In[31]:=
```
fM[v_] := (2 π σ^2)^-3/2 (4 π v^2) Exp[-v^2/(2 σ^2)];
Integrate[fM[v], {v, 0, ∞}, Assumptions → {σ > 0}]
```

Out[32]=
$$1$$

In[117]:=
```
veffA = FullSimplify[(Integrate[fM[v]/((c_s)^2 + v^2)^(6/2), {v, 0, ∞}])^-1/6 /. {σ → M c_s},
  Assumptions → {c_s > 0, σ > 0, M > 0}]
```

Out[117]=
$$\frac{2^{7/12}\, \mathcal{M}^{7/6}\, c_s}{\left(\sqrt{2}\,\left(\mathcal{M}+\mathcal{M}^3\right)+e^{\frac{1}{2\mathcal{M}^2}}\sqrt{\pi}\,\left(-1-2\mathcal{M}^2+\mathcal{M}^4\right)\text{Erfc}\left[\frac{1}{\sqrt{2}\,\mathcal{M}}\right]\right)^{1/6}}$$

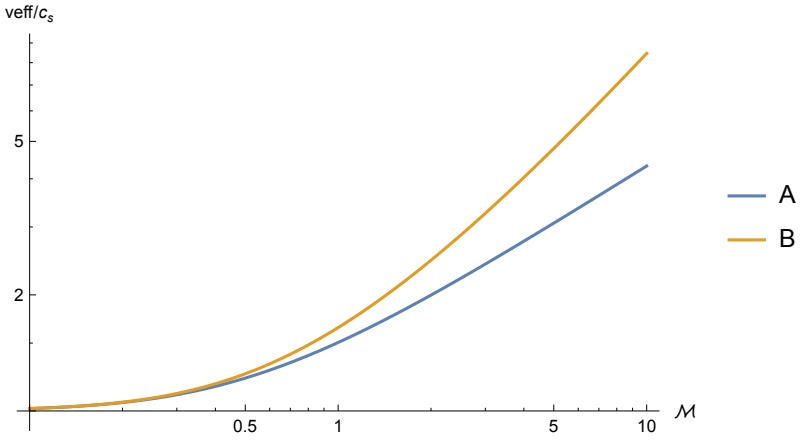
In[118]:=
```
veffB = FullSimplify[(Integrate[fM[v]/((c_s)^2 + v^2)^(3/2), {v, 0, ∞}])^-1/3 /. {σ → M c_s},
  Assumptions → {c_s > 0, σ > 0, M > 0}]
```

Out[118]=
$$\frac{\sqrt{2}\; e^{-\frac{1}{12\mathcal{M}^2}}\, \pi^{1/6}\, \mathcal{M}^{5/3}\, c_s}{\left(\left(1+2\mathcal{M}^2\right)\text{BesselK}\left[0, \frac{1}{4\mathcal{M}^2}\right]-\text{BesselK}\left[1, \frac{1}{4\mathcal{M}^2}\right]\right)^{1/3}}$$

In[119]:=
```
veffAnorm = veffA / c_s;
veffBnorm = veffB / c_s;
```

In[121]:=
```
LogLogPlot[{veffAnorm, veffBnorm}, {M, 0.1, 10},
 PlotLegends → {"A", "B"}, AxesLabel → {"M", "veff/c_s"}]
```

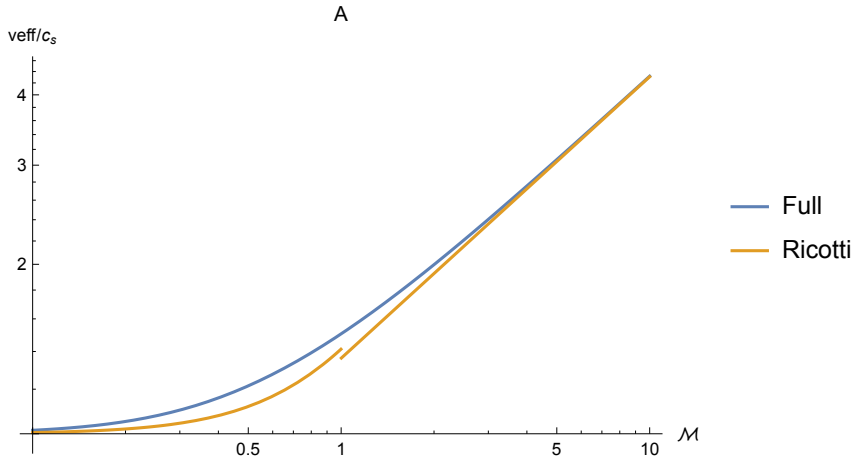
Out[121]=



In[165]:=
```
veffAnormRicotti[M_] := If[M < 1, (1 + M^2)^1/2, (16/(Sqrt[2 π]) M^3)^1/6]
veffBnormRicotti[M_] := If[M < 1, (1 + M^2)^1/2, M (Sqrt[2/π] Log[2/E M])^-1/3]
```

In[167]:=
```
LogLogPlot[{veffAnorm, veffAnormRicotti[ℳ]},
 {ℳ, 0.1, 10}, PlotLegends → {"Full", "Ricotti"},
 AxesLabel → {"ℳ", "veff/cs"}, PlotLabel → "A"]
```

Out[167]=



In[168]:=
```
LogLogPlot[{veffBnorm, veffBnormRicotti[ℳ] },
 {ℳ, 0.1, 10}, PlotLegends → {"Full", "Ricotti"},
 AxesLabel → {"ℳ", "veff/cs"}, PlotLabel → "B"]
```

Out[168]=