

case II:- Partition can be k & $n-k$ for all other k .

This gives us:-

$$T(n) \leq \frac{1}{n} \left[T(\max(1, n-1)) + \sum_{k=1}^{n-1} T(\max(k-1, n-k)) \right] + O(n)$$

here $O(n)$ is for partition

further:

$$T(n) \leq \frac{1}{n} \left(T(n-1) + 2 \sum_{k=\lceil n/2 \rceil}^{n-1} T(k) \right) + O(n)$$

$$\text{since } \max(k-1, n-k) = \begin{cases} k-1 & \text{if } k > \lceil n/2 \rceil \\ n-k & \text{if } k \leq \lceil n/2 \rceil \end{cases}$$

Simplifying this gives us

$$T(n) \leq \frac{2}{n} \sum_{k=\lceil n/2 \rceil}^{n-1} T(k) + O(n)$$

By induction we assume that $T(n) \leq cn$
where $k < n$ and $T(k) \leq ck$

$$\Rightarrow T(n) \leq \frac{2c}{n} \sum_{k=\lceil n/2 \rceil}^{n-1} k + O(n)$$

$$\leq \frac{2c}{n} \left[\sum_{k=1}^{n-1} k + \sum_{k=1}^{\lceil n/2 \rceil} k \right] + O(n)$$

$$\leq \frac{2c}{n} \left[\frac{n(n-1)}{2} - \frac{\lceil n/2 \rceil (\lceil n/2 \rceil - 1)}{2} \right] + O(n)$$

$$\leq \frac{2c}{n} \left[\frac{n^2 - n}{2} - \frac{n^2/4 - 3n/2 + 2}{2} \right] + O(n)$$

$$\leq \frac{2c}{n} \left[\frac{n^2 - n}{2} - \frac{n^2}{4} + \frac{3n}{2} - 2 \right] + O(n)$$

$$\leq \frac{c}{n} \left[\frac{n^2 - n}{2} - \frac{n^2}{4} + \frac{3n}{2} - 2 \right] + O(n)$$

$$\leq c \left[\frac{n-1}{4} - \frac{n}{2} + \frac{3}{2} - \frac{2}{n} \right] + O(n)$$

$$\leq cn + O(n)$$

$$\Rightarrow T(n) = O(n)$$

Average Time Complexity ~~and~~ worst time complexity is still $O(n)$

Steps involved in Selection Problem

Randomised select (A, p, r, i):

if ($p == r$) then return $A[p]$

$q = \text{Randomised partition}(A, p, r)$

$j = q - p + 1$

if ($i == j$) then return $A[q]$

else if ($i < j$) then return randomised select

else random select ($A, q + 1, r, i - j$)

Randomised partition (A, p, r):

$i = \text{random}(p, r)$

swap ($A[i], A[r]$);

partition (A, p, r);

