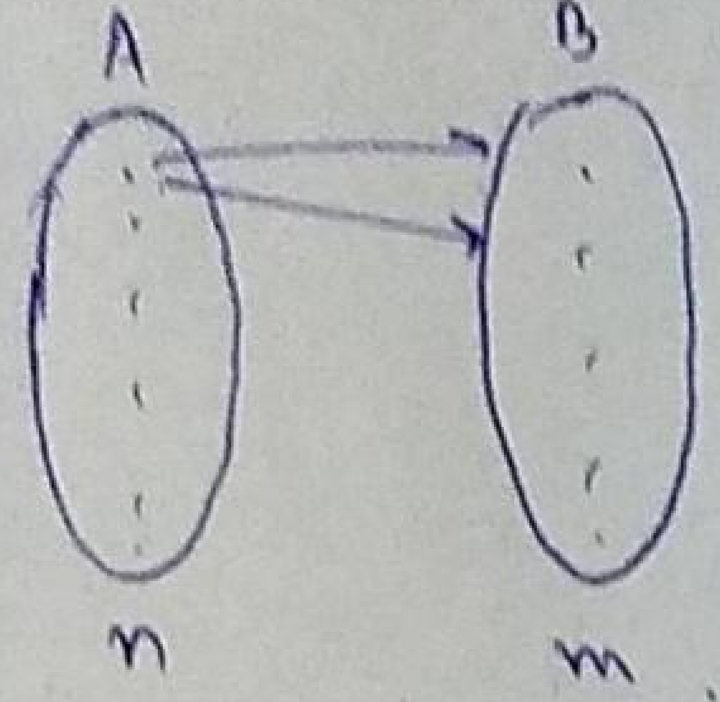
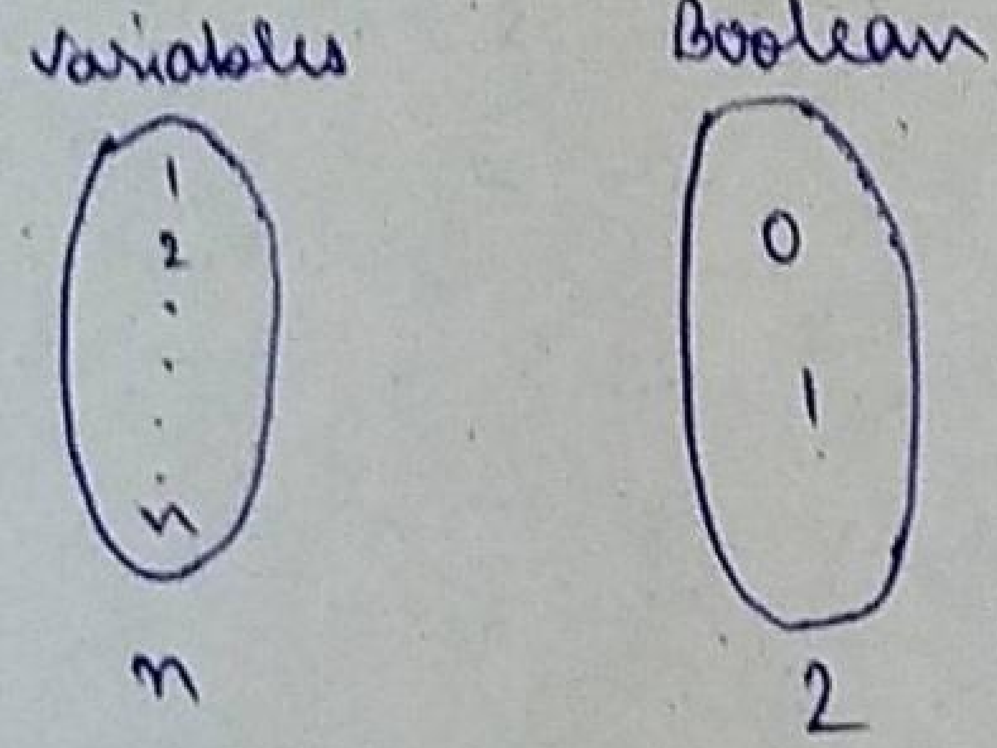
NUMBER OF FUNCTIONS

