INTRODUCTION :- TO MINIMIZATION OF BOOLEAN EXPRESSIONS :-

CRITERIA TO DETERMINE MINIMAL COST :-

a) Minimum no. of appearances of literals

b) Minimum no. of literals in SOP or POS expression.

c) minimum no. of terms in SOP expression, provided there is no other such expression with same no. of terms and fewer literals.

IRREDUNDANT OR IRREDUCIBLE EXPRESSION :-

An SOP expression from a which no term or literal can be deleted without altering its logical value is called an irredundant or irreducable expression.

KMAP METHOD

→ The algebraic procedure of combining various terms and applying to them the rules becomes very tedious as no. of variables increases.

→ KMap provides systematic way to derive the minimal expression.

→ A KMap is modification of truth table

→ n variable map consist of 2^n cells.

→ cycle code is used in the combination as column and row heading.

→ Property of cyclic code is b/w any two successive number there will be only 1 bit change.

→ A collection of 2^m cells, each adjacent to 'm' cells of collection is called a '**subcube**'.

→ The no. of product terms in expression is equal to the no. of subcubes.

COVERING

→ A switching function f is said to cover g and denoted by "f is superset of g", if f assumes true value whenever g does.

→ If 'f' covers 'g' and 'g' covers 'f' then both are equivalent.

→ If g has 'x' min terms and g is a function of on n variable then no. of covering functions for g is $2^{(2^n - x)}$

$$f(w, x, y, z) = wx + yz$$

$x = 16$

$n = 16 - 7 = 9$

$$\# \text{ covering function} = 2^{(2^9 - 16)}$$

Implicants & Prime Implicants

→ If f covers g then g is said to imply f, $g \to f$

$f(a,b,c) = ab + c$

$f_1 = ab$, $f_2 = c$

f_1 and f_2 are implicants of f.

PRIME IMPLICANT : (LONGEST SUBCUBE)

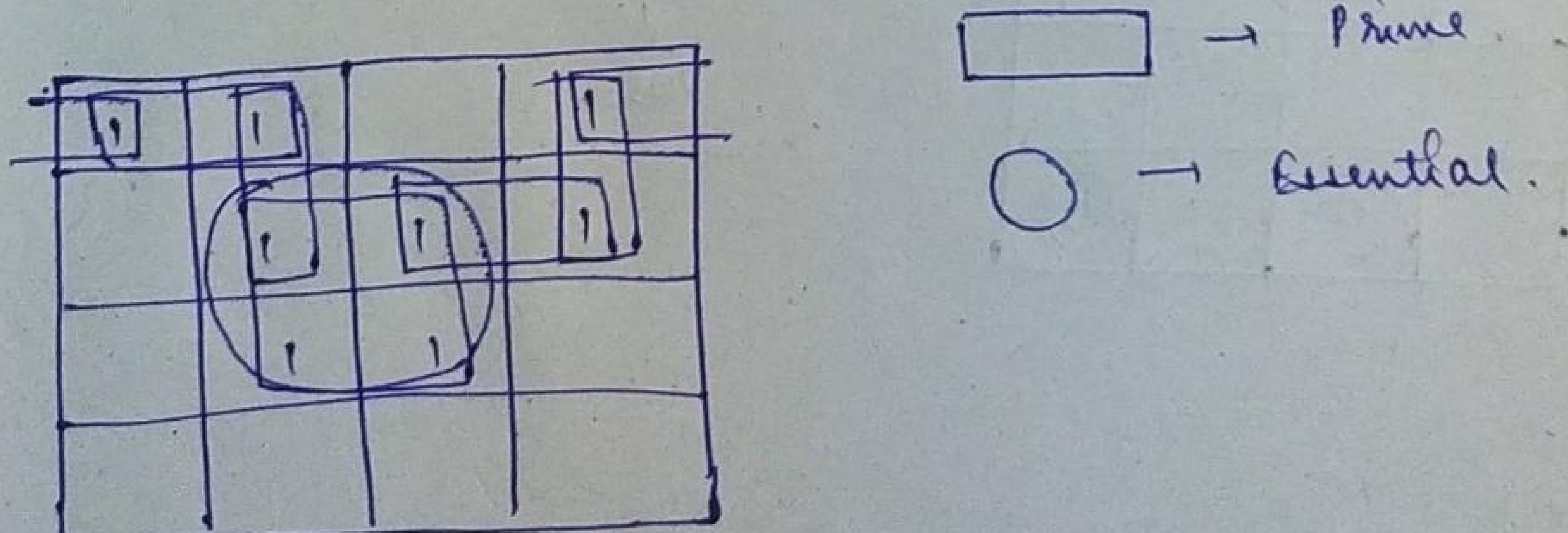
An implicant 'p' of a function 'f' is said to be prime implicant if

