Excercise

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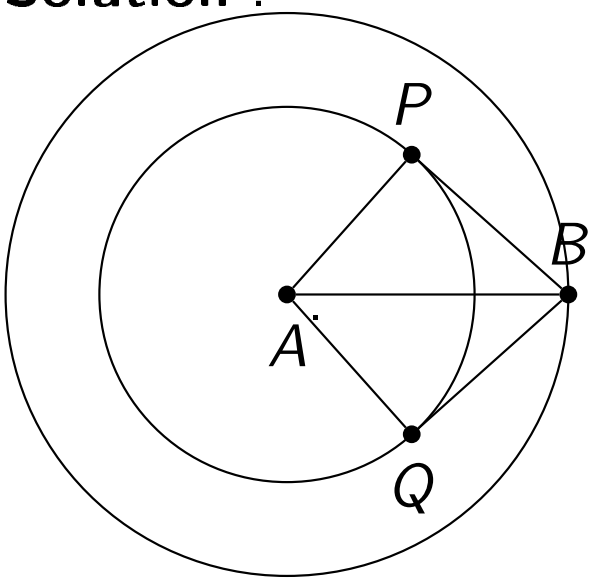
Circle construction

Construct a tangent to a circle of radius 4 units from a point on the concentric circle of radius 6 units.

- **Solution** :