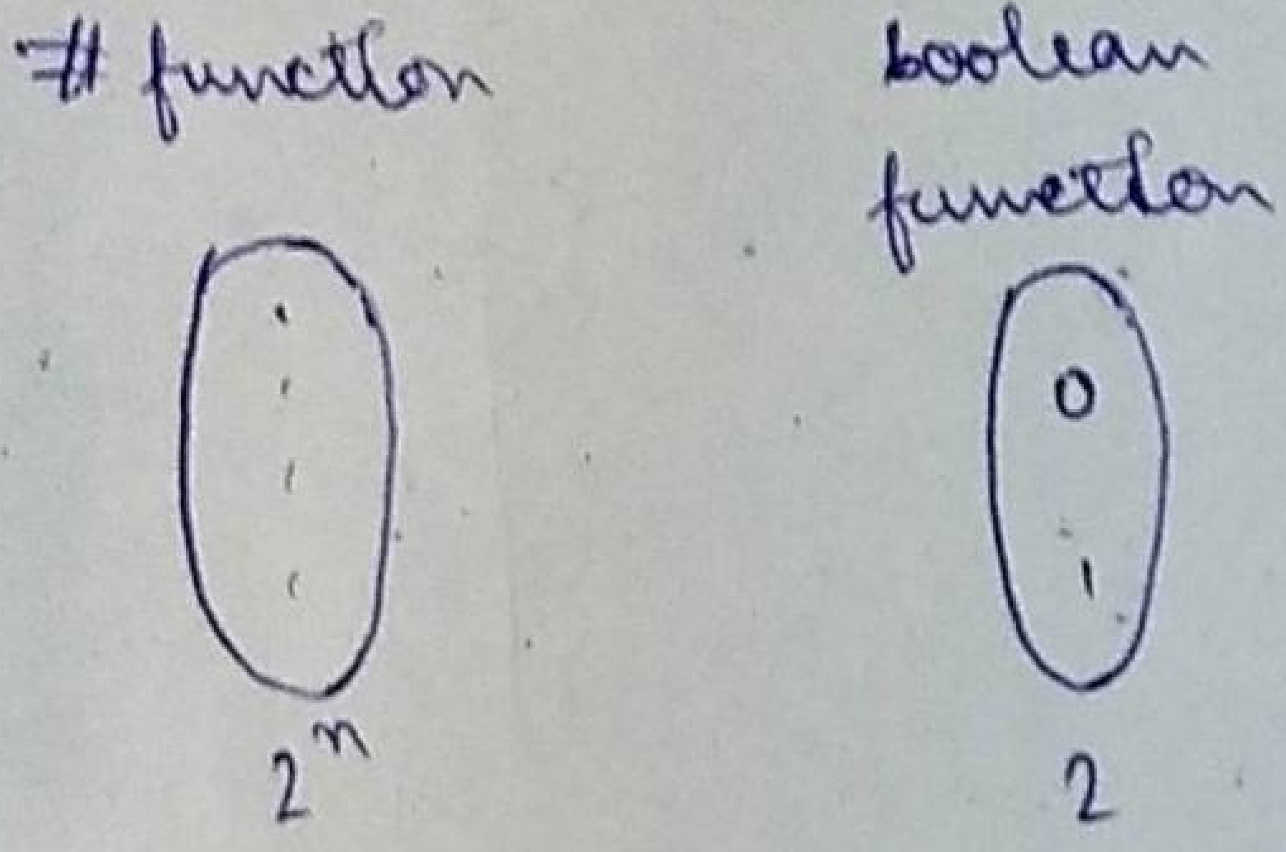


~~Rose~~ # function = m^n

(*) For n boolean variables, how many boolean functions?



function = 2^n



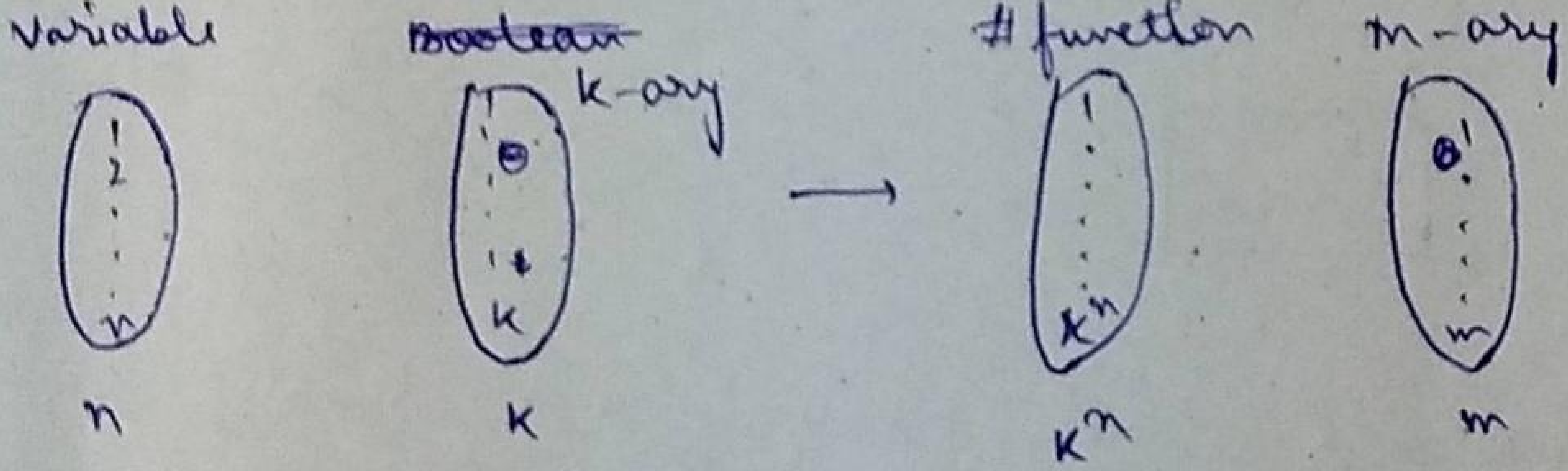
boolean function = 2^{2^n}

So, for n boolean variables,

boolean functions = 2^{2^n}

(*) For n k-ary variables, how many m-ary functions possible?

[m/k-ary → boolean / ternary / quarternary / ..]
(2) (3) (4)



function = k^n

m-ary functions = m^{k^n}

So, for n k-ary variables, number of m-ary function possible are = m^{k^n}

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COUNTING NUMBER OF FUNCTIONS & NEUTRAL FUNCTIONS :

① How many boolean functions are possible with 3 variables such that there are exactly 3 minterms.

→ Total functions for 3 variables (boolean) = $2^3 = 8$.