a) P is a product term

b) deletion of any literal from 'p' results in a new product which is not covered by f.

ESSENTIAL PRIME IMPLICANT :-

A prime implicant 'p' of a function 'f' is said to be an essential prime implicant, if it covers at least one minterm of 'f' which is not covered by any other prime implicant.



→ If there is no essential prime implicant then it is called cyclic Kmap.

PROCEDURE TO FIND MINIMAL SOP :-

YZ \ WX	00	01	11	10
00			1	
01	1	1	1	
11		1	1	1
10		1		

$$f = \Sigma(1, 5, 6, 7, 11, 12, 13, 15)$$

$$= \bar{w}\bar{y}z + wx\bar{y} + wyz + \bar{w}xy$$

Q. which PI is essential ?

C \ AB	00	01	11	10
0		1		1
1	1	1		1

a) $\bar{B}C$, $\bar{A}B$

b) $\bar{A}C$, $\bar{A}B$

c) $\bar{A}B$, $A\bar{B}$

✓d) $\bar{A}B$, $A\bar{B}$, $\bar{B}C$.

MINIMAL POS

$$f(w, x, y, z) = \Pi(5, 6, 9, 10)$$

YZ \ WX	00	01	11	10
00				
01		0		0
11				
10		0		0

$$f = (w+\bar{x}+y+\bar{z})(\bar{w}+x+y+\bar{z})$$
$$(w+\bar{x}+\bar{y}+z)(\bar{w}+x+\bar{y}+z)$$

↓

POS

Q. Find the number of literals in minimum POS and SOP for $f(w,x,y,z) = \prod(1,5,6,7,11,12,13,15)$

yz \ wx	00	01	11	10
00	1	1	0	1
01	0	0	0	1
11	1	0	0	0
10	1	0	1	1

$$POS = (\bar{w}+\bar{x}+y)(w+y+\bar{z})(w+\bar{x}+\bar{y})(\bar{w}+\bar{y}+\bar{z})$$

$$SOP = \bar{w}\bar{y}\bar{z} + w\bar{x}\bar{y} + \bar{w}\bar{x}y + wy\bar{z}$$

No of literals in POS = 12

" " " " SOP = 12.

INTRO TO DON'T CARES :-

→ A function is said to be completely specified if it is given 0 or 1 for every combination of input variable. There exists some functions which are not completely specified.

→ Combinations for which value of a function is not specified are called don't care combination denoted by 'd' or 'ϕ'

→ An incompletely specified function containing k- don't care conditions corresponds to a class of 2^k distinct functions.