PB and QB are the tangents

- **Given** : r1=4 and r2 = 6

$$a = \sqrt{r2^2 - r1^2}$$

$$a = 4.47$$

$$c = r1 and b = r2$$

$$p = \frac{b^2 + c^2 - a^2}{2b}$$

$$p = 2.66$$

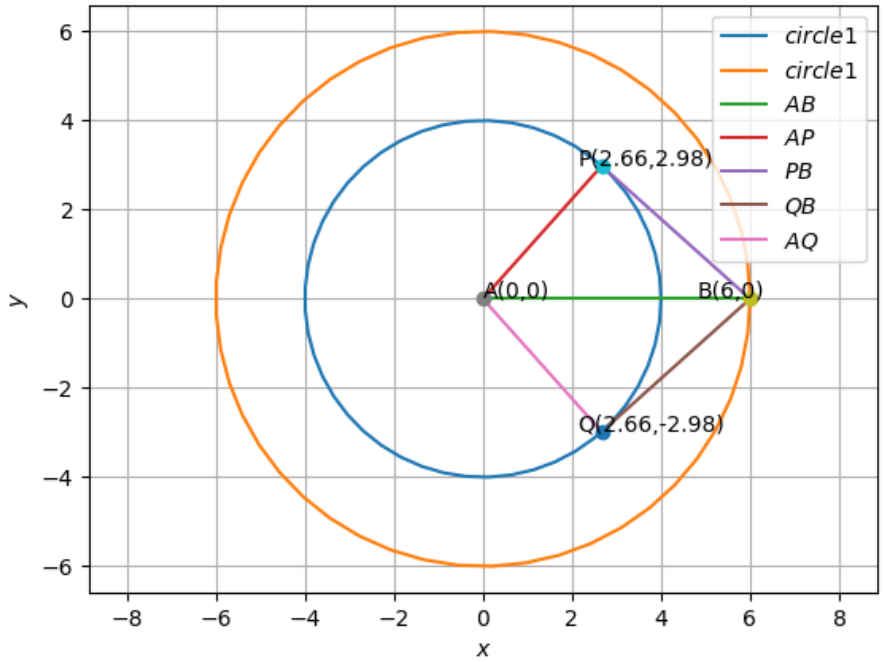
$$q = \sqrt{c^2 - p^2}$$

$$q = 2.98$$

$$AB = r2$$

$$P = (2.66, 2.98)$$

$$Q = (2.66, -2.98)$$

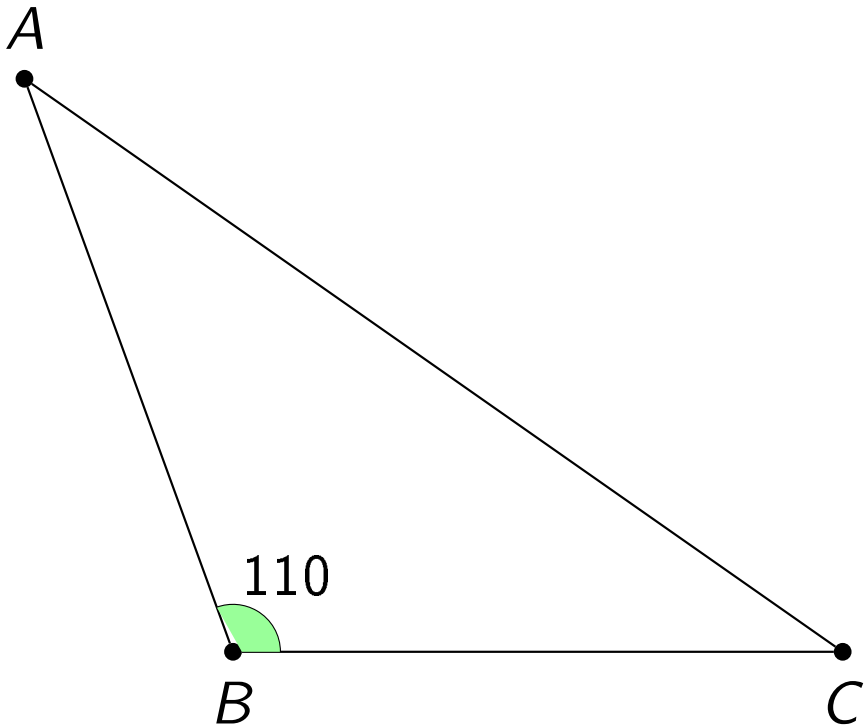


- https://github.com/pratibha444/GEOMETRY/blob/master/figs/CIRCLE_CON.tex
- https://github.com/pratibha444/GEOMETRY/blob/master/CODES/circle/circon.py