Value of minterms = 1

For exactly 3 minterms we get 3 ones out of 8 total boolean functions.

a	b	c	
0	0	0	1
⋮			1
			1
			0
			0
			0
			0
1	1	1	0

$\boxed{\text{So, \# of functions} = {}^8C_3 = 56}$

② For at most 3 minterms,

$\# \text{ of functions} = {}^8C_0 + {}^8C_1 + {}^8C_2 + {}^8C_3$

$= 8 + 28 + 56 + 1 = \boxed{93}$

③ For 'k' boolean variable and exactly 'm' minterms

$\# \text{ of function} = \boxed{{}^{2^k}C_m}$

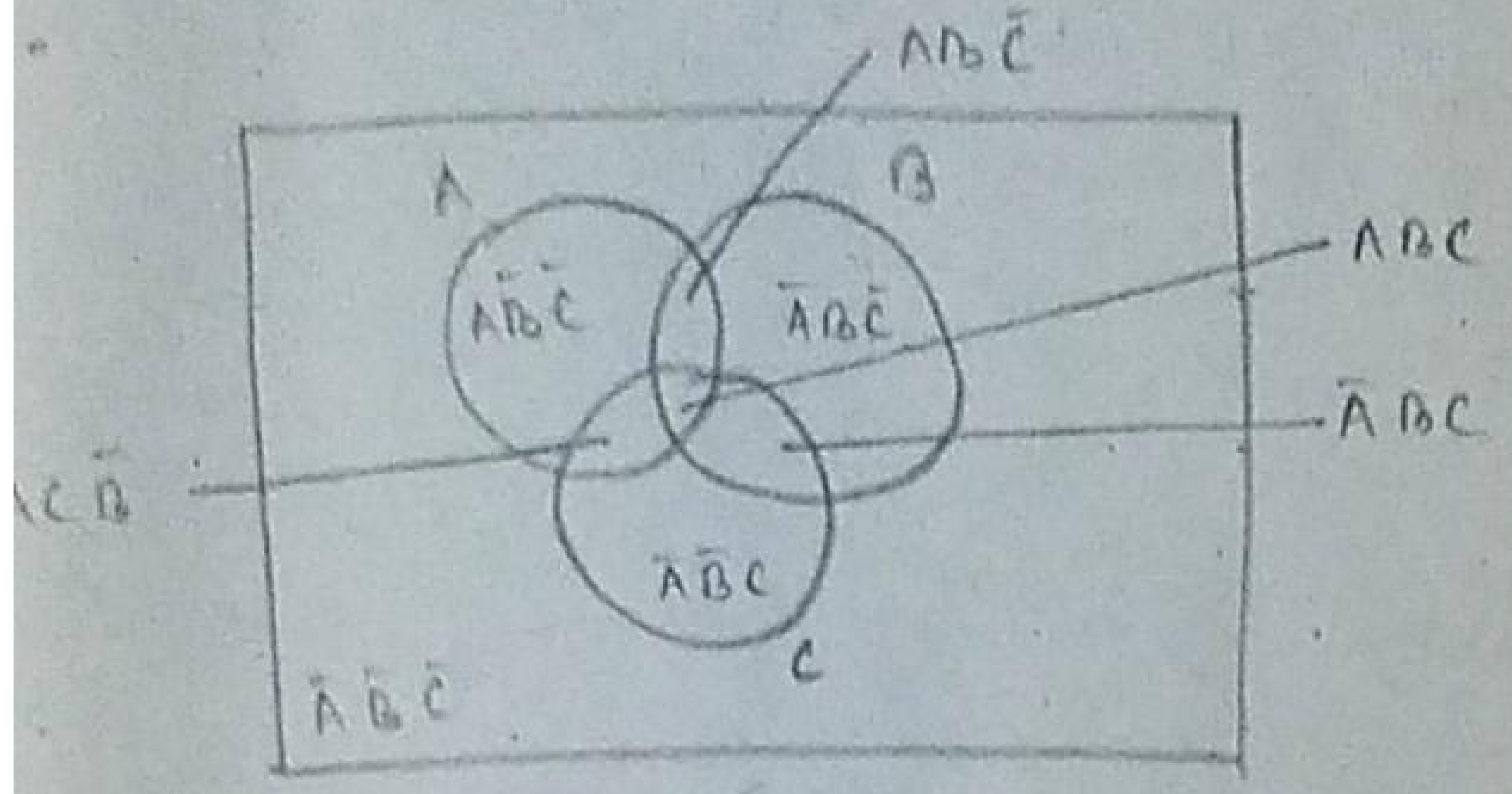
→ NEUTRAL FUNCTION : A neutral function is a function in which # minterms is equal to # maxterm.