EXAMPLES ON DONT CARE :-

1) $f = \Sigma(5, 6, 7, 8, 9) + \phi(10, 11, 12, 13, 14, 15)$

YZ \ WX	00	01	11	10
00			d	1
01		1	d	1
11		1	d	d
10		1	d	d

$f = w + xz + xy$

2)

YZ \ WX	00	01	11	10
00	0	x	0	x
01	x	1	x	1
11	0	x	1	0
10	0	1	x	0

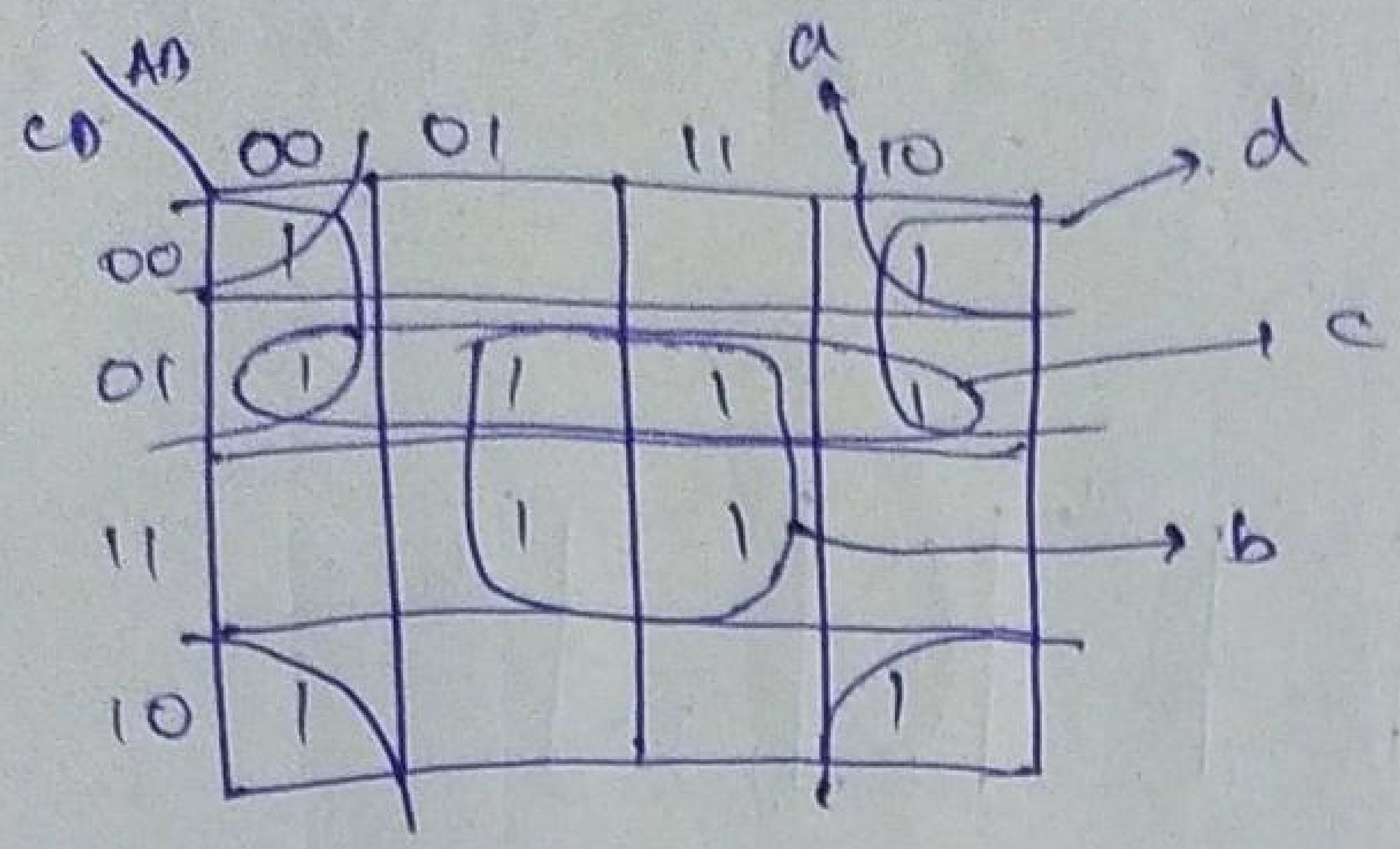
$f = \bar{y}z + xy$

3)