Triangle construction

1. Construct an isosceles triangle in which the lengths of the equal sides is 6.5 and the angle between them is 110°

- **Solution**



- BC = 6.5
- AC = 10.64
- AB = 6.5
- $\angle B = 110$

- **Given**: BC = 6.5 and AB = 6.5
  $\angle$ABC = 110

$$a = 6.5 and c = 6.5$$

$$b = \sqrt{a^2 + c^2 - 2accos(A)}$$

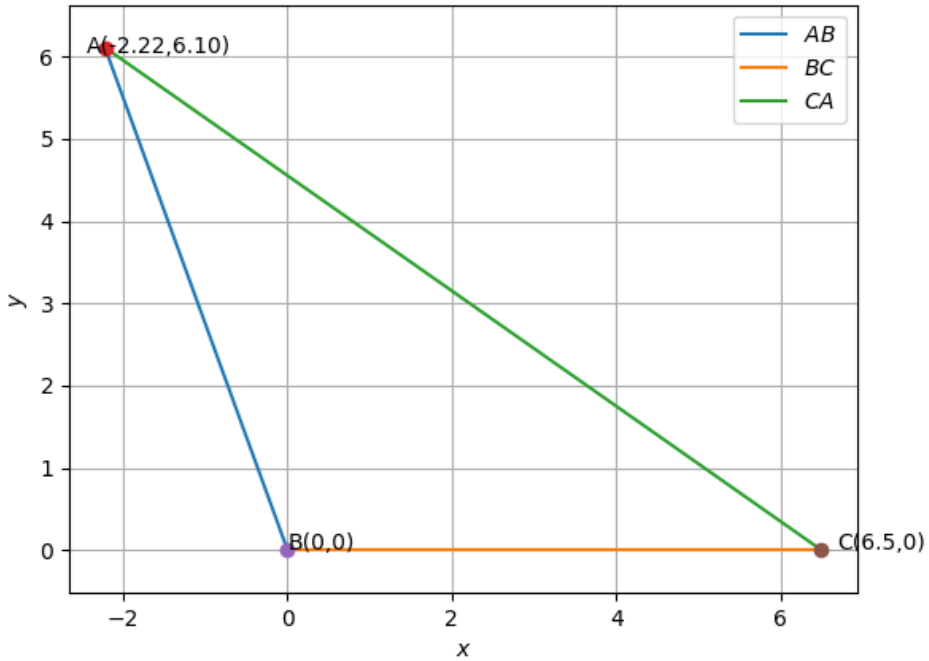
$$b = 10.64$$

$$p = \frac{a^2 + c^2 - b^2}{2a}$$

$$p = -2.22$$

$$q = \sqrt{c^2 - p^2}$$

$$q = 6.10$$

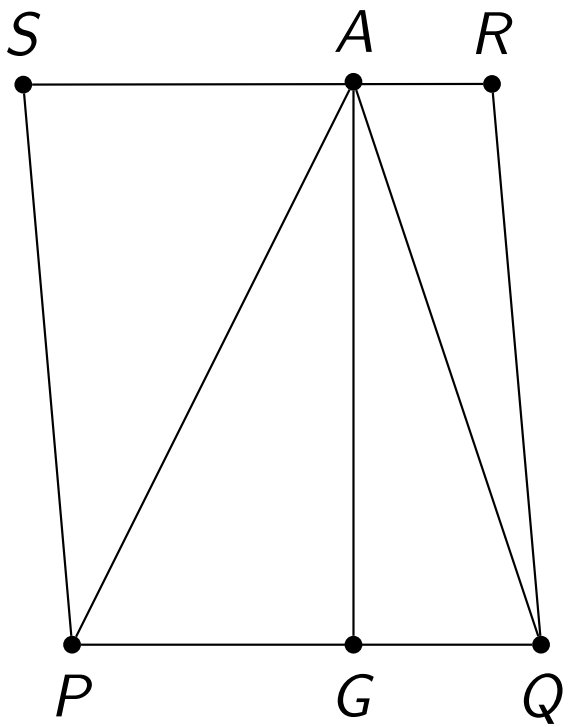


- https://github.com/pratibha444/GEOMETRY/blob/master/figs/tri_iso.tex
- https://github.com/pratibha444/GEOMETRY/blob/master/CODES/triangle/TRI_CON.py

Quadrilateral excercise

2. A farmer was having a field in the form of a parallelogram PQRS . She took any point A on RS and joined it to points P and Q. In how many parts the fields is divided? What are the shapes of these parts? The farmer wants to sow wheat and pulses in equal portions of the field separately. How should she do it?

- **Solution** :



- PQ = 5
- SP = 6
- AG = 6
- $\triangle$ APQ = $\triangle$PAS + $\triangle$ QAR
- $\triangle$ APQ = $\frac{1}{2}$ X l X b
  $\triangle$ APQ = $\frac{1}{2}$ X 6 X 5
  $\triangle$ APQ = $15cm^2$
- $\triangle$ PAS = $\frac{1}{2}$ X l X b
  $\triangle$ PAS = $\frac{1}{2}$ X 6 X 3
  $\triangle$ PAS = $9cm^2$
- $\triangle$ QRA = $\frac{1}{2}$ X l X b
  $\triangle$ QRA = $\frac{1}{2}$ X 6 X 2
  $\triangle$ QRA = $6cm^2$
- 6 + 9 = 15
- Hence Area of $\triangle$APQ = Area of $\triangle$PAS + Area of $\triangle$QAR

- After joining the point A to p and Q, the feild is divided into 3 parts.
- All the three parts are in triangle shape.
  As PQRS is a parallelogram so

$$Area\ of\ PQRS = Area\ of \triangle APS + \triangle ARQ + \triangle PAQ - -(1)$$

Area of triangle is half of parallelogram

if they have same base and lie between same parallel lines.

