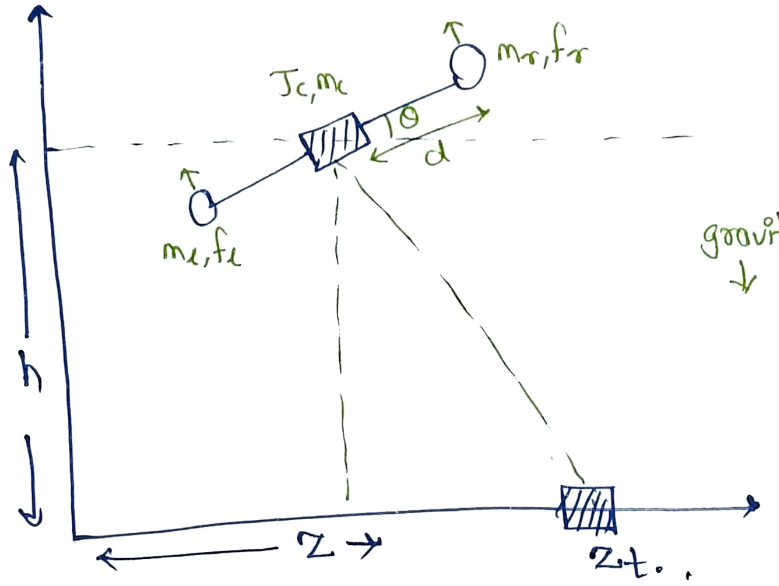


Kinetic Energy

$$P_c = \begin{pmatrix} z_v \\ h \\ \theta \end{pmatrix} \qquad V_c = \dot{P}_c = \begin{pmatrix} \dot{z}_v \\ \dot{h} \\ \dot{\theta} \end{pmatrix} \Rightarrow \begin{pmatrix} \dot{z}_v \\ \dot{h} \\ 0 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ \dot{\theta} \end{pmatrix}$$

$$P_l = \begin{pmatrix} z - d\cos\theta \\ h - d\sin\theta \\ 0 \end{pmatrix} \qquad V_l = \dot{P}_l = \begin{pmatrix} \dot{z} + d\dot{\theta}\sin\theta \\ \dot{h} - d\dot{\theta}\cos\theta \\ 0 \end{pmatrix}$$

$$P_r = \begin{pmatrix} z + d\cos\theta \\ h + d\sin\theta \\ 0 \end{pmatrix} \qquad V_r = \dot{P}_r = \begin{pmatrix} \dot{z} - d\dot{\theta}\sin\theta \\ \dot{h} + d\dot{\theta}\cos\theta \\ 0 \end{pmatrix}$$

$$K.E = \frac{1}{2} m_c V_c^2 + \frac{1}{2} m_l V_l^2 + \frac{1}{2} m_r V_r^2 + \frac{1}{2} J_c \dot{\theta}^2$$

$$= \frac{1}{2} m_c \left(\cancel{(\dot{z} + d\dot{\theta}\sin\theta)^2} + (\dot{z}^2 + \dot{h}^2) \right) + \frac{1}{2} J_c \dot{\theta}^2 + \frac{1}{2}\left((\dot{z} + d\dot{\theta}\sin\theta)^2 + (\dot{h} - d\dot{\theta}\cos\theta)^2 \right) + \frac{1}{2} m_r \left((\dot{z} - d\dot{\theta}\sin\theta)^2 + (\dot{h} + d\dot{\theta}\cos\theta)^2 \right)$$

$$= \frac{1}{2} m_c (\dot{z}^2 + \dot{h}^2) + \frac{1}{2} J_c \dot{\theta}^2 + \frac{1}{2} m_l \left(\dot{z}^2 + d^2\dot{\theta}^2 \sin^2\theta + \dot{h}^2 + d^2\dot{\theta}^2\cos^2\theta + 2\dot{z}\dot{\theta} d\sin\theta - 2\dot{h}\dot{\theta} d\cos\theta \right)$$

$$+ \frac{1}{2} m_r \left(\dot{z}^2 + d^2\dot{\theta}^2 \sin^2\theta - 2\dot{z}\dot{\theta} d\sin\theta + \dot{h}^2 + d^2\dot{\theta}^2\cos^2\theta + 2\dot{h} d\dot{\theta}\cos\theta \right)$$

$$\boxed{K.E. = \frac{1}{2} m_c (\dot{z}^2 + \dot{h}^2) + \frac{1}{2} J_c \dot{\theta}^2 + \frac{1}{2}(m_l + m_r)(\dot{z}^2 + \dot{h}^2) + \frac{1}{2}(m_l + m_r)(d^2\dot{\theta}^2)}$$

$$P.E = P_0 + m_c g h + m_\ell g(h - d\sin\theta) + m_r g(h + d\sin\theta)$$ ②

equation of motion.

$$L = K.E - P.E.$$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot q}\right) - \frac{\partial L(q)}{\partial q} = \tau - B\dot q$$

$$L = \frac{1}{2}(m_c + m_\ell + m_r)(\dot z^2 + \dot h^2) + \left(\frac{1}{2}J_c + \frac{1}{2}(m_\ell + m_r)d^2\right)\dot\theta^2$$
$$- P_0 - m_c g h + m_\ell g(h - d\sin\theta) + m_r g(h + d\sin\theta)$$

$$\cancel{\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot q}\right)} \Rightarrow \cancel{(m_c + m_\ell + m_r)\ddot z} =$$

$$\tau - B\dot q = \begin{pmatrix} -(f_\ell + f_r)\sin\theta - \mu\dot z \\ (f_r + f_\ell\cos\theta)\ \cancel{(m_\ell + m_r + m_c)g} \\ (f_r - f_\ell)d \end{pmatrix}$$

$$(m_c + m_\ell + m_r)\ddot z = -(f_\ell + f_r)\sin\theta - \mu\dot z \quad — ①$$

$$(m_c + m_\ell + m_r)\ddot h + (m_c + m_\ell + m_r)g = (f_\ell + f_r)\cos\theta. \quad — ②$$

$$(J_c + 2(m_\ell + m_r)d^2)\ddot\theta = (f_\ell - f_r)d. \quad — ③$$

here, $m_r = m_\ell$. let. $m_r + m_\ell \rightarrow 2m$.

$$\begin{pmatrix} 2m + m_c & 0 & 0 \\ 0 & 2m + m_c & 0 \\ 0 & 0 & (J_c + 2md^2) \end{pmatrix} \begin{pmatrix} \ddot z \\ \ddot h \\ \ddot\theta \end{pmatrix} = \begin{pmatrix} -(f_\ell + f_r)\sin\theta - \mu\dot z \\ +(f_r + f_\ell)\cos\theta - (2m + m_c)g \\ (f_r - f_\ell)d \end{pmatrix}$$

linearization.

