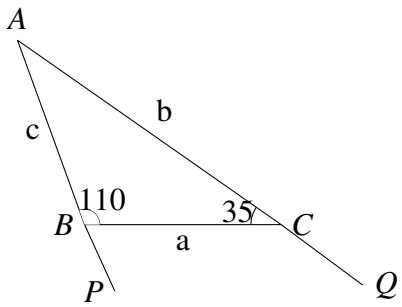
To prove : PQRS is a rectangle

Proof:

- ∠P = ∠R (opposite sides of parallelogram are equal)
- ∠P + ∠R = 180° (sum of opposite angles of a cyclic quadrilateral is 180°)
- 2∠P = 180°
- ∠P = 90 °
  Hence PQRS is a rectangle as in rectangle one angle is 90°.

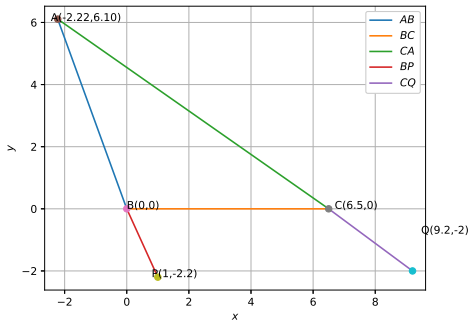
Triangle excercise

5 Sides AB and AC of △ABC are extended to points P and Q respectively.Also,∠PBC < ∠QCB.Show that AC>AB .



- a= 6.5,b=6.5,c=10.64

- $c^2 = b^2 + a^2$ - 2abcos(B)
- cos(C)= -($\frac{c^2-b^2-a^2}{2ab}$)
- cos(C) = 35°



https://github.com/pratibha444/GEOMETRY/blob/master/figs/TRI_EX.tex

https://github.com/pratibha444/GEOMETRY/blob/master/CODES/triangle/TRI_ISOP.py

- **Given** : P and Q are extended to AB and AC respectively.
  And ∠PBC < ∠QCB –(1)
  So,
  ∠PBC + ∠ABC = 180°
  ∠QCB + ∠ACB = 180°
  ∠PBC = 180 - ∠ABC –(2)
  ∠QCB = 180 - ∠ACB –(3)

  By substituting the value of ∠PBC and ∠QCB in equation 1

  180 - ∠ABC < 180 - ∠ACB
  -∠ABC < -∠ACB

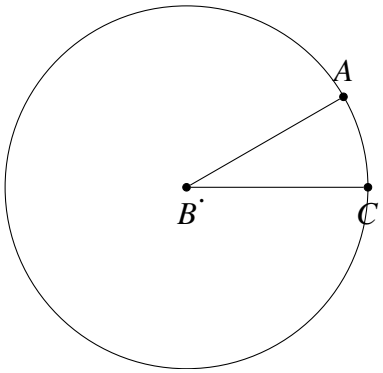
  multiplying both the sides by '-'

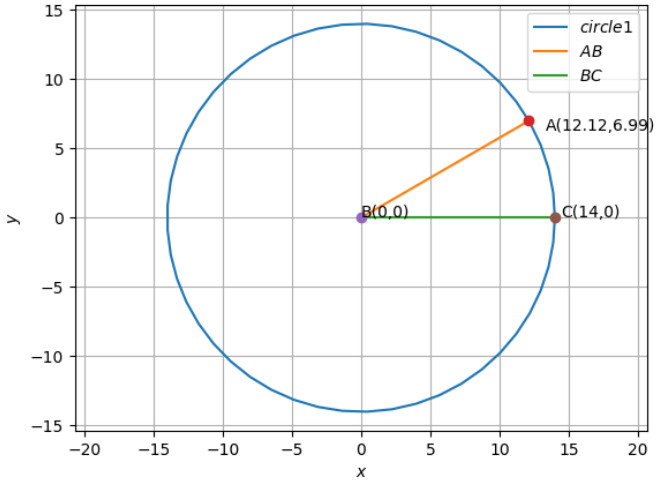
  -(-∠ABC) > -(-∠ACB)
  ∠ABC > ∠ACB
  (sides opposite to greater angle is longer)

Miscellaneous Exercises

6 The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.

- **Solution** :
- Given r = 14





- In 60 minutes minute hand covers 360°
  For 5 minutes 6° × 5 = 30°
- Here θ = 30 ° and r = 14cm
- Area of sector = $\frac{\theta}{360} \times \pi\ r^2$ =51.31cm^2.

https://github.com/pratibha444/GEOMETRY/blob/master/figs/clock.tex

https://github.com/pratibha444/GEOMETRY/blob/master/CODES/newclock.py

Quadrilateral construction

7 Can you construct a quadrilateral PQRS with PQ=3, RS=3, PS=7.5, PR=8 and SQ=4?

- **Given**: Quadrilateral PQRS with PQ = 3 cm RS = 3 cm PS = 7.5 cm and SQ = 4

cm

- We know from triangle inequality theorem that sum of any two sides is greater than the third side ,

- But here we have PQ = 3 cm PS = 7.5 cm SQ = 4 cm (AS we get PQS triangle in quadrilateral PQRS where PQ and PS are sides and SQ is diagonal of quadrilateral .

  And

  PQ + SQ = 3 + 4 = 7

  and

  PS = 7.5 , So

  PQ + SQ <PS ,
  But that equation is not true for another triangle.
  So, we can't construct the given quadrilateral.