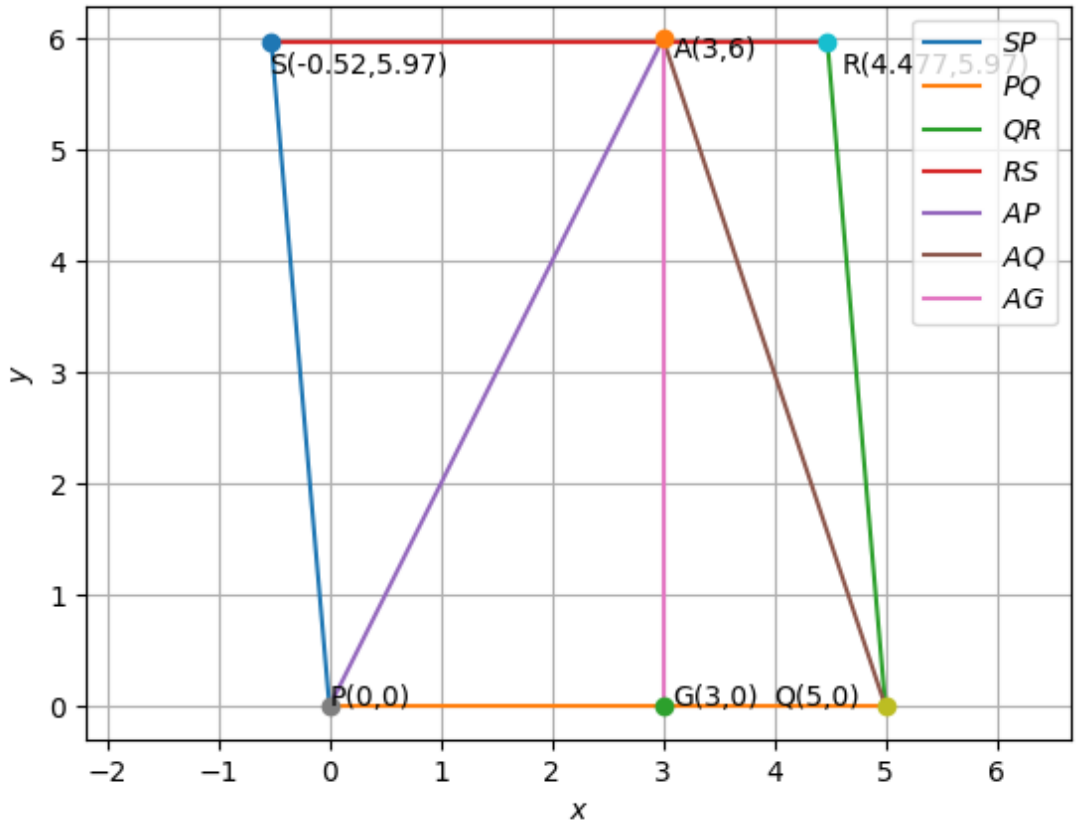
$$Area\ of \triangle PAQ = \frac{1}{2} Area\ of\ PQRS$$

$$\triangle PAQ = \frac{1}{2} area(\triangle APS + \triangle ARQ + \triangle APQ$$

$$2\triangle PAQ - \triangle PAQ = area(\triangle APS + \triangle ARQ)$$

$$\triangle PAQ = \triangle APS + \triangle ARQ$$

Hence the farmer can sow wheat in $\triangle PAQ$ *and pulses in* $\triangle APS$ *and* $\triangle ARQ$

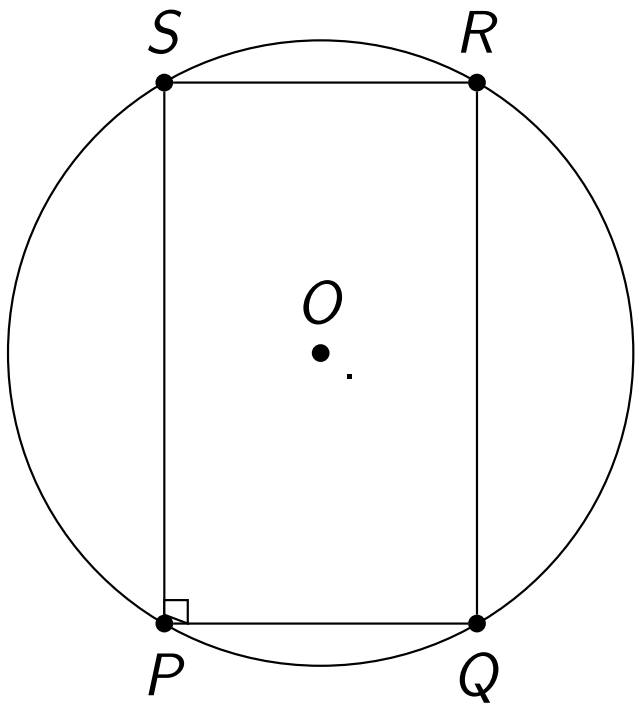


- https://github.com/pratibha444/GEOMETRY/blob/master/figs/FARM.tex
- https://github.com/pratibha444/GEOMETRY/blob/master/CODES/quad/QUAD_EXCERCISE.py

Circle excercise

3. Prove that a cyclic parallelogram is a rectangle.

- Solution :



