Scalable Positional Analysis for Studying Evolution of Nodes in Networks

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Position and Role

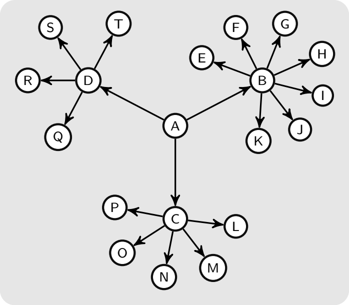
- Position and Roles
  - **Position** refers to set of individuals (actors or nodes) who are embedded similarly in the network
  - **Role** is the pattern of relation that exists between actors or different positions
- Positional Analysis
  - Partition actors (nodes) in the network into discrete subgroups based on some kind of structural similarity
  - Structural equivalence, Automorphic equivalence, Regular Equivalence are few such equivalence relations from social sciences; RolX from Henderson et al.
  - Equitable partitions is a similar notion from graph isomorphism literature

Why Positional Analysis?

- A very intuitive way of understanding interactions in networks
- Nodes having similar structural signature tend to have similar behaviour
- Important tool in the analysis of social networks [1]
- Blockmodels - To facilitate further processing on social networks by obtaining a reduced representation
- Position plays important role in evolution of networks
- Mining “roles” from network data easy for humans, difficult to automate!
- Need new methods to do this in a scalable fashion

[1] S. Wasserman and K. Faust, Social Network Analysis: methods and applications. Cambridge University Press, 1994

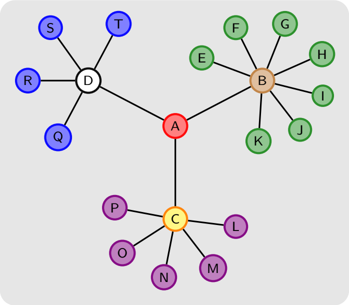
Teacher - Teaching Assistant (TA) - Student Network



Example: Teacher - TA - Student Network

- Possible **positions/blocks** of actors (nodes) in the current Example:
  - Position/Block 1 (Teacher): {A}
  - Position/Block 2 (TAs): {B,C,D}
  - Position/Block 3 (Students): {E,F,G,...,R,S,T}

Equitable Partition [1] on Teacher - TA - Student N/w

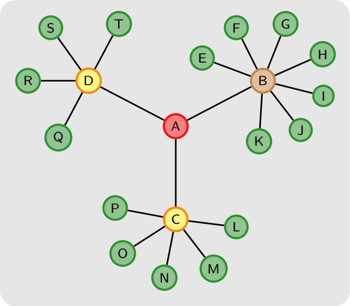


Equitable Partition of Teacher - TA - Student N/w

- For 2 actors (nodes) to be at **same position/block** under Equitable partition equivalence relation
  - They need to be connected to other positions with **exactly the same degree!**
- Equitable Partitioning leads to **7 positions/blocks** of the Teacher - TA - Student N/w:
  - {{A},{B},{C},{D},{E,F,G,H,I,J,K},{L,M,N,O,P},{Q,R,S,T}}

[1] B. D. McKay, "Practical graph isomorphism," Congressus Numerantium, vol. 30, pp. 45-87, 1981

ϵ - Equitable Partition [2] on Teacher - TA - Student N/w



1 - Equitable Partition of Teacher - TA - Student N/w ($\epsilon = 1$)

- For 2 actors (nodes) to be at **same position/block** under the ϵ - Equitable partition equivalence relation
    - They get a **freedom of ϵ degrees** in number of connections to every other position/block
- ϵ - Equitable Partitioning with $\epsilon = 1$ leads to **4 positions/blocks** of the Teacher - TA - Student N/w:
    - {{A},{B},{C,D},{E,F,G,I,J,K,L,M,N,O,P,Q,R,S,T}}

[2] Kate and Ravindran, 2009

Advantages of ϵ-Equitable Partition

- Two nodes are equivalent if the strengths of connections to corresponding blocks is within ϵ of each other
  - Can consider a zero strength connection equivalent to an ϵ strength connection
- Advantages:
  - More inclusive
  - ϵ of zero yields an equitable partition
  - Corresponds to intuitive notions
- Problem:
  - Uniqueness?