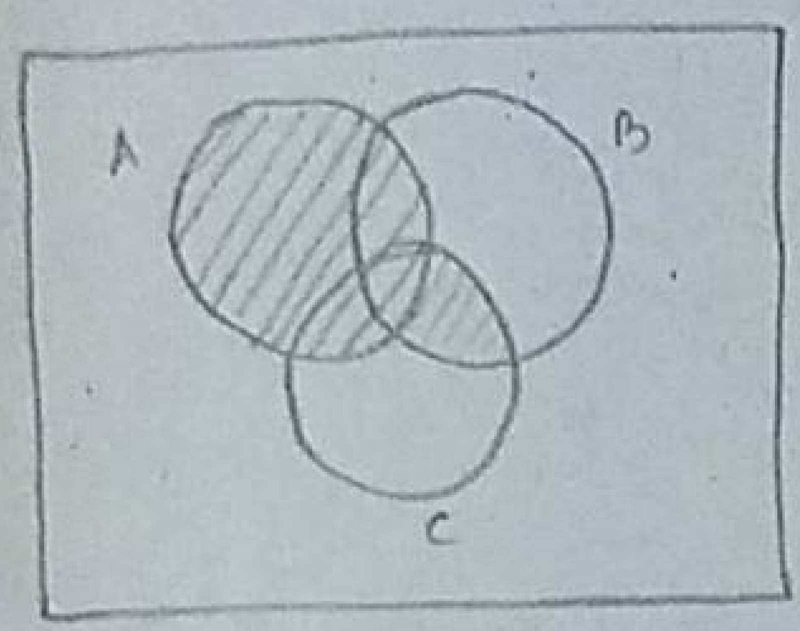
→ For n variables, i.e, 2^n input, and

$\# \text{minterm} = \# \text{maxterm} = 2^n/2 = 2^{n-1}$

Then $\boxed{\text{no. of neutral function} = {}^{2^n}C_{2^{n-1}}}$ (for n boolean variables)

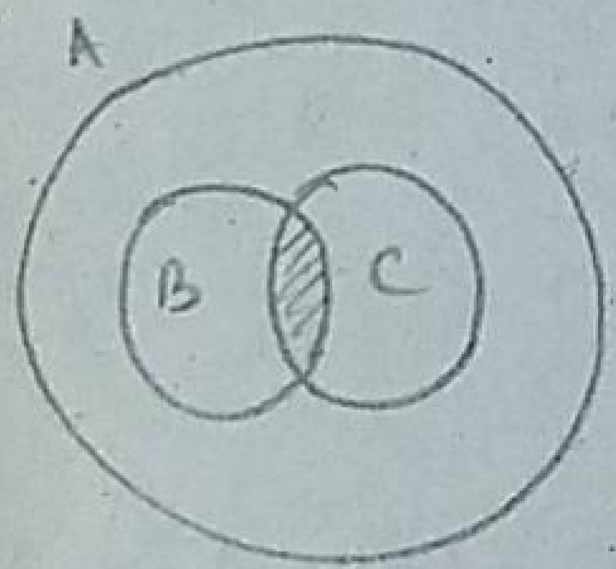
VENN DIAGRAM REPRESENTATION



→ 

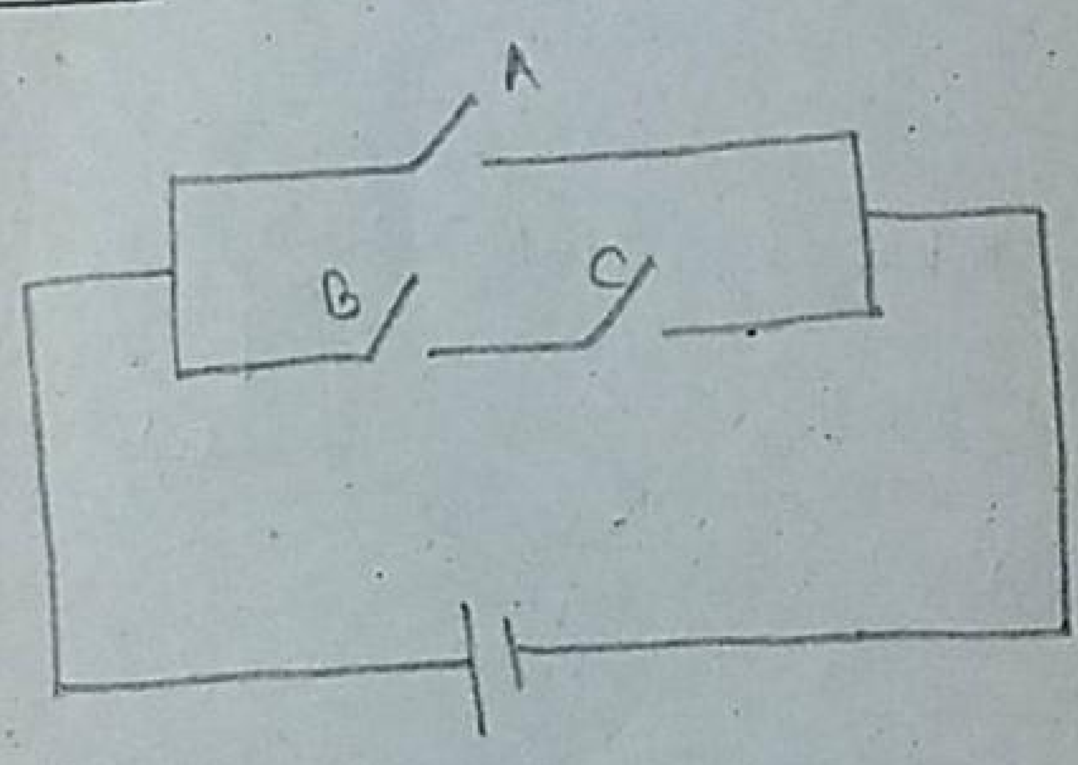
$\longrightarrow A + BC\bar{A}$.

# of possible shadings $= 2^{2^3} = 256$

→ 

$\longrightarrow A B . C$ [In this venn diagram $\bar{A}$ terms are not possible]

CONTACT REPRESENTATION :-



→ Every boolean function can be represented with the help of serial and parallel contact.