- PQ = 2cm and PS = 4cm, radius = 4

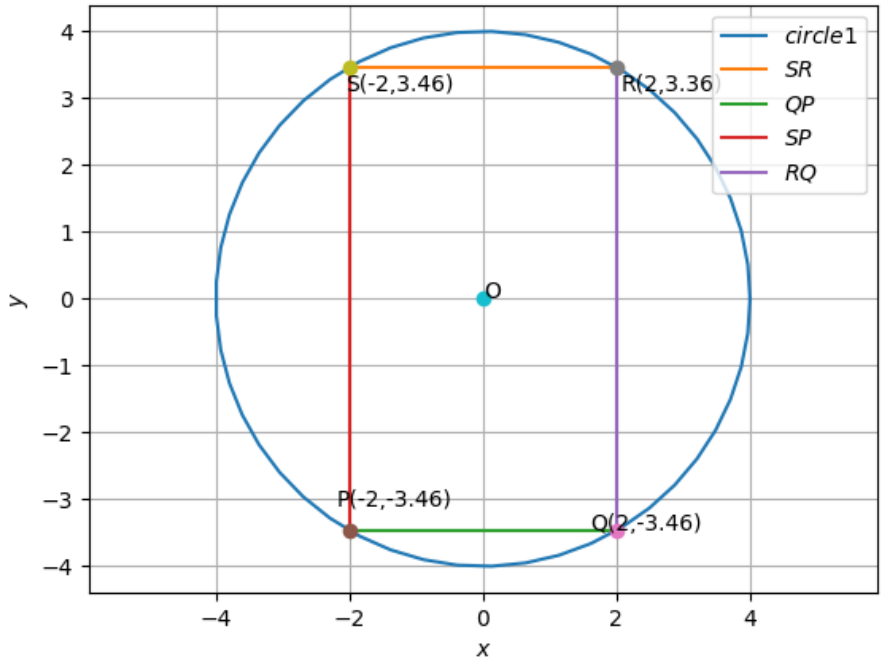
- **Calculation**

$$a = 2$$

$$c = 4$$

$$b = \sqrt{c^2 - a^2}$$

$$b = 3.46$$



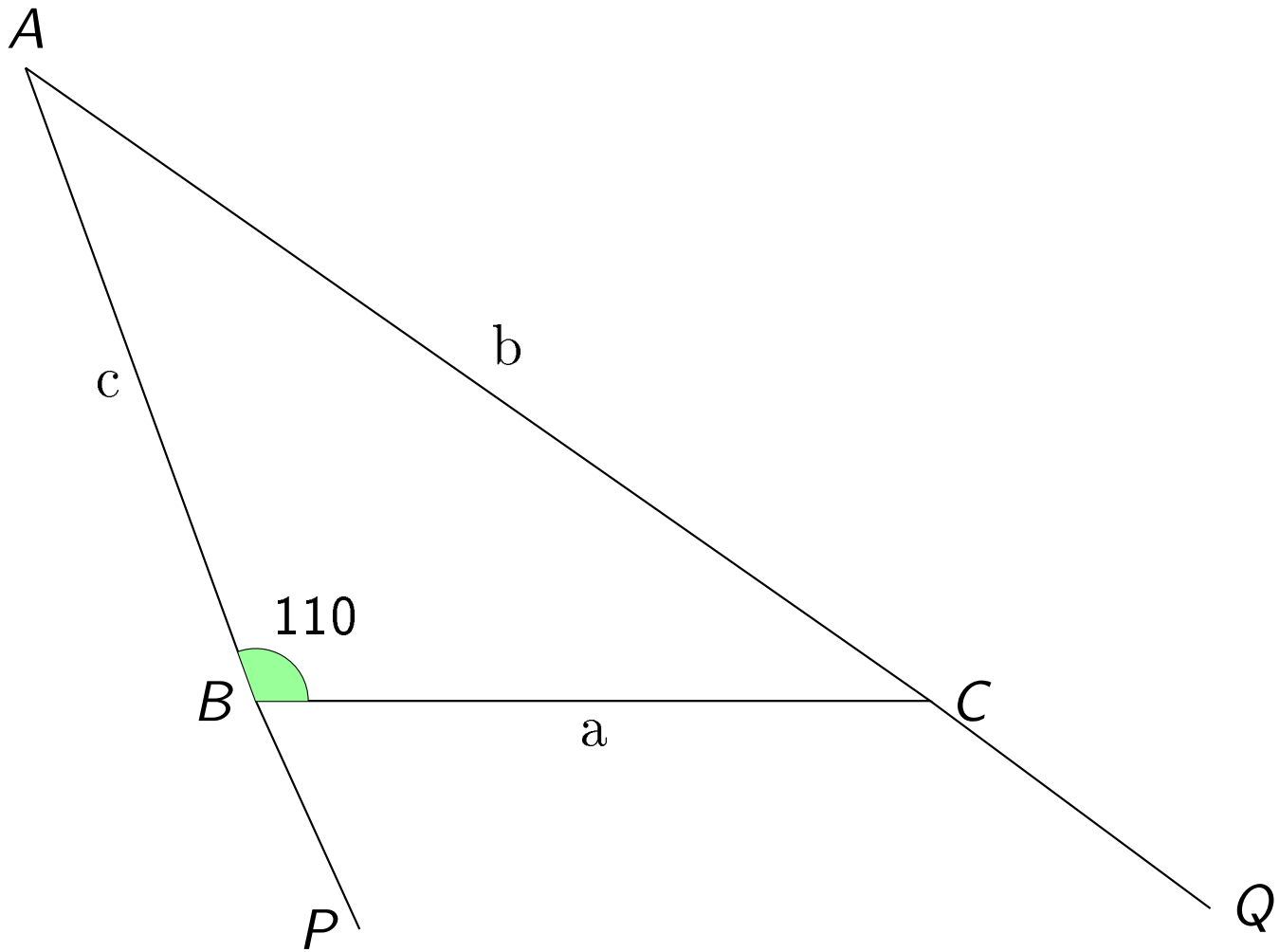
- https://github.com/pratibha444/GEOMETRY/blob/master/figs/CYCPA.tex
- https://github.com/pratibha444/GEOMETRY/blob/master/CODES/quad/QUADP.py