Fast ϵ-Equitable Partition Algorithm

- New algorithm with better heuristics to find ϵEP
- Based on modification of McKay's Equitable Partition [3] Algorithm
- Running time complexity of proposed algorithm is
  - $O(n^2 \log n)$ for *Sparse Graphs*
  - $O(n^3)$ for *Dense Graphs*
  - Efficient in Practice
  - Complexity of ϵEP Algorithm from [2] is $O(n^3)$ for all class of graphs

Fast ϵ-Equitable Partition Algorithm

Algorithm 1 Fast ϵ-Equitable Partition

Input: graph G, *ordered unit* partition π, epsilon ϵ
Output: ϵ-equitable partition π

active = indices(π)
while (active $\neq \phi$) **and** (π is **not** $discrete$) **do**
  $idx = min$(active)
  active = active $\setminus \{idx\}$
  $f(u) = deg_{\mathrm{G}}(u, \pi[idx])\ \forall u \in V$
  $\pi' = \mathrm{SPLIT}(\pi, f, \epsilon)$
  active = active $\cup$ [*ordered* indices of newly *split* cells from π', while replacing (*in place*) the indices from π which were *split*]
  $\pi = \pi'$
end while

Scalable ϵ-Equitable Partition Algorithm

- Each iteration of the Fast ϵEP Algorithm translates nicely to MapReduce [5] paradigm
  - Implemented the most computationally intensive step of the ϵEP algorithm, computation of function f , as a $\textsc{Map}$ operation. f maps every vertex $u \in V$ to its degree to current active cell C_a
  - Algorithm is *iterative* in nature! Frameworks introduce high job setup overhead times across the iterative MapReduce steps
  - Implemented a bare minimum MapReduce framework using open source tools
  - Execution time empirical studies on random power-law graphs for the proposed algorithm show encouraging results

[5] J. Dean and S. Ghemawat, MapReduce: Simplified Data Processing on Large Clusters, Communications of the ACM, 51 (2008), pp. 107113.

Scalable ϵ-Equitable Partition Algorithm

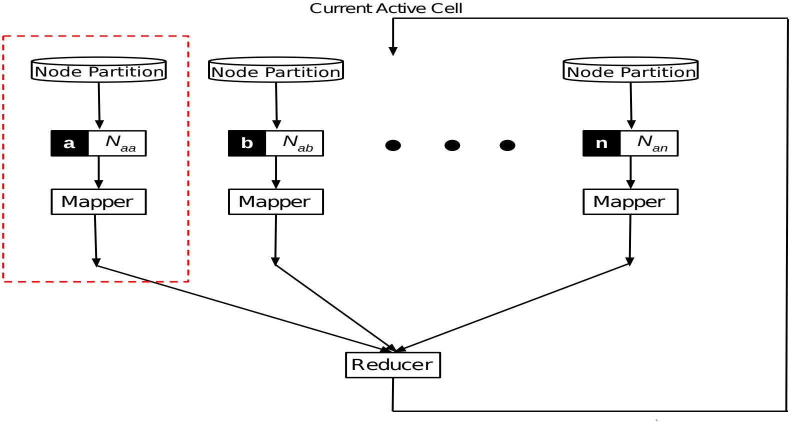
Algorithm 2 MapReduce step of the Parallel ϵ-Equitable Partition

class MAPPER
    method INITIALIZE()
        $C_a \leftarrow$ *Current Active Cell*
    method MAP(id n, vertex N)
        $d \leftarrow |N.AdjacencyList \cap C_a|$
        EMIT(id n, value d)

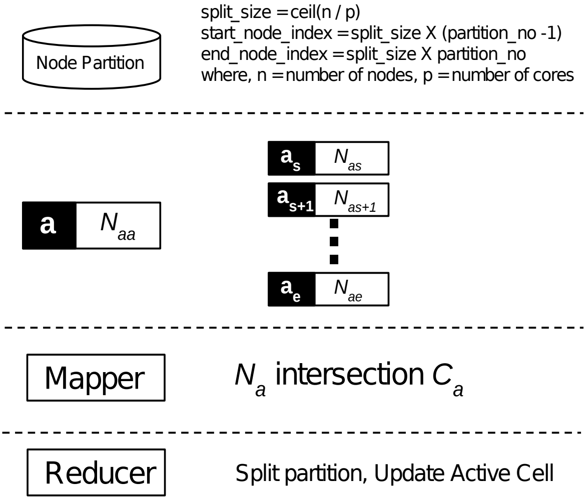
---

class REDUCER
    method REDUCE()
        SPLIT(π, f, ϵ)
        UPDATE(*active*)
        UPDATE(π)