XY \ WZ	00	01	11	10
00		x		
01		x	1	1
11	1	1	1	1
10		x		

$f = wy + xy$

Q. How many minimal expression are possible?

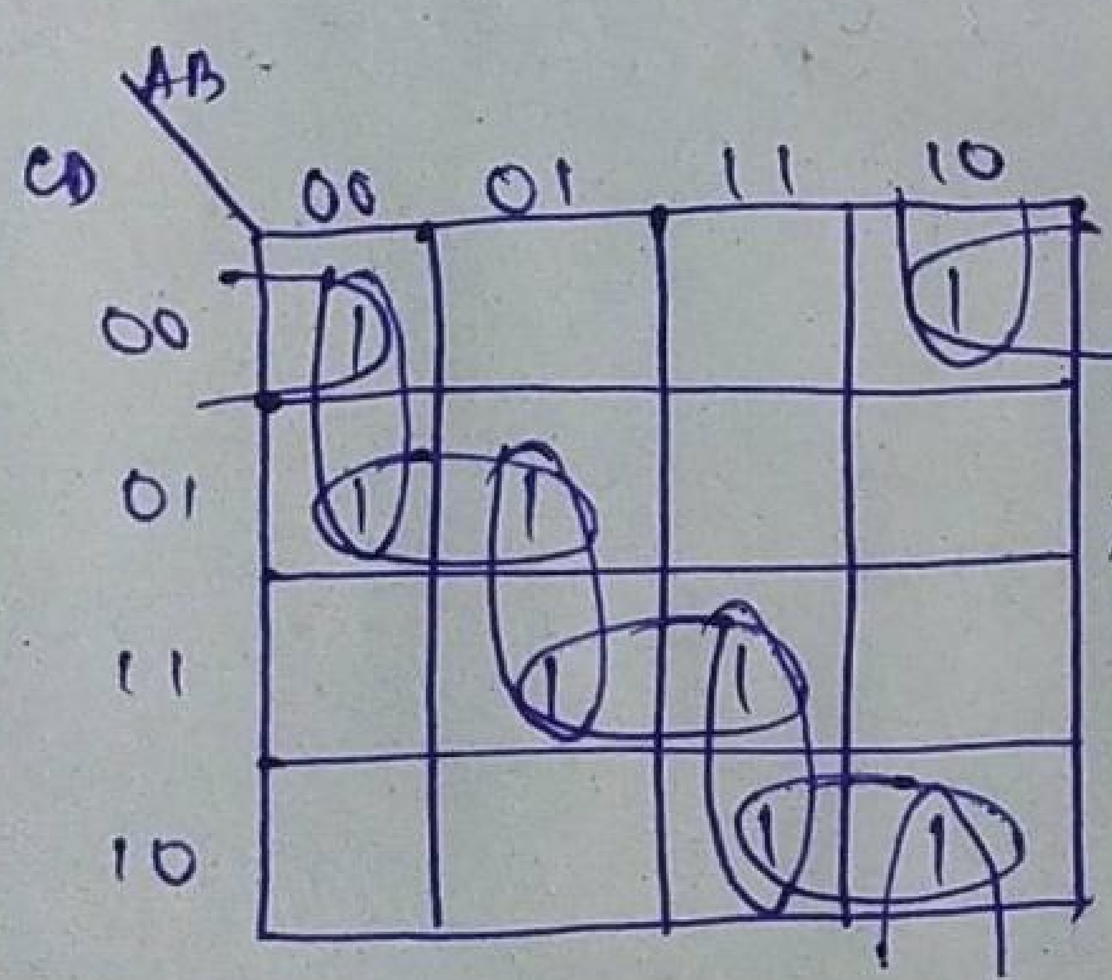


	✓ 0	c 1	✓ 2	✓ 5	✓ 7	✓ 8	c 9	✓ 10	✓ 13	✓ 15
E → a	1		1			1		1		
E → b				1	1				1	1
E → c		1		1			1		1	
d	1	1				1	1			