To prove : PQRS is a rectangle
Proof:

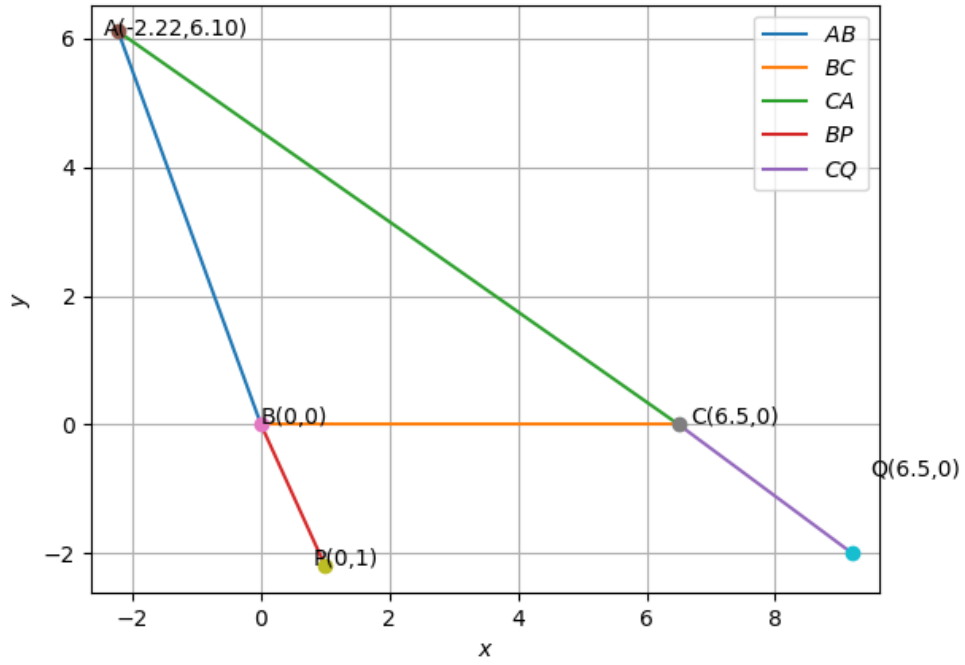
- $\angle P = \angle R$ (opposite sides of parallelogram are equal)
- $\angle P + \angle R = 180^\circ$ (sum of opposite angles of a cyclic quadrilateral is 180°)
- $2\angle P = 180^\circ$
- $\angle P = 90\ ^\circ$
  Hence PQRS is a rectangle as in rectangle one angle is 90°

Triangle excercise

1. Sides AB and AC of $\triangle ABC$ are extended to points P and Q respectively.Also,$\angle PBC < \angle QCB$.Show that AC>AB .