equilibrium.

$$\dot{x} = 0 = \begin{pmatrix} \dot{z} \\ \dot{h} \\ \dot{\theta} \\ \ddot{z} \\ \ddot{h} \\ \ddot{\theta} \end{pmatrix} = 0.$$

$\dot{z} = \dot{h} = \dot{\theta} = 0$

from (1). (2) & (3).

$$-(f_l + f_r)\sin\theta - \mu\dot{z} = 0$$

$$(f_l + f_r)\cos\theta - (2m + m_c)g = 0$$

$$(f_l - f_r)d = 0.$$

- let. $F = (f_l + f_r)$ & $\tau = d(f_r - f_l)$

$$-F\sin\theta - \mu\dot{z} = 0$$

$$F\cos\theta - (2m + m_c)g = 0$$

$$\tau = 0.$$

So.

$$\tau_e = 0, \quad F_e = (2m + m_c)g, \quad \theta_e = n\pi$$

- linearized eqn

$$2\left(m + \frac{m_c}{2}\right)\ddot{\tilde{z}} = -(2m + m_c)g\tilde{\theta} - \mu\dot{\tilde{z}} \quad \text{— (4)}$$

$$2\left(m + \frac{m_c}{2}\right)\ddot{\tilde{h}} = \tilde{F} \quad \text{— (5)}$$

$$\left(I_c + 2md^2\right)\ddot{\tilde{\theta}} = +\tilde{\tau} \quad \text{— (6).}$$

Laplace.

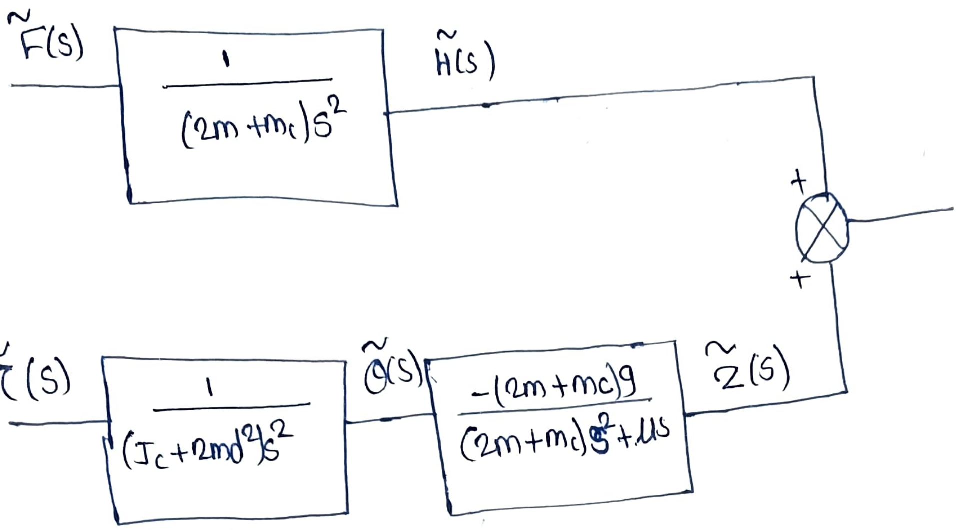
$$2\left(m + \frac{m_c}{2}\right)s^2\tilde{Z}(s) + s\mu\tilde{Z}(s) = -(2m + m_c)g\tilde{\theta}(s)$$

$$2\left(m + \frac{m_c}{2}\right)s^2\tilde{H}(s) = \tilde{F}(s)$$

$$\left(J_c + 2md^2\right)s^2\tilde{\theta}(s) = \tilde{\tau}(s)$$

Transfer function.

$$\boxed{\begin{aligned} \frac{\tilde{z}(s)}{\tilde{\theta}(s)} &= \frac{-(2m+m_c)g}{(2m+m_c)s^2+\mu s} \\ \frac{\tilde{\theta}(s)}{\tilde{\tau}(s)} &= \frac{+1}{(J_c+2md^2)s^2} \\ \frac{\tilde{H}(s)}{\tilde{F}(s)} &= \frac{1}{(2m+m_c)s^2} \end{aligned}}$$



state space model.

longitudinal States

$$\tilde{x}_{lon} = (\tilde{h}, \dot{\tilde{h}})^T \qquad \tilde{u}_{lon} = F.$$

$$\dot{\tilde{x}}_{lon} = A\tilde{x}_{lon} + B\tilde{u}_{lon}$$

$$\tilde{y}_{lon} = C\tilde{x}_{lon} + D\tilde{u}_{lon}$$

$$\dot{\tilde{x}} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} \tilde{h} \\ \dot{\tilde{h}} \end{pmatrix} + \begin{pmatrix} 0 \\ \frac{1}{(2m+m_c)} \end{pmatrix}\tilde{u}$$

$$\tilde{y}_{lon} = \begin{pmatrix} 1 & 0 \end{pmatrix}\begin{pmatrix} \tilde{h} \\ \dot{\tilde{h}} \end{pmatrix}$$

lateral

$$\tilde{x}_{lat} = (\tilde{z}, \tilde{\theta}, \dot{\tilde{z}}, \dot{\tilde{h}})^T \qquad \tilde{y}_{lat} = (\tilde{z}, \tilde{\theta})^T \qquad \tilde{u}_{lat} = \tilde{\tau}$$

$$\dot{\tilde{x}}_{lat} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -g & \frac{\mu}{2m+m_c} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \tilde{z} \\ \tilde{\theta} \\ \dot{\tilde{z}} \\ \dot{\tilde{\theta}} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{J_c + 2md^2} \end{pmatrix} \tilde{u}$$

$$\tilde{y}_{lat} = (\tilde{z}, \tilde{\theta})^T$$

$$\tilde{y}_{lat} = \begin{pmatrix} \tilde{z} \\ \tilde{\theta} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \tilde{z} \\ \tilde{\theta} \\ \dot{\tilde{z}} \\ \dot{\tilde{\theta}} \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tilde{u}$$