Scalable ϵ-Equitable Partition MapReduce



Scalable ϵ-Equitable Partition MapReduce Step



Scalable ϵEP Algorithm: Computational Aspects

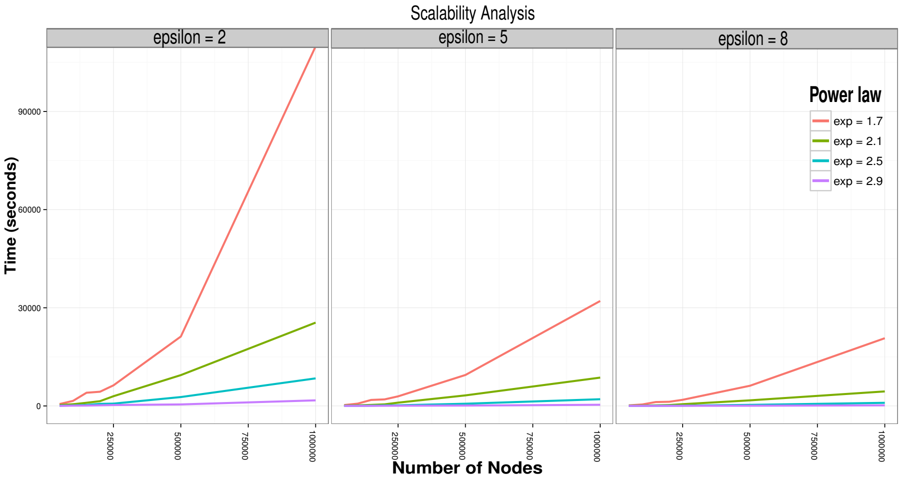
- Complexity bound for sparse graphs is $O(\frac{n^2}{p} \log n)$, where p is the number of cores
- Total additional overhead $i \times c$, data copy cost c for i iterations
- Data copy cost c is small and constant in average, since we copy only the active cell data C_a. Node data is cached on each compute node, hence doesn't require copy overhead
- i is proportional to size of the graph G, inversely proportional to ϵ

γ	2.9	2.5	2.1
Complexity	$O(n)$	$O(n \log n)$	$O(n^2)$

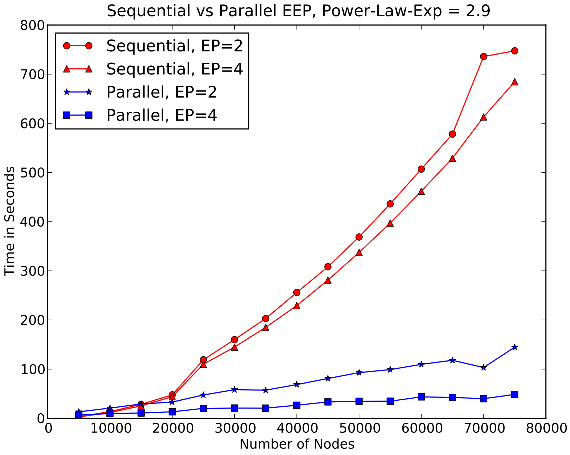
Table: Curve Fitting Results, $\epsilon = 5$

- For $\gamma = 2.1$, difference between sum squared residual for the curves $n \log n$ and n^2 was negligible

Scalability Analysis



EEP: Sequential vs Parallel on Sparse Graphs, $\gamma = 2.9$



Node Evolution Studies

- Datasets Used - Facebook (New Orleans Regional Network) [6] and Flickr [7]

Table: Facebook Dataset Details

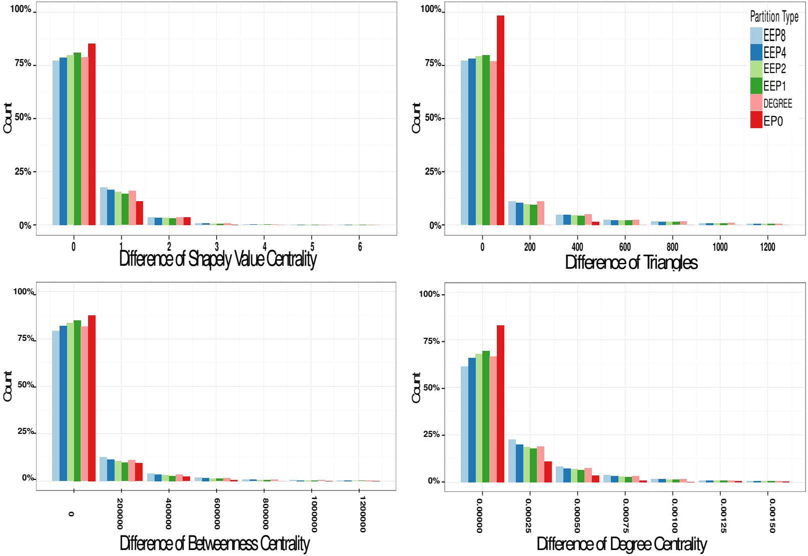
Graph	Vertices	Edges	Upto Date
1	15273	80005	2007-06-22
2	31432	218292	2008-04-07
3	61096	614796	2009-01-22