- a= 6.5,b=6.5,c=10.64
- cos(C)=-$\frac{c^2-b^2-a^2}{2ab}$
- cos(C)=1



- https://github.com/pratibha444/GEOMETRY/blob/master/figs/TRI_EX.tex
- https://github.com/pratibha444/GEOMETRY/blob/master/CODES/triangle/TRI_ISOP.py

- **Given** : P and Q are extended to AB and AC respectively. And $\angle PBC < \angle QCB$ –(1)
  So,

$$\angle PBC + \angle ABC = 180^\circ$$
$$\angle QCB + \angle ACB = 180^\circ$$

$$\angle PBC = 180 - \angle ABC - -(2)$$
$$\angle QCB = 180 - \angle ACB - -(3)$$

By substituting the value of $\angle PBC$ and $\angle QCB$ in equation 1

$$180 - \angle ABC < 180 - \angle ACB$$
$$-\angle ABC < -\angle ACB$$

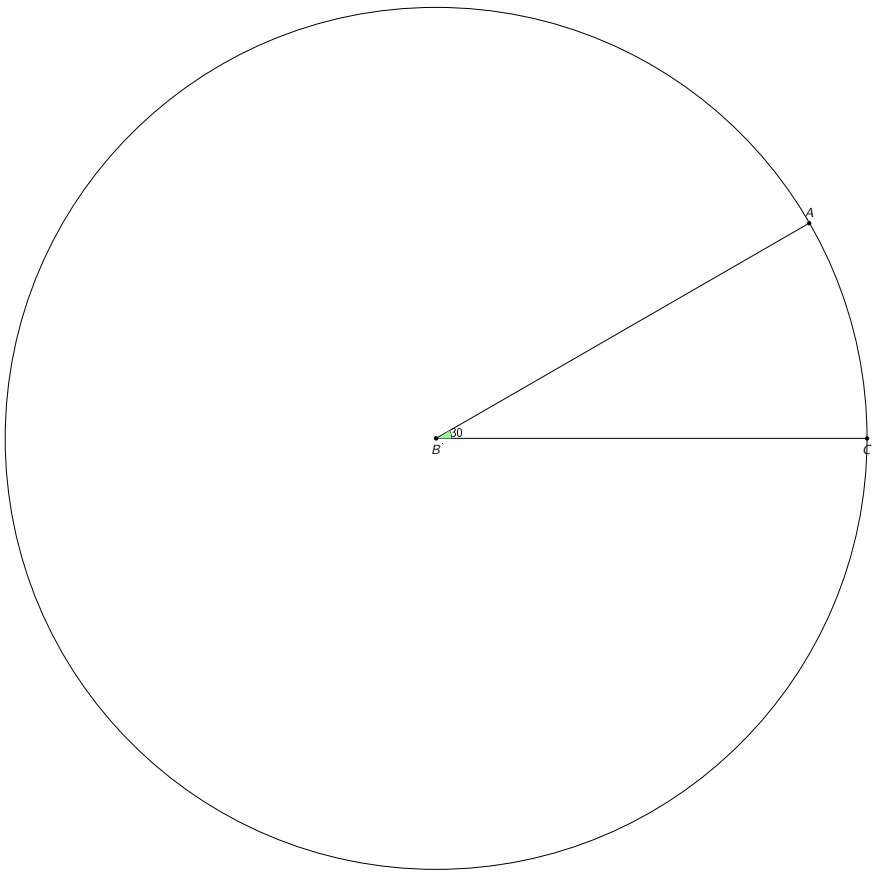
multiplying both the sides by '-'