Homework 7

Open loop poles

$$\frac{\tilde{H}(s)}{\tilde{F}(s)} = \frac{1}{(2m + m_c)s^2}$$

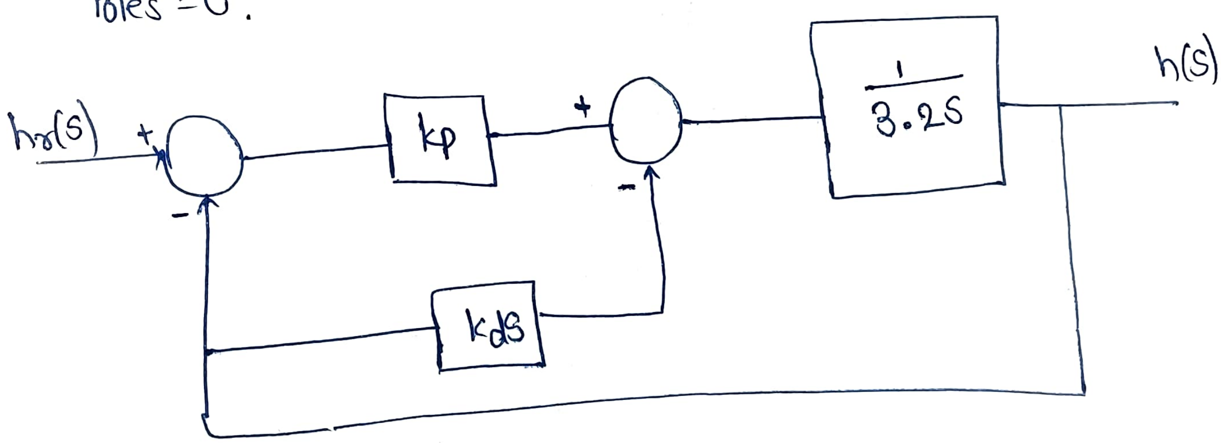
$m = m_r + m_e = 0.3 + 0.3 = 0.6$

$m_c = 2kg.$

$$= \frac{1}{(0.6 \times 2 + 2)s^2}$$

$$= \frac{1}{3.2 s^2}$$

Poles $= 0$.



$$h(s) = \left(\frac{1}{3.2s^2}\right)\left(k_p\,(h_r(s) - h(s)\right) - k_D s\, h(s)$$

$$3.2\, s^2\, h(s) = k_p\, h_r(s) - k_p h(s) - k_D s\, h(s)$$

$$3.2 s^2\, h(s) + k_p h(s) + k_D s\,(h(s)) = k_p\, h_r(s)$$

$$(3.2\, s^2 + k_p + k_D s)\, h(s) = k_p\, h_r(s)$$

$$h(s) = \frac{k_p}{(3.2 s^2 + k_p + k_D s)}\; h_r(s)$$

$$h(s) = \frac{k_p/3.2}{\left(s^2 + \frac{k_p}{3.2} + \frac{k_D}{3.2} s\right)}\, h_r(s)$$

$$\Delta_{cl} = s^2 + s/3.2\; k_D + \frac{1}{3.2} k_p$$

Poles. $P_1 = -0.2$ & $P_2 = -0.3$.

$$\Delta_{cl}^{d} = (s+0.2)\,(s+0.3)$$

$$= s^2 + 0.5s + 0.6.$$

Comparing. Δ_{cl} & Δ_{cl}^{d}

$$s^2 + \frac{s}{3.2} k_D + \frac{1}{3.2} k_p = s^2 + 0.5\,s + 0.6$$

$$k_D = 0.5 \times 3.2 = 1.6.$$

$$k_p = 0.6 \times 3.2 = 1.92.$$

$$\boxed{k_D = 1.6 \;\;\&\;\; k_p = 1.92.}$$

Homework 8.

(a) $\zeta = 0.707 \qquad t_r = 8sec$

$$\omega_n = \frac{2.2}{t_r} = \frac{2.2}{8} = 0.275$$

desired characteristic polynomial.

$$\Delta_{cl}^{d} = s^2 + 2\zeta\omega_n s + \omega_n^2 = s^2 + 0.38885\,s + 0.075625$$

$\hookrightarrow P_1 = -0.0205$ & $P_2 = -0.36831$.

&

$$\Delta_{cl} = s^2 + \frac{s}{3.2}k_D + \frac{1}{3.2}k_P.$$

by Comparison.

$$k_{Dh} = 0.38885 \times 3.2 = 1.244 \qquad \& \qquad k_{Ph} = 0.075625 \times 3.2 = 0.241984$$

(b)

$t_{r\theta} = 0.8$ & $\zeta_\theta = 0.707 \Rightarrow \omega_{n\theta} = \cancel{0.275}\ 2.75.$

lateral transfer funⁿ..

$$\frac{\tilde{Z}(s)}{\tilde{\tau}(s)} = \frac{\tilde{Z}(s)}{\tilde{\theta}(s)}\frac{\tilde{\theta}(s)}{\tilde{\tau}(s)}$$

$$= \frac{-(2m+m_c)g}{(J_c+2md^2)s^2((2m+m_c)s^2+\mu s)}$$

- inner loop.

$$\frac{\tilde{\theta}(s)}{\tilde{\tau}(s)} = \frac{1}{(J_c+2md^2)s^2} \Rightarrow \frac{1}{(0.0084+2\times0.6\times(0.35^2))s^2} = \frac{1}{6.435s^2}$$

desired characteristic polynomial.

$$\Delta_{cl}^{d} = s^2 + 2\zeta\omega_n s + \omega_n^2$$
$$= s^2 + 3.8885s + 7.562$$

$$\theta(s) = \frac{1}{6.435s^2}\left(k_P(\theta_r(s) - \theta(s)) - k_D s(\theta(s))\right)$$

$$6.435(s^2)\theta(s) = k_P\theta_r(s) - k_P\theta(s) - k_D s\,\theta(s)$$

$$(6.435(s^2) + sk_D + k_P)\,\theta(s) = k_P\theta_r(s).$$

$$\theta(s) = \frac{k_P}{(6.435 s^2 + s k_D + k_P)} \theta_r(s)$$

$$= \frac{k_P/6.435}{\left(s^2 + \frac{s}{6.435} k_D + \frac{k_P}{6.435}\right)} \theta_r(s).$$

$$\Delta_{c1} = s^2 + \frac{s}{6.435} k_D + \frac{k_P}{6.435}$$

Comparing Δ_{c1}^d & Δ_{cl}

$k_D = 3.8885 \times 6.435 = 25.$

$k_P = 7.562 \times 6.435$

$\boxed{k_{D\theta} = 25}$ & $\boxed{k_{P\theta} = 48.664}$

ⓒ <u>DC gain.</u>

$$\theta(s) = \frac{\frac{+k_P}{6.435}}{\left(s^2 + \frac{s}{6.435} k_D + \frac{k_P}{6.435}\right)} \theta_r(s)$$