a + b + c or a + b + d $\boxed{\text{Answer} = 2}$

BRANCHING FOR MINIMIZING CYCLIC FUNCTIONS :-

B → C , H → G



	✓ 0	✓ 1	✓ 5	✓ 7	✓ 8	✓ 10	✓ 14	✓ 15
✓ $A = \bar{W}\bar{X}\bar{Y}$	×	×						
× $B = \bar{W}\bar{Y}Z$		×	×					
✓ $C = \bar{W}XZ$			×	×				
$D = XYZ$				×				×
✓ $E = WXY$							×	×
$F = WY\bar{Z}$						×	×	
✓ $G = W\bar{X}\bar{Z}$					×	×		
× $H = \bar{X}\bar{Y}\bar{Z}$	×				×			

$\boxed{A + C + G + E}$
↓
Minimal Expression

Implicant & Prime Implicant difference :-

Q) which of the following can be a prime Implicant of a function $f(A, B, C)$

a) $ab + c$

b) $bc + a$

c) $ac + b$,

d) ab

$c \backslash ab$	00	01	11	10
0			1	
1	1	1	1	1

$ab + c$

addition of two prime Implicants

prime implicant..

Q) which can't be PI of $f(A, B, C)$?

a) $\{ab, bc, ca\}$,

b) $\{b\bar{c}a, ba, bc\}$

c) $\{\bar{a}, b, ca\}$

d) $\{\bar{a}\bar{b}, ab\}$

(*) a and ab can't be together i.e., if x is a PI anything containing x should not be a PI.

CONVERTING A FUNCTION INTO SELF DUAL :-

Q. What min terms have to be added to make the following a self dual?

$$f(A,B,C,D) = \bar{A}BC + (AC+B)D$$

→ $f(A,B,C,D) = \bar{A}BC + ACD + BD$

Mutually exclusive : (0, 15) (1, 14) . . . . . (7, 8)

$f = \bar{A}BC(D+\bar{D}) + A(B+\bar{B})CD + (A+\bar{A})B(C+\bar{C})D$

$= \underset{7}{\bar{A}BCD} + \underset{6}{\bar{A}BC\bar{D}} + \underset{15}{ABCD} + \underset{11}{A\bar{B}CD} + \underset{13}{AB\bar{C}D} + \underset{5}{\bar{A}B\bar{C}D}$

No mutual exclusive term is present.

$f = \bar{A}BCD + \bar{A}BC\bar{D} + ABCD + A\bar{B}CD + AB\bar{C}D + \bar{A}B\bar{C}D + \underbrace{\underset{1}{\bar{A}\bar{B}\bar{C}D} + \underset{3}{\bar{A}\bar{B}CD}}_{\text{added}}$

COMBINING FUNCTIONS HAVING DON'T CARES :-

Q. How many functions does $f_1 f_2$ and $f_1 + f_2$ represent?

$f_1(a,b,c) = \Sigma(0,2,4) + \Sigma_\phi(3,5,7)$

$f_2(a,b,c) = \Sigma(2,3) + \Sigma_\phi(1,6,7)$

	f_1	f_2	$f_1 f_2$	f_1+f_2
0	1	0	0	1
1	0	ϕ	0	ϕ
2	1	1	1	1
3	ϕ	1	ϕ	1
4	1	0	0	1
5	ϕ	0	0	ϕ
6	0	ϕ	0	ϕ
7	ϕ	ϕ	ϕ	ϕ

$\# \phi = 2$, # functions $= 2^2$

$\# \phi = 4$

function $= 2^4$

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PRIME IMPLICANTS & DON'T CARES :-

Q. The number of PI, EPI & redundant PI for

$$f(A,B,C) = \sum(2,5,6,7)$$

C \ AB	00	01	11	10
0		1	1	8
1			1	1

PI = 3

EPI = 2

redundant PI = 1

Q. A function $f(A,B,C) = \sum(3,5,6)$ is minimized to $A+BC$, then what are the don't cares?

→ $f(A,B,C) = \sum(3,5,6)$

C \ AB	00	01	11	10
0			1	d
1		1	d	1

don't cares = 4 & 7 = Σ_{ϕ}

i.e, $A\bar{B}\bar{C}$, ABC.