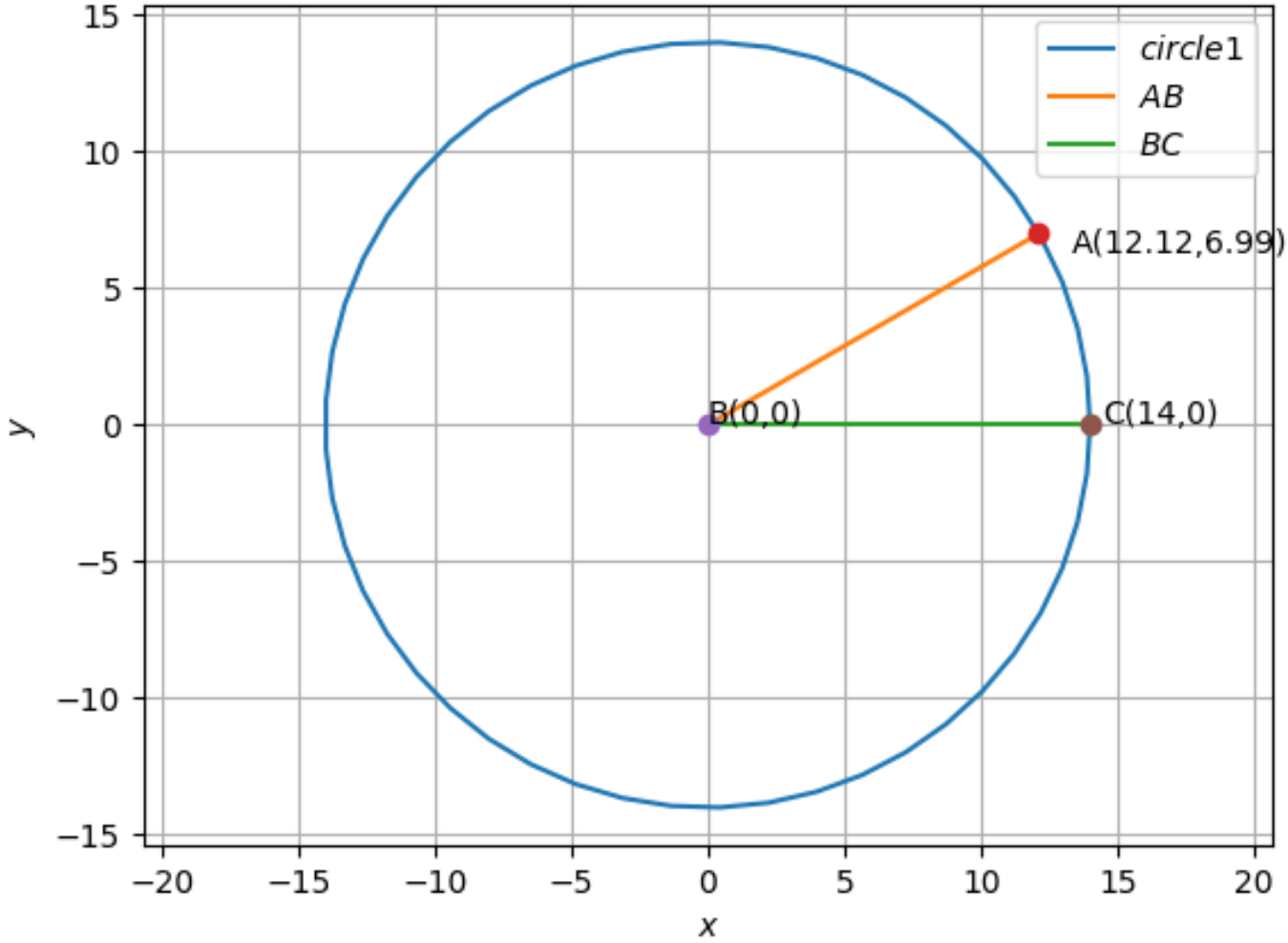
$$-(-\angle ABC) > -(-\angle ACB)$$
$$\angle ABC > \angle ACB$$

(sides opposite to greater angle is longer)

2. The length of the minute hand of a clock is 14 cm. Find the area swept by the minute hand in 5 minutes.

- **Solution** :
- Given r = 14




circle1
AB
BC
A(12.12,6.99)
B(0,0)
C(14,0)
15
10
5
0
−5
−10
−15
−20
−15
−10
−5
0
5
10
15
20
x
y

In 60 minutes minute hand covers 360°
For 5 minutes $6^\circ \times 5 = 30^\circ$
Here $\theta = 30^\circ$ and r = 14cm

$$\text{Area of sector} = \frac{\theta}{360} \times \pi r^2 = 51.31 cm^2.$$

- https://github.com/pratibha444/GEOMETRY/blob/master/figs/clock.tex
- https://github.com/pratibha444/GEOMETRY/blob/master/CODES/newclock.py

Quadrilateral construction

3. Can you construct a quadrilateral PQRS with PQ=3, RS=3, PS=7.5, PR=8 and SQ=4?

   - **Given**: Quadrilateral PQRS with PQ = 3 cm RS = 3 cm PS = 7.5 cm and SQ = 4 cm
     We know from triangle inequality theorem that sum of any two sides is greater than the third side ,
     But here we have PQ = 3 cm PS = 7.5 cm SQ = 4 cm (AS we get PQS triangle in quadrilateral PQRS where PQ and PS are sides and SQ is diagonal of quadrilateral .
     And
     PQ + SQ = 3 + 4 = 7
     and
     PS = 7.5 , So
     PQ + SQ >PS , But that equation is not true for any type of triangle .
     So, we can't construct the given quadrilateral.