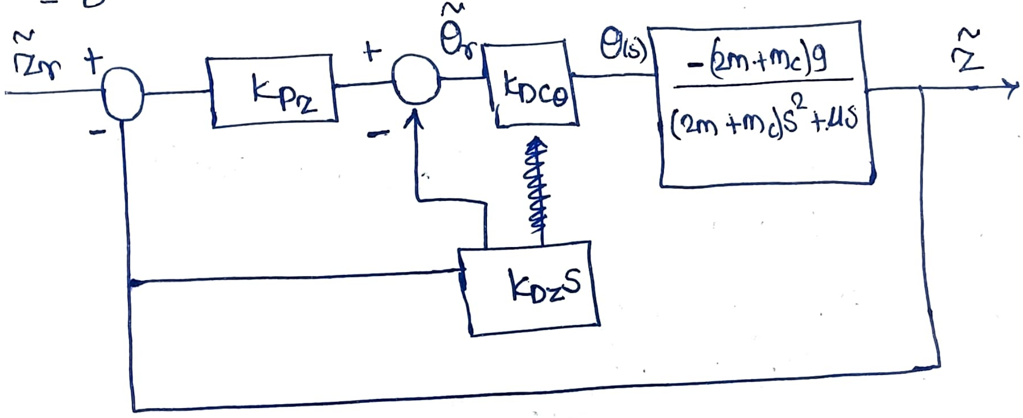
now

$$\lim_{s \to 0} \theta(s) = 1.$$

$k_{DC\theta} = 1.$

ⓓ $t_{rz} = 10 t_{r\theta} = 8$

$\zeta_z = 0.707. \Rightarrow \omega_{nz} = \frac{2.2}{8} = 0.275.$



Since. $k_{DC\theta} = 1.$

$$\tilde{z}(s) = \left(\frac{-(2m+m_c)g}{(2m+m_c)s^2+\mu s}\right)\left(k_{Pz}\left(z_r(s) - z(s)\right) - k_{Dz}s(z(s))\right)$$

&

$$\Delta_{cl}^d = s^2 + 2\zeta\omega_n s + \omega_n^2$$
$$= s^2 + 0.38885 s + 0.075625.$$

(9)

$$Z(s) = \left(\frac{-g}{s^2 + \frac{\mu}{(2m+m_c)} s}\right)\left(k_{pz}\left(Z_r(s) - Z(s)\right)\right) - k_{Dz} s\left(Z(s)\right)$$

$g = 9.81\, m/s^2$ $\qquad m_c = 2\,kg.$

$m_l + m_r = 2m = 2 \times 0.3\,kg = 0.6\,kg$

$\mu = 0.13\, kg/s$

$$\left(\frac{-g}{s^2 + \frac{\mu}{(2m+m_c)} s}\right) \Rightarrow \left(\frac{-9.81}{s^2 + \frac{0.13}{(2\times 0.6+2)} s}\right) = \left(\frac{-9.81}{s^2 + 0.0406\, s}\right)$$

$$Z(s) = \left(\frac{-9.81}{s^2 + 0.0406 s}\right)\left(k_{pz}\left(Z_r(s) - Z(s)\right) - k_{Dz} s\left(Z(s)\right)\right)$$

$$-\left(\frac{s^2 + 0.0406 s}{9.81}\right) Z(s) + k_{pz} Z(s) + k_{Dz} s\left(Z(s)\right) = k_{pz}\left(Z_r(s)\right)$$

$$\left(s^2 + (-9.81\, k_{Dz} + 0.0406) s - 9.81 k_{pz}\right) Z(s) = -9.81\, k_{pz}\left(Z_r(s)\right)$$

$$Z(s) = \frac{-9.81\, k_{pz}\left(Z_r(s)\right)}{\left(s^2 + (0.0406 - 9.81\, k_{Dz}) s - 9.81 k_{pz}\right)}$$

$$\Delta_{cl} = s^2 + (0.0406 - 9.81 k_{Dz}) s - 9.81\, k_{pz}.$$

- Comparing Δ_{cl} & Δ_{cl}^d.

$$(0.0406 - 9.81\, k_{Dz}) = 0.38885.$$

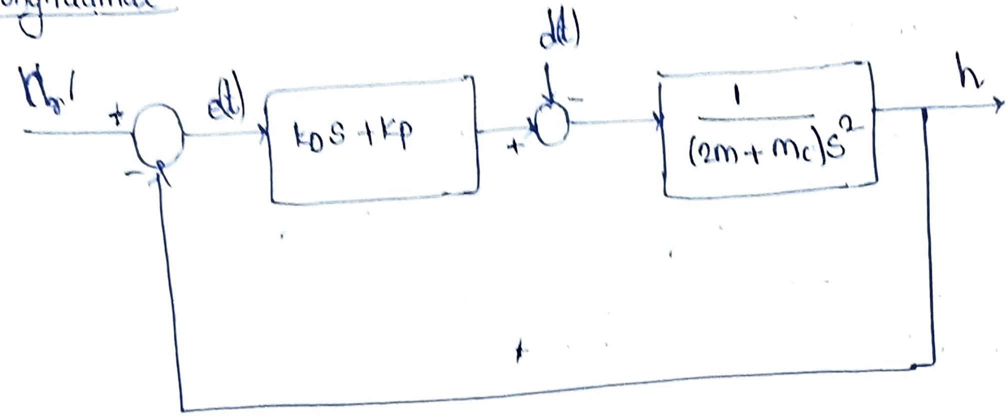
$$-9.81\, k_{Dz} = 0.3479$$

$$\boxed{k_{Dz} = -0.035.}$$

$$-9.81\, k_{pz} = 0.075625$$

$$\boxed{k_{pz} = -0.007708}$$

① longitudinal



$$P(s)C(s) = \left(\frac{1}{(2m+m_c)s^2}\right)(k_D s + k_P) \Rightarrow M_a = \lim_{s\to 0} P(s)C(s)\, s^2$$

System have two poles at zero. so system is type 2.

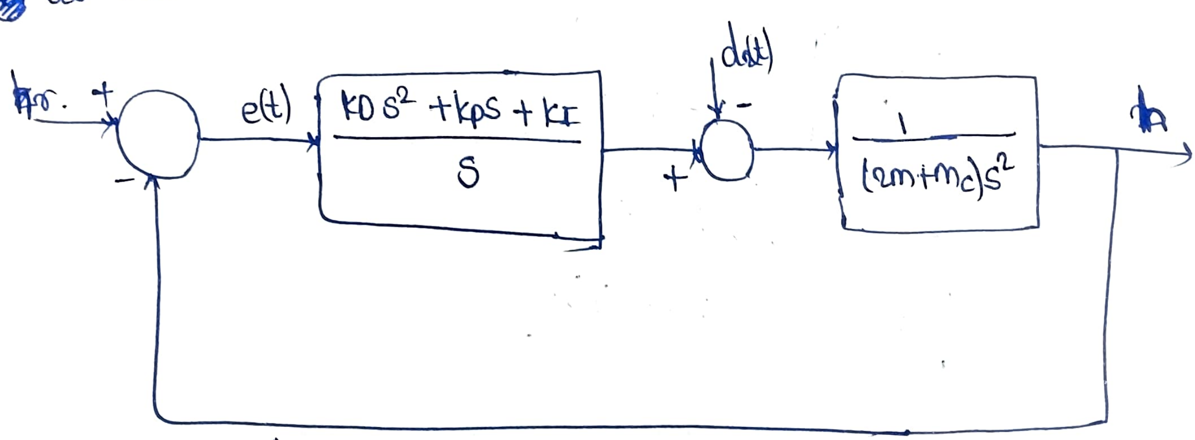
so.