NUMBER OF MINIMAL EXPRESSIONS :-

Q. In a KMap it was found out that EPI are covering all terms except 2 min terms. Those 2 min terms are in turn covered by 3 non essential PI each. What is the no. of minimal SOP expressions?

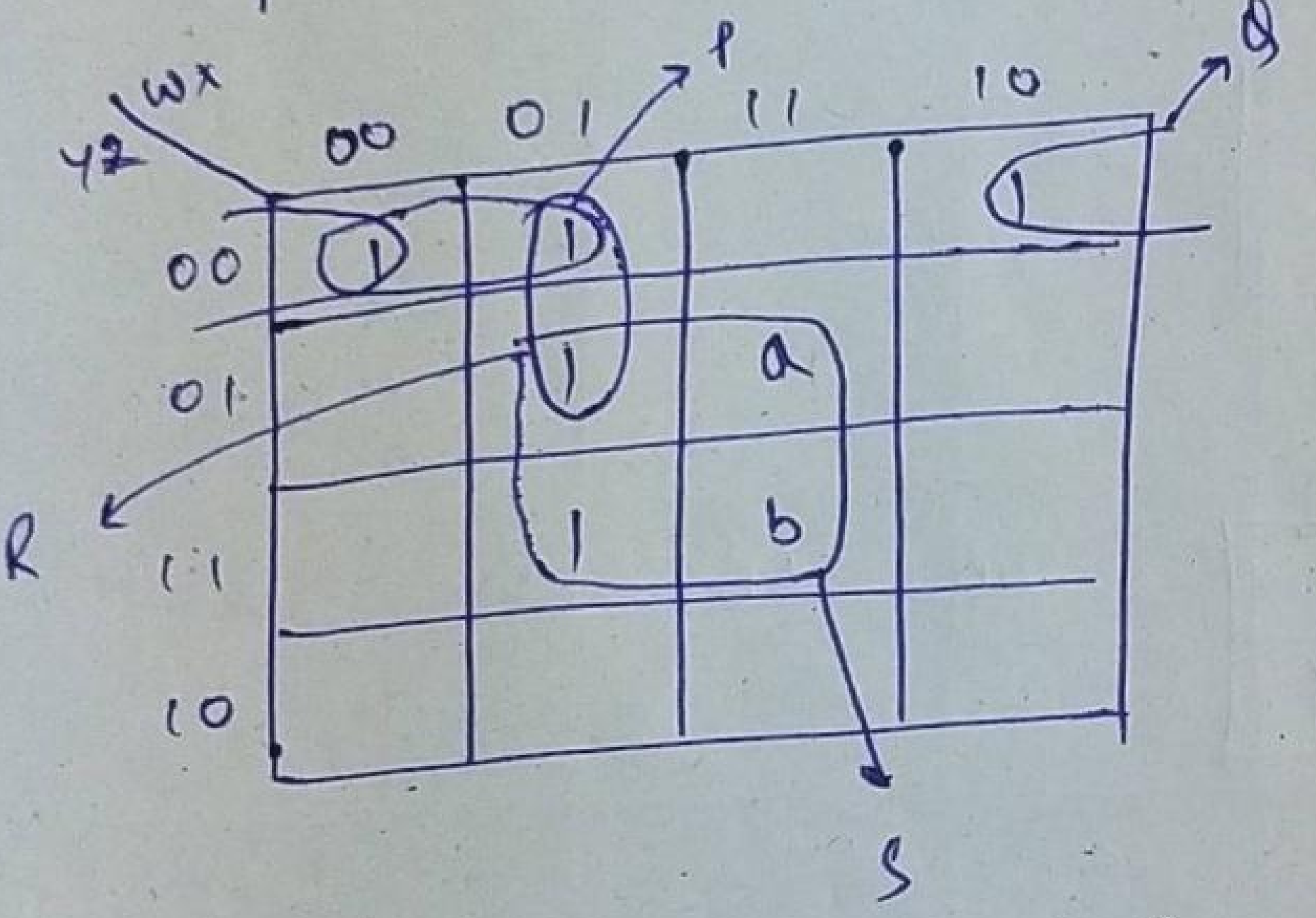
→ # minimal SOP = # EPI + $\sum_{i=1}^{n}$ # NEPI for each minterm.