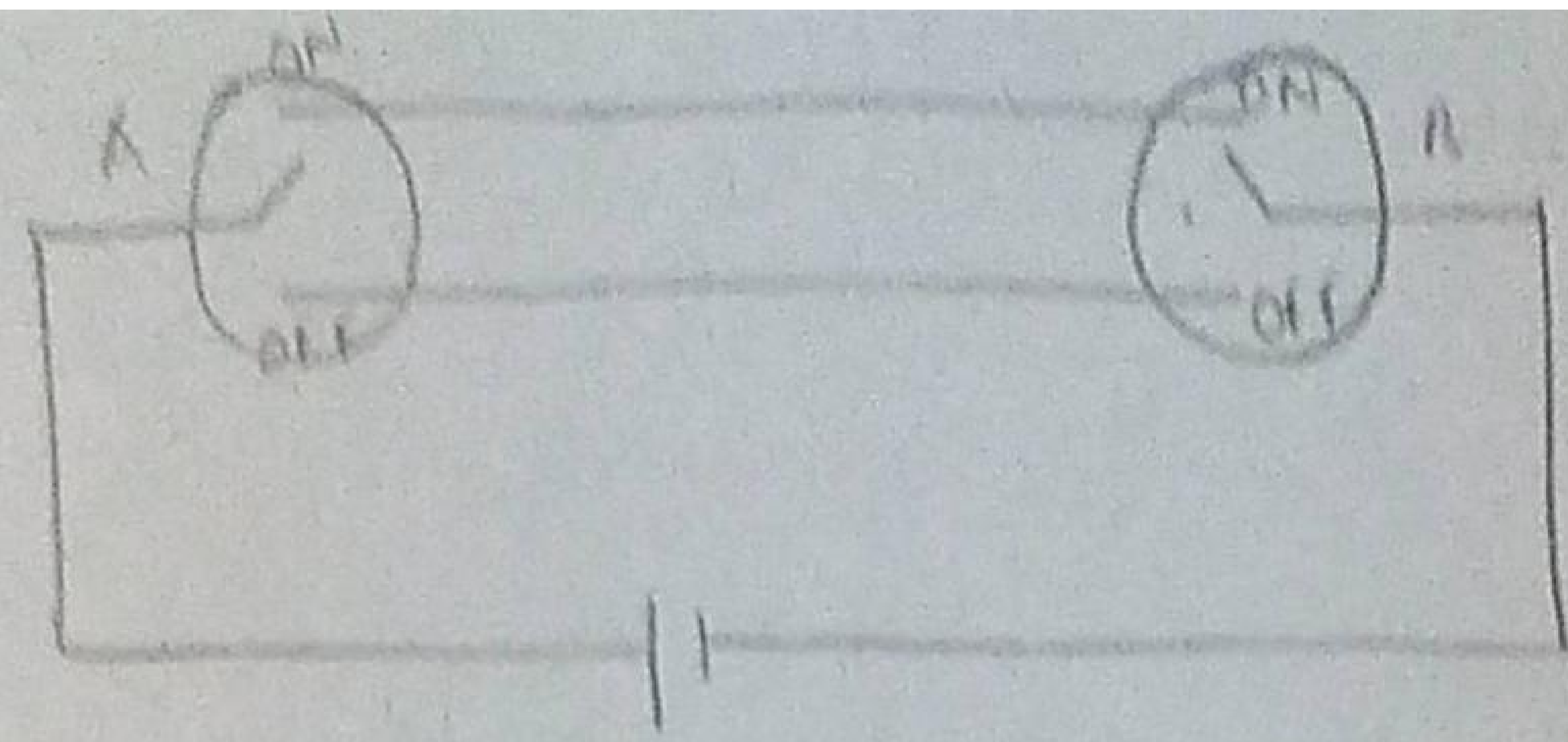
→ B and C (serial contact)

A and C (Parallel contact)

→ Serial contacts are performing 'AND' operation.

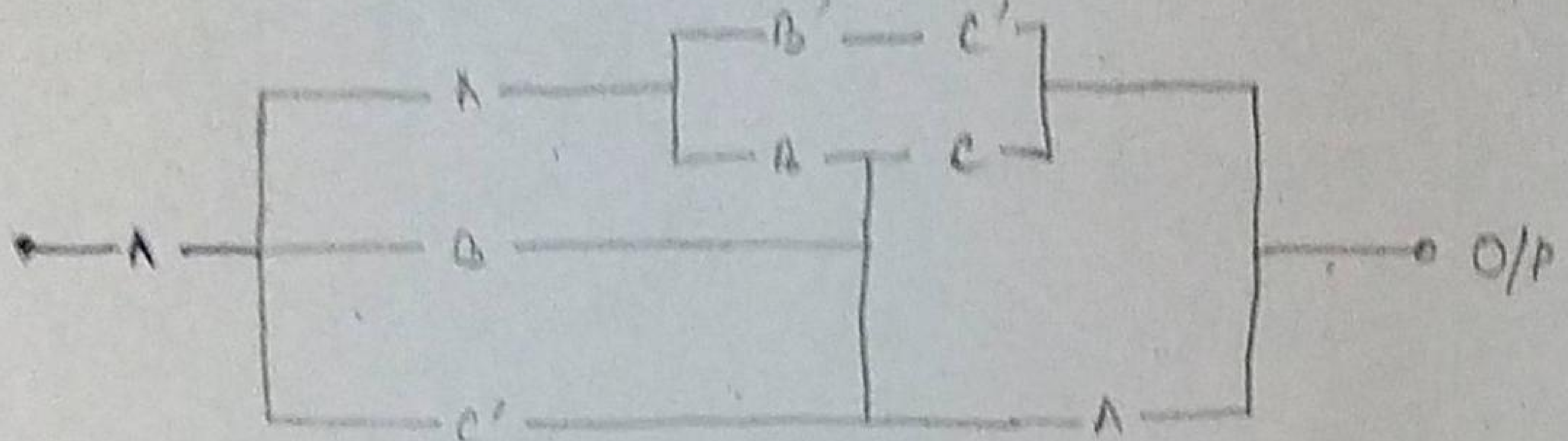
→ Parallel contacts " " 'OR' " .