System type	step $\left(\frac{1}{s}\right)$	ramp $\left(\frac{1}{s^2}\right)$	Parabola $\left(\frac{1}{s^3}\right)$
2	0	0	$\frac{1}{M_a}$

So.

$$\lim_{t\to\infty} e(t) = \frac{1}{M_a} = \frac{1}{\lim_{s\to 0}\left(\frac{k_D s + k_P}{(2m+m_c)\cancel{s^2}}\right)\cancel{s^2}} = \frac{2m+m_c}{k_P}$$

② for PID.



$$P(s)C(s) = \left(\frac{k_D s^2 + k_P s + k_I}{(2m+m_c)s^3}\right)$$

now system have three poles at zero so system type is. type 3

System type	step $(\frac{1}{s})$	Ramp $(\frac{1}{s^2})$	Parabola
3	0	0	0

type with respect to input distourbance.

PD

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0}\left(\frac{P(s)}{1+P(s)C(s)}\right)\left(\frac{1}{s^q}\right)$$

$$= \lim_{s\to 0}\left(\frac{\frac{1}{(2m+m_c)s^2}}{1+\frac{1}{(2m+m_c)s^2}(k_Ds+k_P)}\right)\left(\frac{1}{s^q}\right)$$

$$= \lim_{s\to 0}\left(\frac{1}{(2m+m_c)s^2+k_Ds+k_P}\right)\left(\frac{1}{s^q}\right)$$

$$= \lim_{s\to 0}\left(\frac{1}{k_P}\right)\left(\frac{1}{s^q}\right)$$

so. system type is zero with r.t input distourbance.

PID

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0}\left(\frac{\frac{1}{(2m+m_c)s^2}}{1+\frac{1}{(2m+m_c)s^2}\left(\frac{k_Ds^2+k_Ps+k_I}{s}\right)}\right)\left(\frac{1}{s^q}\right)$$

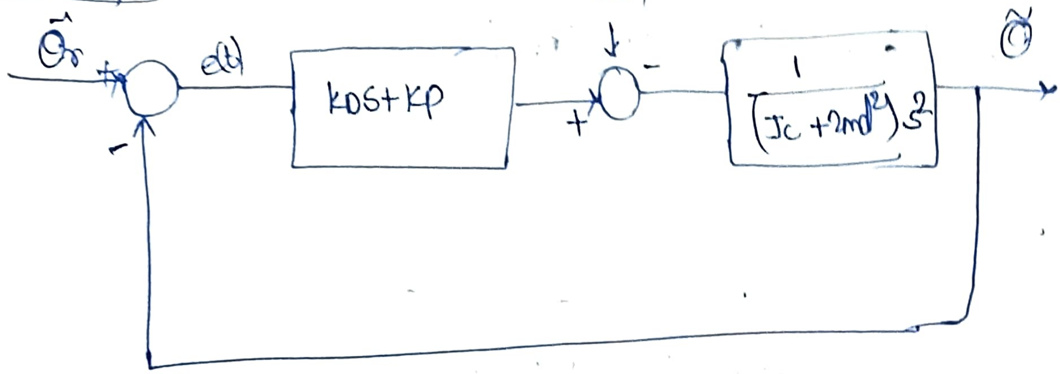
$$= \lim_{s\to 0}\left(\frac{s}{(2m+m_c)s^2+k_Ds^2+k_Ps+k_I}\right)\left(\frac{1}{s^q}\right)$$

$$= \lim_{s\to 0}\left(\frac{1}{(2m+m_c+k_D)s^2+k_Ps+k_I}\right)\frac{1}{s^{q-1}}$$

so. system type is 1. for PID.

(b) inner loop.

PD



$$P(s)C(s) = \frac{1}{(J_c + 2md^2)s^2}\left(k_D s + k_P\right)$$

system type is 2.

system type	step$\left(\frac{1}{s}\right)$	ramp$\left(\frac{1}{s^2}\right)$	Parabola$\left(\frac{1}{s^3}\right)$
2	0	0	$\frac{1}{M_a}$

Parabola

$$\lim_{t\to\infty} e(t) = \frac{1}{M_a} = \frac{1}{\lim_{s\to 0}\left(\frac{k_D s + k_P}{(J_c + 2md^2)s^2}\right)\times s^2} = \left(\frac{J_c + 2md^2}{k_P}\right)$$

input disturbance.

PD

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0}\left(\frac{P(s)}{1 + P(s)C(s)}\right)\frac{1}{s^q}$$

$$= \lim_{s\to 0}\left(\frac{\frac{1}{(J_c + 2md^2)s^2}}{1 + \frac{k_D s + k_P}{(J_c + 2md^2)s^2}}\right)\frac{1}{s^q}$$

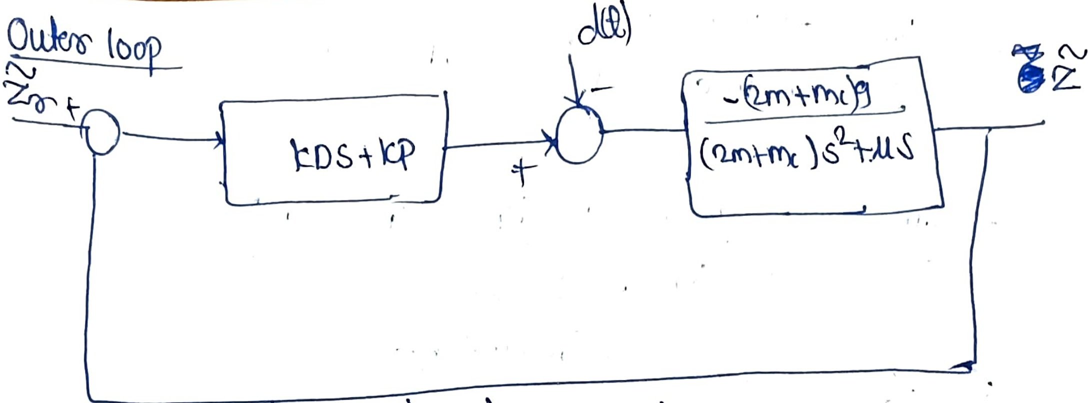
$$= \lim_{s\to 0}\left(\frac{1}{(J_c + 2md^2)s^2 + k_D s + k_P}\right)\left(\frac{1}{s^q}\right)$$

$$= \lim_{s\to 0}\left(\frac{1}{s^q}\right)$$

System type is zero.

