

★ Basic properties of Switching Algebra :-

1) The basic element of switching algebra is a Boolean variable.

2) A Boolean variable can take two values $\{0, 1\}$.

3) Main operators in switching algebra are

+ (OR)			· (AND)			- (NOT)		also called complement
0	0	0	0	0	0	0	1	
0	1	1	0	1	0			
1	0	1	1	0	0	1	0	
1	1	1	1	1	1			

4) Basic Properties :-

a) Idempotency :- (checking various combinations from truth table)

$$x \cdot x = x$$

$$x + x = x$$

$$x + 1 = 1$$

$$x + 0 = x$$

$$x \cdot 0 = 0$$

$$x \cdot 1 = x$$

b) Commutativity :- (changing position of operand, check the value)

$$x + y = y + x$$

$$x \cdot y = y \cdot x$$

NOTE: Associativity is checked when there is some ambiguity.

$$a + b + c \Rightarrow op_1 \text{ } \textcircled{op_1} \text{ } op_2 \text{ } \textcircled{op_2} \text{ } op_3$$

Should op_2 be associated with op_1 or op_3 ?

If op_2 is associated with op_1 it is called left associative

and for op_3 it is called right associative.

This happens when two operators of same priority are present

c) Associativity :- (Checking if left and right associative evaluation are same)

$$(x + y) + z = x + (y + z)$$

$$(x \cdot y) \cdot z = x \cdot (y \cdot z)$$

d) Complement :- (Evaluating with the complement value)

$$x + \bar{x} = 1$$

$$x \cdot \bar{x} = 0$$

e) Distributivity :-

$$x \cdot (y + z) = xy + xz$$

$$x + yz = (x + y)(x + z)$$

NOTE: A quick process to check the expression is

put $x=0$ and check the L.H.S & R.H.S.

In every couple equation, $1 \rightarrow 0$, $0 \rightarrow 1$, $+$ \rightarrow \cdot , \cdot \rightarrow $+$

This property is known as 'duality'. 'Principle of Duality'.

★ Switching Expressions and Simplification:

1) Switching Expression is a finite number of combinations of switching variables and constants {0, 1} by means of switching operations (+, ·, NOT)

Ex: ~~abc~~ $x + \bar{x}yz + x\bar{z}$, $a + bc + \bar{b}d$

NOTE: Every occurrence of a variable in its true or complement form is called a literal.

$x + \bar{x}yz + x\bar{z}$ has 3 variables but 6 literals.

2) Properties for simplifying switching expression:

a) Absorption:

$$\Rightarrow x + xy = x$$

$$\Rightarrow x + x'y = (x + x')(x + y) = x + y \quad [\text{Distributivity}]$$

b) Dual:

$$x(x' + y) = x \cdot x' + x \cdot y = 0 + xy = xy$$

c) Consensus Theorem:

$$xy + \bar{x}z + yz = xy + \bar{x}z \quad [yz \text{ is redundant}]$$

A variable concatenated with another variable and its complement concatenated with a 3rd variable and again 2nd & 3rd variables are ANDed and the whole is ORed. Then the final term is redundant.

NOTE: If the value of expression does not depend on any term then that term is called redundant.

$$\Rightarrow xy + \bar{x}z + yz$$

$$= xy(1) + \bar{x}z(1) + yz(1)$$

$$= xy + \bar{x}z + (x + \bar{x})yz$$

$$= xy + \bar{x}z + xyz + \bar{x}yz$$

$$= xy(1+z) + \bar{x}z(1+y)$$

$$= xy(1) + \bar{x}z(1) = xy + \bar{x}z \quad (\text{Proved})$$

SIMPLIFICATION

$$\textcircled{1} \quad \bar{x}\bar{y}z + yz + xz$$

$$= (x + y + \bar{x}\bar{y})z$$

$$= (x + y + \overline{x+y})z$$

$$= z$$

NOTE: In switching algebra, cancellation of variables is not

allowed i.e., $a+b = a+c \rightarrow$ ['a' can not be cancelled]

$$a \cdot b = a \cdot c \quad \nearrow$$

④ De Morgan's Law and Simplification:

$$1) \overline{xy} = \bar{x} + \bar{y}$$

$$2) \overline{x+y} = \bar{x} \bar{y}$$

i.e., for $f(a, b, c, 0, 1, +, \cdot)$

then complement of $f = \bar{f} = f'(\bar{a}, \bar{b}, \bar{c}, 1, 0, \cdot, +)$

SIMPLIFICATION

$$1) (x+y) [x'(y'+z')] + x'y' + x'z'$$

$$= (x+y) [x + y \cdot z] + x'y' + x'z'$$

$$= (x+y)(x+y)(x+z) + \overline{x+y} + \overline{x+z}$$

$$= (x+y)(x+z) + [(x+y)(x+z)]'$$

$$= 1. \text{ Answer}$$

* Switching Functions :-

1) Canonical Form :-

In this form, every combination that results value of function as 1 should be present in the term.

$$f = a + bc$$

$$\Rightarrow f = \bar{a}bc + a\bar{b}\bar{c} + a\bar{b}c + ab\bar{c} + abc$$

* Canonical Sum of Products :-

→ A product term which contains each of 'n' variables as factors either in complemented or uncomplemented form is called a min term.

→ A min term given the value '1' of exactly one combination of the variables.

→ The sum of all min terms of 'f' for which 'f' assumes '1' is called 'canonical sum of products' or

'disjunctive normal form'.

⑤ Canonical Product of Sums :-

→ A sum term which contains each of 'n' variables as factor either in complemented or uncomplemented form is called a maxterm.

→ A maxterm gives the value of '0' for exactly one combination of the variables.

→ The product of all maxterms of 'f' for which 'f' assumes '0' is called 'canonical product of sums' or 'conjunctive normal form'.

a	b	c	f
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	0

$$f = \prod (0, 3, 5, 7)$$

$$\Rightarrow f = (a+b+c)(a+\bar{b}+\bar{c})(\bar{a}+\bar{b}+\bar{c})(\bar{a}+\bar{b}+c)$$

* Functional properties :-

- The SOP or POS canonical form of a switching function is unique.
- Two switching functions $f_1(x_1, \dots, x_n)$ and $f_2(x_1, \dots, x_n)$ are said to be logically equivalent if and only if both functions have same value for each and every combination of (x_1, x_2, \dots, x_n) .
- Two switching functions are equivalent if their canonical pos or sop are identical.