→ The given circuit will pass current if either A is closed or both B and C are closed i.e, $\boxed{A + BC}$



→ $AB + \bar{A}\bar{B}$ → XNOR

→ Identify the boolean expression given by following circuit.



a) find the valid forward path

b) Perform OR among them

FORWARD PATH : Any part starting from I/P and ending at O/P without forming a cycle.

VALIDITY : No path should contains a variable in both true and complemented form.

b) → $AA\bar{B}\bar{C} + AABC + AABA + ABC + ABA + \underbrace{A\bar{C}C}_{\text{Invalid .}} + A\bar{C}A$

= Simplify it and get result.

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NESTED FUNCTION :-

→ In the following simultaneous boolean expressions, what are the values of w, x, y, z.

$x + y + z = 1$

$xy + \bar{w}\bar{z} = 0$

$x\bar{w} + y\bar{z} = 1$

OPTIONS.

	w	x	y	z
(a)	0	0	0	1
(b)	1	1	0	1
(c)	0	1	0	1
(d)	1	0	0	0

In this type of question we need to check each of the values in the MCQ with the simultaneous equations given in the question and find the correct option.

→ If $f(A, B) = \bar{A} + B$ then find $f(f(x+y, y), z)$

↳ This is called nested function.

$f(f(x+y, y), z)$

$= f(\overline{x+y} + y, z) = f(\bar{x}\bar{y} + y, z)$

$= f((\bar{x}+y)(\bar{y}+y), z) = f(\bar{x}+y, z)$

$= f\ (\overline{\bar{x}+y}) + z = \boxed{x\bar{y} + z}$ Ans..

NAND GATE & PROPERTIES

'↑' → This is symbol for NAND

A	B	$A \uparrow B$
0	0	1
0	1	1
1	0	1
1	1	0

→ Properties :-

a) Identity : $A \uparrow A \overset{?}{=} A$

A	$A \uparrow A$
0	1
1	0

$A \uparrow A = \overline{A}$

So it is not following identity.

b) Commutative :

$A \uparrow B = \overline{A \cdot B} = \overline{B \cdot A}$ { since AND is commutative }

$= B \uparrow A$

$\because A \uparrow B = B \uparrow A$, NAND is commutative.

c) Associativity :

$A \uparrow (B \uparrow C) = (A \uparrow B) \uparrow C$ { ? }

For, $A = 0$, L.H.S = 1, R.H.S = $\overline{C}$

So, NAND is not associative.

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NOR GATE & PROPERTIES

A	B	$A \downarrow B = \overline{A+B}$
0	0	1
0	1	0
1	0	0
1	1	0

Properties :-

a) Idempotency : Idempotency is when $A \odot A = A$
$\odot \rightarrow$ any operator.

$A \downarrow A = \overline{(A+A)} = \overline{A}$

So, nor is not idempotent.

b) Commutative : $A \downarrow B = B \downarrow A$

$A \downarrow B = \overline{A+B} = \overline{B+A} = B \downarrow A$

So, nor is commutative.

c) Associative : $A \downarrow (B \downarrow C) = (A \downarrow B) \downarrow C$

For $A = 1$

$1 \downarrow (B \downarrow C) = 1 \downarrow \overline{(B+C)} = 0$ (L.H.S.)

R.H.S $= (1 \downarrow B) \downarrow C = \overline{1+B} \downarrow C = 0 \downarrow C = \overline{C}$

So, NOR is not associative.

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Ex-OR GATE & PROPERTIES

A	B	$A \oplus B$
0	0	0
0	1	1
1	0	1
1	1	0

[∵ If associativity holds true for both 0 and 1 substitution, we need to check all values in truth table].

Properties :-

a) Not Idempotent

b) Commutative

c) Associative

$$\boxed{\text{NOTE : } \overline{B \oplus C} = \bar{B} \oplus C = B \oplus \bar{C}}$$

Ex-NOR Gate & Properties

A	B	$A \odot B$
0	0	1
0	1	0
1	0	0
1	1	1

Properties :-

a) Not Idempotent

b) Commutative

c) Associative

$$\boxed{\text{NOTE : } \overline{A \odot B} = \bar{A} \odot B = A \odot \bar{B}}$$

1) $A \odot B = \bar{A} \oplus B = A \oplus \bar{B}$

2) $A \oplus B = \bar{A} \odot B = A \odot \bar{B}$

PROPERTIES OF EX-OR & EX-NOR

$A \oplus B = 1$ whenever a) number of 1's is odd

b) sum % 2 is 1

$\odot = 1$ when a) number of 0's is even

If n is even:

If # 1's is odd => # 0's is also odd

If # 1's is even => # 0's is also even

So, $\oplus$ and $\odot$ act as complement for even n.

When n is odd $\oplus$ and $\odot$ are equal to each other.