13

ⓒ Outer loop



$$P(s)\,C(s) = \left(\frac{-(2m+m_c)g}{(2m+m_c)s^2+\mu s}\right)(k_D s + k_P)$$

System type is 1.

System type	step $\left(\frac{1}{s}\right)$	ramp $\left(\frac{1}{s^2}\right)$	Parabola $\left(\frac{1}{s^3}\right)$
1	0	$\frac{1}{M_v}$	∞

$$\lim_{t\to\infty} e(t) = \frac{1}{M_v} = \frac{1}{\lim\limits_{s\to 0}\left(\frac{-(2m+m_c)g}{(2m+m_c)s^2+\mu s}\right)(k_D s+k_P)(s)}$$

$$= \frac{1}{\lim\limits_{s\to 0}\left(\frac{-(2m+m_c)g}{(2m+m_c)s+\mu}\right)(k_D s+k_P)}$$

$$= \frac{\mu}{-(2m+m_c)g\,k_P}.$$

for PID.

$$P(s)\,C(s) = \left(\frac{-(2m+m_c)g}{(2m+m_c)s^2+\mu s}\right)\frac{(k_D s^2 + k_P s + k_I)}{s}$$

System type is. 2..

system type	Step $\left(\frac{1}{s}\right)$	ramp $\left(\frac{1}{s^2}\right)$	Parabola $\left(\frac{1}{s^3}\right)$
2	0	0	$\frac{1}{M_a}$

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0}\left(\frac{-(2m+m_c)g}{(2m+m_c)s^2+\mu s}\right)\left(\frac{k_D s^2+k_P s+k_I}{s}\right)$$

$$= \lim_{s\to 0}\left(\frac{-(2m+m_c)g\,(k_D s^2+k_P s+k_I)}{s^{\cancel{2}}((2m+m_c)s+\mu)}\right)\cancel{s}$$

$$= \cancel{\infty} \rightarrow \cancel{\infty}\ \ \frac{-(2m+m_c)gK_I}{\mu}.$$

input disturbance. for both PD & PID.

PDI

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0}\left(\frac{P(s)}{1+P(s)C(s)}\right)\frac{1}{s^q}.$$

$$= \lim_{s\to 0}\left(\frac{\dfrac{-(2m+m_c)g}{((2m+m_c)s^2+\mu s)}}{1+\left(\dfrac{-(2m+m_c)g(k_D s^2+k_P s+K_I)}{s^2((2m+m_c)s+\mu)}\right)}\right)\left(\frac{1}{s^q}\right)$$

$$= \lim_{s\to 0}\left(\frac{-(2m+m_c)gs}{s^2((2m+m_c)s+\mu)-(2m+m_c)g(k_D s^2+k_P s+K_I)}\right)\left(\frac{1}{s^q}\right)$$

$$= \lim_{s\to 0}\left(\frac{-\cancel{(2m+m_c)g}}{-\cancel{(2m+m_c)g}K_I}\right)\frac{1}{s^{(q-1)}}$$

$$= \lim_{s\to 0}\left(\frac{1}{K_I}\right)\frac{1}{s^{q-1}}$$

System type is 1.

PD

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0}\left(\frac{P(s)}{1+P(s)C(s)}\right)\frac{1}{s^q}$$

$$= \lim_{s\to 0}\left(\frac{\dfrac{-(2m+m_c)g}{((2m+m_c)s^2+\mu s)}}{1+\dfrac{-(2m+m_c)g}{s((2m+m_c)s+\mu)}(k_D s+k_P)}\right)\left(\frac{1}{s^q}\right)$$

$$= \lim_{s \to 0} \left(\frac{-(2m+mc)g}{s((2m+mc)s+\mu) - (2m+mc)g(kDs+kp)} \right) \frac{1}{s^q}$$

$$= \lim_{s \to 0} \left(\frac{1}{kp} \right) \frac{1}{s^q}$$

system type is zero.

State space equations

longitudinal

$$x_{lon} = \left(\tilde{h} \quad \dot{\tilde{h}}\right)^T$$

$$x_{lon} = \begin{pmatrix} \tilde{h} \\ \dot{\tilde{h}} \end{pmatrix} = \begin{pmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{pmatrix} = \begin{pmatrix} \tilde{x}_2 \\ \dfrac{1}{(2m+m_c)} \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{pmatrix} + \begin{pmatrix} 0 \\ \dfrac{1}{(2m+m_c)} \end{pmatrix} \tilde{u}$$

$$\tilde{y}_{lon} = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{pmatrix} = \begin{pmatrix} \tilde{h} \\ \dot{\tilde{h}} \end{pmatrix}$$

lateral

$$\tilde{x}_{lat} = \left(\tilde{z} \quad \tilde{\theta} \quad \dot{\tilde{z}} \quad \dot{\tilde{\theta}}\right)^T$$

$$\tilde{x}_{lat} = \begin{pmatrix} \tilde{z} \\ \tilde{\theta} \\ \dot{\tilde{z}} \\ \dot{\tilde{\theta}} \end{pmatrix} = \begin{pmatrix} \tilde{x}_3 \\ \tilde{x}_4 \\ \tilde{x}_5 \\ \tilde{x}_6 \end{pmatrix} = \begin{pmatrix} \tilde{x}_5 \\ \tilde{x}_6 \\ -g \cancel{\tilde{x}_4} \tilde{\theta} - \dfrac{\mu}{(m_c+2m)} \dot{\tilde{z}} \\ \tilde{u} / 2\left(m + \frac{m_c}{2}\right) \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -g & -\dfrac{\mu}{(m_c+2m)} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \tilde{x}_3 \\ \tilde{x}_4 \\ \tilde{x}_5 \\ \tilde{x}_6 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ \dfrac{1}{(2m+m_c)} \end{pmatrix} \tilde{u}$$

$$\tilde{y}_{lat} = \begin{pmatrix} \tilde{z} \\ \tilde{\theta} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \tilde{x}_3 \\ \tilde{x}_4 \\ \tilde{x}_5 \\ \tilde{x}_6 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \end{pmatrix} \tilde{z}$$

Controllability matrix:

$y_r = (1 \quad 0\;0 \quad 0)x \longrightarrow$ reference output.