$= 3 + 3 + 3 = 9.$

PRIME IMPLICANT CHART

Q. In a prime implicant chart representing a boolean expression $f(w, x, y, z)$ columns represent minterms and rows represent PI, identify P, Q, R, S & a, b.

	0	4	5	7	8	a	b
P	✓	✓					
Q	✓				✓		
R		✓	✓				
S			✓	✓		✓	✓



$P = \overline{w}\,\overline{y}\,\overline{z}$

$Q = \overline{x}\,\overline{y}\,\overline{z}$

$R = \overline{w} x \overline{y}$

$a = 13$

$b = 15$

$S = xz$

VARIABLE ENTRANT MAP (VEM)

$f(A, B, C) = \cancel{\Sigma(1, 3, 5, 7)}\ \Sigma(1, 2, 5, 6)$

A	B	C	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	0

A	B	f
0	0	C
0	1	$\overline{C}$
1	0	C
1	1	$\overline{C}$

In terms C, 0, 1

B \ A	0	1
0	C	C
1	$\overline{C}$	$\overline{C}$

→ size reduced.

MINIMIZATION USING VEM

1) Set all variables in cell as '0' and obtain SOP expression

2) a) Make one variable in the cell as '1' and obtain SOP by making earlier ~~expressions~~ minterms (1's) as don't cares.

   b) multiply above SOP with the concerned variable

3) Repeat step 2 until all variables are covered.

4) SOP of VEM is obtained by ORing the previous SOP expressions

→ Ex:

C \ AB	00	01	11	10
0	D	1	$\bar{D}$	$\bar{D}$
1	D	1	0	∅

Step 1:

C \ AB	00	01	11	10
0	0	1	0	0
1	0	1	0	∅

SOP = $\bar{A}B$

Step 2:

Assuming $D = 1$, $\bar{D} = 0$

C \ AB	00	01	11	10
0	1	∅	0	0
1	1	∅	0	∅

SOP = $\bar{A}$

Assuming $D = 0$, $\bar{D} = 1$

C \ AB	00	01	11	10
0	0	∅	1	1
1	0	∅	0	∅

SOP = $A\bar{C}$

Step 3: $\bar{A}B + \bar{A}D + A\bar{C}\bar{D}$

EXAMPLE ON VEM

1) $f(A, B, C) = \Sigma(3, 5, 6, 7)$ is realized by following VEM then find P, Q, R, S.

B \ A	0	1
0	P	S
1	R	Q

A	B	C	f
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

A	B	f
0	0	0
0	1	C
1	0	C
1	1	1

$\boxed{P = 0, \quad Q = 1; \quad S = R = C}$

B \ A	0	1
0	0	C
1	C	1

$\longrightarrow$

B \ A	0	1
0	0	0
1	0	1

$\Rightarrow AB$

$\downarrow$

B \ A	0	1
0	0	1
1	1	Ø

$\Rightarrow A + B$

$\boxed{AB + AC + BC} \longleftarrow$