$A \oplus B$

B \ A	0	1
0		1
1	1	

$A \odot B$

B \ A	0	1
0	1	
1		1

$A \oplus B \oplus C$

C \ AB	00	01	11	10
0		1		1
1	1		1	

$A \odot B \odot C$

C \ AB	00	01	11	10
0		1		1
1	1		1	

ExOR ⟶ # minterm == # maxterm and 1's where #1's is od

Ex NOR ⟶ # minterm == # maxterm and 1's where # 0's is even.

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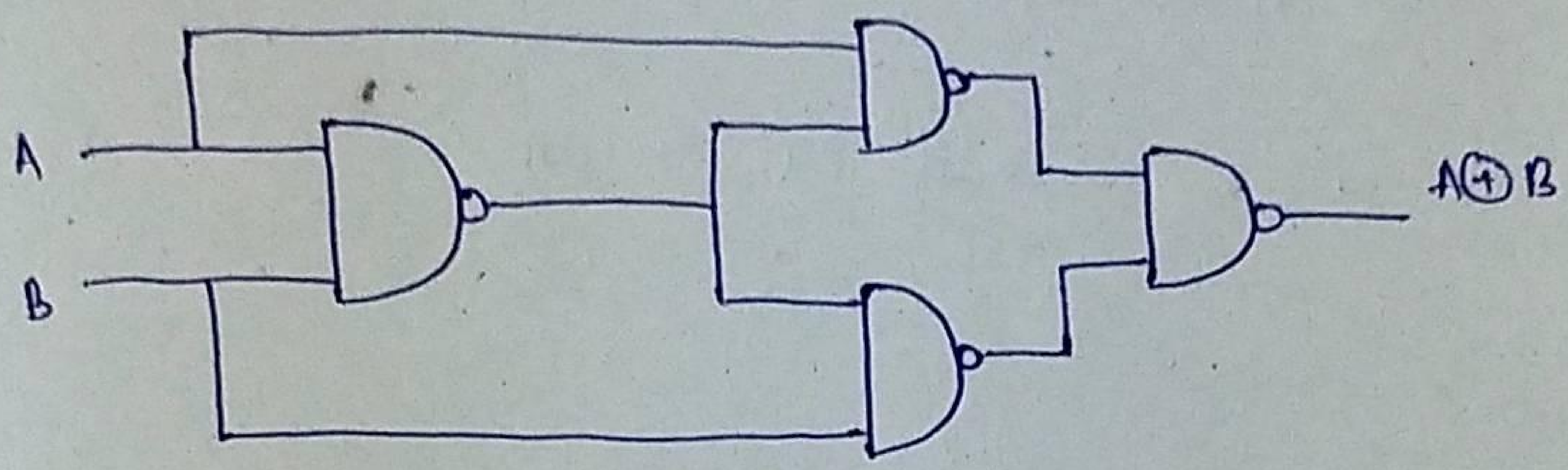
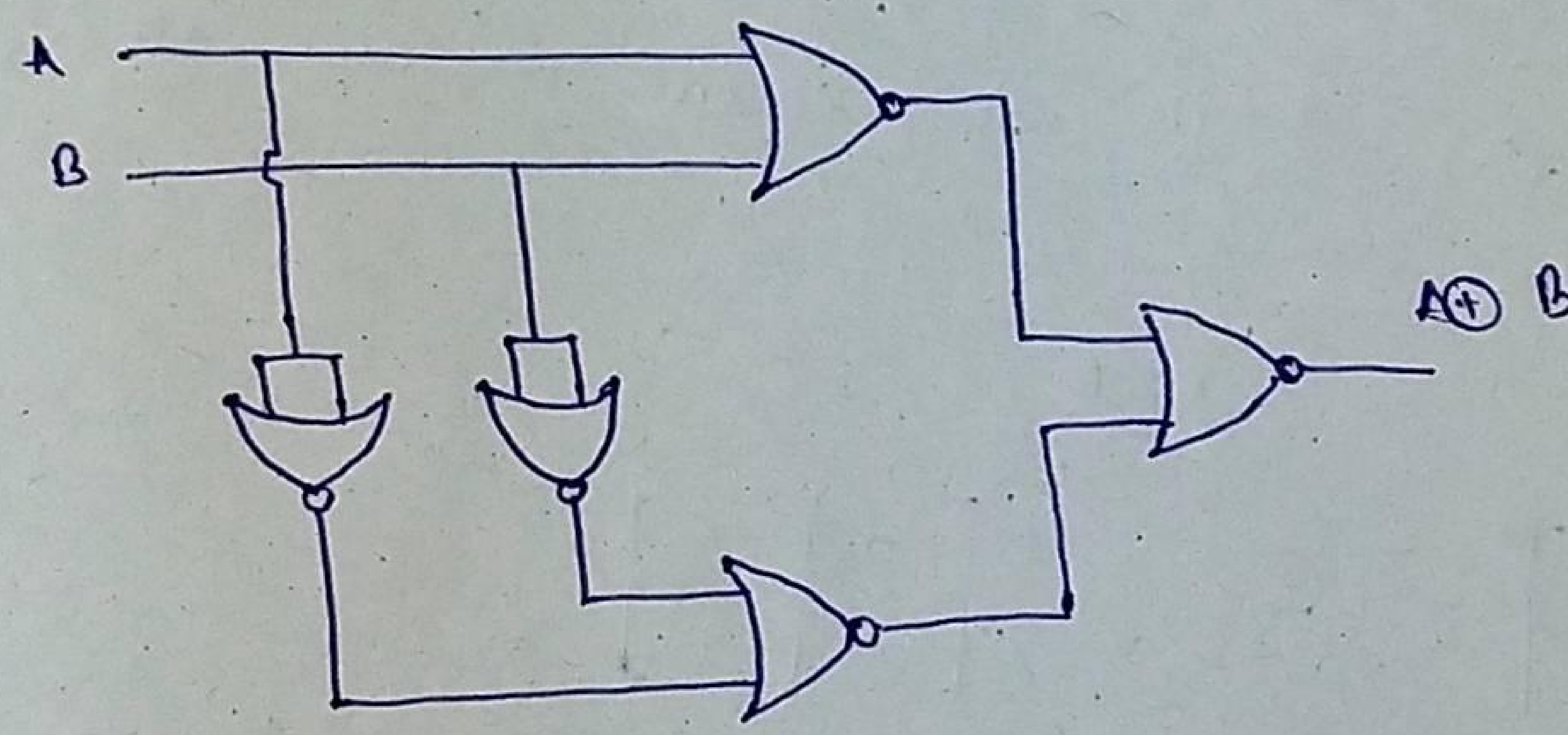
MINIMUM NUMBER OF GATES FOR Ex-OR & Ex-NOR