$C_{AB} = [B \quad AB \quad A^2B \quad A^3B]$

$$= \begin{pmatrix} 0 & 0 & 0 & -119.7801 \\ 0 & 12.21 & 0 & 0 \\ 0 & 0 & -119.7801 & 5.9890 \\ 12.21 & 0 & 0 & 0 \end{pmatrix}.$$

$\det(C_{AB}) = -2.1390 \times 10^6 \neq 0$. so. its Controllable.

For. logi.

$$C_{AB\,long} = \begin{pmatrix} 0 & 0.3846 \\ 0.3846 & 0 \end{pmatrix}$$

$\det(C_{AB\,long}) = -0.1479 \neq 0$. so. Controllable.

Openloop characteristic polynomial

$\Delta_{ol}(s) = \det(sI - A)$

$$= \begin{pmatrix} s & 0 & 0 & 0 \\ 0 & s & 0 & 0 \\ 0 & 0 & s & 0 \\ 0 & 0 & 0 & s \end{pmatrix} - \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -g & \frac{-\mu}{(2m+m_c)} & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$= \det \begin{pmatrix} s & 0 & -1 & 0 \\ 0 & s & 0 & -1 \\ 0 & +9.81 & (s+0.05) & 0 \\ 0 & 0 & 0 & s \end{pmatrix}$$

$\Delta_{ol}(s) = \left(s^3(20s+1)\right)/20 = \cancel{20}\, s^4 + \frac{1}{20}s^3 = s^4 + 0.05s^3$

which says

$a_A = (0.05, \quad 0 \quad 0 \quad 0)$

$$A_h = \begin{pmatrix} 1 & 0.05 & 0 & 0 \\ 0 & 1 & 0.05 & 0 \\ 0 & 0 & 1 & 0.05 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

desire close loop polynomial

$$\Delta_{cl}^{d}(s) = (s^2 + 2\xi_\theta \omega_{n\theta} s + \omega_{n\theta}^2)(s^2 + 2\xi_z \omega_{nz} s + \omega_{nz}^2)$$

$\cancel{(s^2 +}$

$\xi_z = 0.707 \qquad \omega_{nz} = 0.275.$

$\xi_h = 0.707 \qquad \omega_{nh} = 0.275.$

$\xi_\theta = 0.707 \qquad \omega_{n\theta} = 2.75.$

$$\Delta_{cl}^{d}(s) = (s^2 + 3.8885 s + 7.562)(s^2 + 0.38885 s + 0.075625)$$

$$= (s^4 + 0.38885 s^3 + 0.075625 s^2 + 3.8885 s^3 + 1.5120 s^2 + 0.29406 s + 7.562 s^2 + 2.9421 s + 0.57193)$$

$$= (s^4 + 4.27735 s^3 + 9.849625 s^2 + 3.23616 s + 0.57193)$$

$$\alpha = (4.27735, 9.849625, 3.23616, 0.57193)$$

gain.

$$K = (\alpha - a_A) A_A^{-1} C_{AB}^{-1} = (-0.0048 \cdot 0.7894 \quad -0.0230 \quad 0.3462)$$

$$\boxed{k_{r2} = \frac{-1}{C_r (A - BK)^{-1} B} = -1/209.3351.} = -4.777 \times 10^{-3}$$

Lognitudinal

Controlability matrix.

$$y_r = (1 \quad 0)\, x.$$

$$A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 0 \\ 0.3846 \end{pmatrix}$$

$$C_{AB} = \begin{pmatrix} 0 & 0.3846 \\ 0.3846 & 0 \end{pmatrix}$$

$$\det(C_{AB}) = -0.1479. \quad \text{so Controlable.}$$

Open loop characteristic.

$$\Delta_{OL}(s) = \det\left(\begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}\right)$$

$$= \det\begin{pmatrix} s & -1 \\ 0 & s-1 \end{pmatrix}$$

$$= s(s-1) + 0$$

$$= s^2 - s.$$

$$\Delta_{ol}(s) = s^2 - s.$$

$$a_A = (-1 \quad 0)$$

$$A_A = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$$

desire loop.

$$\Delta_{cl}^{d}(s) = s^2 + 2\xi\omega_n s + \omega_n^2 = s^2 + 0.38885s + 0.075625.$$

$$\alpha = (0.38885\,,\, 0.075625).$$

gain

$$k = (\alpha - a_A)\, A_A^{-1}\, C_{AB}^{-1} = (3.8078 \quad 3.6112)$$

$$k_{rA} = \frac{-1}{C_r(A-BK)^{-1}B} = 3.8080.$$

$k_{rz} = -4.777 \times 10^{-3}$ & $k_{rh} = 3.8080.$

Home work 12

$C_r = \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix}$

$$A_1 = \begin{pmatrix} A & 0 \\ -C_r & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & -9.81 & -0.05 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$B_1 = \begin{pmatrix} B \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 12.21 \\ 0 \end{pmatrix}$$

$$C_{A_1B_1} = \begin{pmatrix} 0 & 0 & 0 & -119.7801 & 5.9890 \\ 0 & 12.2100 & 0 & 0 & 0 \\ 0 & 0 & -119.7801 & 5.9890 & -0.2995 \\ 12.21 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 119.7801 \end{pmatrix}$$

$\det(C_{A_1B_1}) = -2.5620 \times 10^{8}.$

Open loop characteristic.

$$\Delta_{o1}(s) = \det(sI - A_1).$$

$$= \det\begin{pmatrix} s & 0 & -1 & 0 & 0 \\ 0 & s & 0 & -1 & 0 \\ 0 & +9.81 & (s+0.05) & 0 & 0 \\ 0 & 0 & 0 & s & 0 \\ \cancel{0}+1 & 0 & 0 & 0 & s \end{pmatrix}$$

$$= s^4(s + 0.05)$$

$$= s^5 + 0.05\, s^4$$

$$a_{A_1} = (0.05,\ 0\ \ 0\ \ 0\ \ 0)$$

$$A_{A_1} = \begin{pmatrix} 1 & 0.05 & 0 & 0 & 0 \\ 0 & 1 & 0.05 & 0 & 0 \\ 0 & 0 & 1 & 0.05 & 0 \\ 0 & 0 & 0 & 1 & 0.05 \\ 0 & 0 & 0 & 0 & 1 \end{pmatrix}$$

Close loop poles are.

$$P_{1,2} = -1.9618 \pm 2.1319i$$

$$P_{3,4} = -0.1768 \pm 0.1921i$$

let $P_I = -10$.

$$\Delta^d_{cl}(s) = s^5 + 14.2772 s^4 + 52.61212 s^3 + \cancel{101.7 s^4}\ 101.7274 s^2 + 32.92643\, s + 5.7211$$

$$\alpha = (14.2772 \quad 52.61212 \quad 101.7274 \quad 32.92643 \quad 5.7211)$$

Augmented gain.

$k_J = (\alpha - a_{A_1}) A_{A_1}^{-1} C_{A_1B_1}^{-1}$

$= (-0.2749 \quad 4.2514 \quad -0.8276 \quad 1.1652 \quad 0.0478)$

$k = k_1(1:4) = (-0.2749 \quad 4.2514 \quad -0.8276 \quad 1.1652)$

$k_I = k_1(5) = 0.0478.$

logitudinal

$C_r = (1\,0)$

$$A_1 = \begin{pmatrix} A & 0 \\ -C_r & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}$$

$$B_1 = \begin{pmatrix} B \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0.3846 \\ 0 \end{pmatrix}$$

$$C_{A_1B_1} = \begin{pmatrix} 0 & 0.3846 & 0 \\ 0.3846 & 0 & 0 \\ 0 & 0 & -0.3846 \end{pmatrix}$$

$\det(C_{A_1B_1}) = 0.0569.$

Open loop

$\Delta_{o1}(s) = \det(sI - A)$

$$= \det\left(\begin{pmatrix} s & 0 & 0 \\ 0 & s & 0 \\ 0 & 0 & s \end{pmatrix} - \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix}\right)$$

$$= \det\begin{pmatrix} s & -1 & 0 \\ 0 & s & 0 \\ 1 & 0 & s \end{pmatrix} = s^3.$$

$$\alpha_{A_1} = \begin{pmatrix} 0 & 0 & 0 \end{pmatrix}$$

$$A_{A_1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Close loop.

$P_{i_{12}} = -0.1944 \pm 0.1945i$.

let. $P_I = \cancel{-10}$. -0.1

$$\Delta_{cl}^{d}(s) = s^3 + 0.4888s^2 + 0.114501s + 0.00756$$

$$\alpha = \begin{pmatrix} 0.4888 & 0.114501 & 0.00756 \end{pmatrix}$$

Augmented gain

$$k_1 = (\alpha - a_{A_1}) A_{A_1}^{-1} C_{A_1 B_1}^{-1}$$

$$= \begin{pmatrix} 0.4888 & 0.114501 & 0.00756 \end{pmatrix} \left(A_{A_1}^{-1} C_{A_1 B_1}^{-1} \right)$$

$$k_1 = \begin{pmatrix} 0.2977 & 1.2709 & -0.0197 \end{pmatrix}$$

$$k = k_1(1\ 2) = \begin{pmatrix} 0.2977 & 1.2709 \end{pmatrix}$$

$$k_I = k_1(3) = -0.0197.$$

logitudinal

$$A = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 0 \\ 0.3846 \end{pmatrix} \qquad C = (1\,0)$$

$$O_{A,C} = \begin{pmatrix} C \\ CA \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \neq 0 \quad \text{so. observable.}$$

Open characteristic.

- $\Delta_{ol}(s) = \det(sI - A) = \det\begin{pmatrix} s & -1 \\ 0 & s-1 \end{pmatrix}$

$$= s^2 - s.$$

$$a = (-1 \quad 0)$$

$$A = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$$

desire loop

- $\Delta^{d}_{cl}(s) = s^2 + 0.38885s + 0.075625$

$\beta = (0.38885 ; 0.075625)$

gain.

$$L = O_{AC}^{-1} (A^T)^{-1} (\beta - a)^T$$

$$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \left((0.38885, 0.075625) - (-1, 0) \right)^T$$

$$= \begin{pmatrix} \cancel{1.94425} \\ \cancel{1.890625} \end{pmatrix} = \begin{pmatrix} 1.38885 \\ 1.464475 \end{pmatrix}$$

The observer is.

$$\boxed{\dot{\hat{x}} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \hat{x} + \begin{pmatrix} 0 \\ 0.3846 \end{pmatrix} u + \begin{pmatrix} 1.38885 \\ 1.464475 \end{pmatrix} \left(y - (1 \ 0)\hat{x}\right)}$$

Lateral

$$A = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -9.81 & 0.05 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \qquad B = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 12.21 \end{pmatrix}$$

$$C = \begin{pmatrix} 1 & 0 & 0 & 0 \end{pmatrix}$$

$$O_{AC} = \begin{pmatrix} C \\ CA \\ CA^2 \\ CA^3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -9.8100 & 0.05 & 0 \\ 0 & -0.4905 & 0.0025 & -9.81 \end{pmatrix}$$

Open loop

$$\Delta_{OL}(s) = \det(sI - A)$$

$$= \det \begin{pmatrix} s & 0 & -1 & 0 \\ 0 & s & 0 & -1 \\ 0 & 9.81 & (s+0.05) & 0 \\ 0 & 0 & 0 & s \end{pmatrix}$$

$$= s^4 + 0.05s^3$$

$$\alpha = \begin{pmatrix} 0.05 & 0 & 0 & 0 \end{pmatrix}$$

$$A_A = \begin{pmatrix} 1 & 0.05 & 0 & 0 \\ 0 & 1 & 0.05 & 0 \\ 0 & 0 & 1 & 0.05 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

desire close loop.

$$A^{d}_{cl}(s) = (s^4 + 4.27735 s^3 + 9.849625 s^3 + 8.236168 + 0.572193)$$

$$\beta = (4.27735, 9.849625, 3.23616, 0.572193)$$

gain

$$L = O_{Ac}^{-1} (A^T)^{-1} (\beta - a)^T$$

$$= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.0051 & -0.1019 & 0 \\ 0 & 1.00 & 0 & 0 \\ 0 & 0 & 0.0051 & -0.1019 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 & 0 \\ -0.050 & 1 & 0 & 0 \\ 0.0025 & -0.050 & 1 & 0 \\ -0.001 & 0.0025 & -0.05 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 4.22735 \\ 9.849625 \\ 8.23616 \\ 0.572193 \end{pmatrix}$$

$$= \begin{pmatrix} 4.2273 \\ -0.2316 \\ +9.6383 \\ -0.0303 \end{pmatrix}$$

The observer is.

$$\boxed{\dot{\hat{x}} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & -9.81 & 0.05 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \hat{x} + \begin{pmatrix} 0 \\ 0 \\ 0 \\ 12.21 \end{pmatrix} u + \begin{pmatrix} 4.2273 \\ -0.2316 \\ 9.6383 \\ -0.0303 \end{pmatrix} \left(y - \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix}^T \hat{x} \right)}$$