① How many NAND gates are required for Ex-OR?



So, Number of NAND gates required is 4.

② How many NOR gate are required for Ex-OR? → 5



③ # of NAND gate for Ex-NOR → ~~4~~ 5

④ # of NOR gate for Ex-NOR → ~~5~~ 4

FUNCTIONALLY COMPLETE OPERATIONS

A set of operations is said to be functionally complete (or) universal if and only if every switching function can be expressed by means of operations in it.

→ The set $\{+, \cdot, -\}$ is clearly function complete
(complement)

→ The set $\{+, -\}$ is said to be functionally complete.

→ The set $\{\cdot, -\}$ " " " " " "

NOTE: A set is functionally complete if we can derive a set which is already functionally complete.

EXAMPLES

① $f(A, B, C) = \bar{A} + B\bar{C}$

$f(A, A, A) = \bar{A} + A\bar{A} = \bar{A}$ → complement is derived

$f(f(A,A,A), B, f(B,B,B)) = (\bar{\bar{A}}) + B\bar{\bar{B}} = A + B \cdot B = A+B$

↳ OR is implemented

So, $(+, -)$ is getting derived.

② $f(A, B) = \bar{A} + B$

$f(A, A) = \bar{A} + A = 1$, $f(B, B) = 1$

$f(A, 0) = \bar{A}$ → complement implementation

$f(f(A,0), B) = (\bar{\bar{A}}) + B = A+B$ → Or implemented

$(+, -)$ is getting derived

Whenever a function is taking support from 0 or 1, then it is called partially functionally complete.

③ $f(A, B) = \bar{A} B$

$f(A, A) = \bar{A} A = 0$, $f(B, B) = 0$.

$f(\bar{A}, 1) = \bar{A}$ → complement done.

$f(f(\bar{A}, 1), B) = A B$ → And done.

$(\cdot, -)$ is ~~got~~ derived.

∴ The $f(A, B)$ is partially functionally complete with help of 1)

④ $f(A, B, C) = AB + BC + CA$

In given expression if there is no complement then it is not functionally complete.

⑤ $f(x, y) = \bar{x}y + x\bar{y}$

$f(x, 1) = \bar{x}.1 + x.0 = \bar{x}$ → $(-)$ DONE

$(+)$ or $(\cdot)$ is not implemented

So, f is not FC.

⑥ Any boolean function can be defined with which of following operations.

a) $\oplus$, NOT ✗

b) $\oplus$, 1, OR ✓

c) $\oplus$, 1, NOT ✗

d) $\odot$, 1, NOT

SELF DUAL FUNCTIONS

→ Whenever a function is equal to its dual then it is called self dual.

$f(A, B, C) = AB + BC + CA$

$f_d(A, B, C) = (A+B)(B+C)(C+A) = AB + BC + CA.$

→ A boolean function is dual if :-

a) If it is neutral (no. of minterms = no. of maxterms)

b) The function does not contain two mutually exclusive terms.

Mutually Exclusive pairs → (0, 7), (1, 6), (2, 5), (3, 4)

$f = 2 \times 2 \times 2 \times 2 = 16$

No. of pair $= 2^{n-1}$

→ ① $f(A, B, C) = \sum(0, 2, 3)$ ✗

This function don't contain mutually exclusive value but it is not neutral.

② $f(A, B, C) = \sum(0, 1, 6, 7)$ ✗ → contain mutually exclusive

③ $f(A, B, C) = \sum(0, 1, 2, 4)$ ✓ → Neutral + Non mutually exclusive

④ $f(A, B, C) = \sum(3, 5, 6, 7)$ ✓ → " "

INTRO TO ELECTRONIC GATES

1) Electronic gates generally receive voltage as input and produces voltage as output.

2) The precise value of voltages ~~is~~ are not significant towards determination of logical operation of gates

3) The significant point is that voltages are restricted b/w two limits.

4) Two valued variables may be used to represent these voltages

5) If we associate constant '1' with high voltage and '0' with low voltage, it is called (+)ve logic system and for vice versa it is called (-)ve logic system.

(*) POSITIVE & NEGATIVE LOGIC SYSTEMS ARE DUALS OF EACH OTHER