Table: Flickr Dataset Details

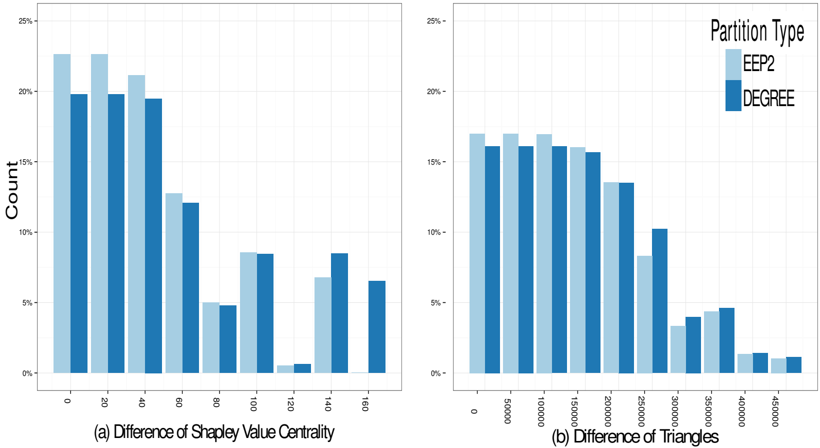
Graph	Vertices	Edges	Upto Date
1	1277145	6042808	2006-12-03
2	1856431	10301742	2007-05-19

[6] B. Viswanath, A. Mislove, M. Cha, and K. P. Gummadi, On the Evolution of User Interaction in Facebook, in Proceedings of the 2nd ACM workshop on Online Social Networks, ACM, 2009.
[7] A. Mislove, H. S. Koppula, K. P. Gummadi, P. Druschel, and B. Bhattacharjee, Growth of the Flickr Social Network, in Proceedings of the first workshop on Online Social Networks, ACM, 2008, pp. 2530.

Results: Co-Evolving node pairs for Facebook Graph $G_1 \to G_3$



Results: Co-Evolving node pairs for Flickr Graph



Results: Evolution of Positions

Epsilon	0	1	2	3	4	5	6	7	8	d*
$G1 \rightarrow G2$	59.59	66.19	76.60	83.00	86.57	89.43	91.29	92.88	94.18	86.93
$G1 \rightarrow G3$	54.11	57.17	69.33	76.61	80.85	84.37	86.60	88.95	90.72	79.42
$G2 \rightarrow G3$	56.88	67.18	76.80	82.12	85.55	87.99	89.87	91.48	92.93	78.11

Table: Percentage of ϵEP overlap using the Partition Similarity Score for time evolving graphs of the Facebook Network. ϵ varied from 0 to 8, $\epsilon = 0$ corresponds to an equitable partition. d* denotes the partition based on degree.

$$sim(\pi_t, \pi_{t+\delta t}) = \frac{1}{2}\left[\left(\frac{N - |\pi_t \cap \pi_{t+\delta t}|}{N - |\pi_t|}\right) + \left(\frac{N - |\pi_t \cap \pi_{t+\delta t}|}{N - |\pi_{t+\delta t}|}\right)\right] \quad (1)$$

Conclusion and Future Work

- Conclusions
  - Proposed a new algorithm with better heuristics to find ϵ-equitable partition of a graph
  - Proposed a scalable version using MapReduce paradigm
- Future Scope of Work
  - Explore the implied advantage of our Parallel ϵEP Algorithm to find the *coarsest equitable partition* of very large graphs for an $\epsilon = 0$.
  - Explore algorithms for positional analysis of very large graphs using vertex-centric computation paradigms

Thank you!

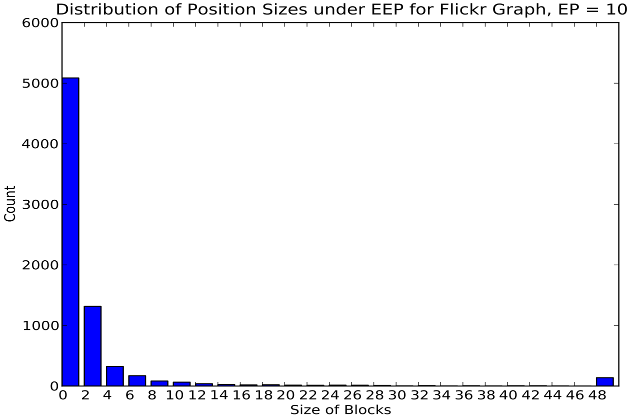
Thank you!

Split function for Fast ϵEP Algorithm

Algorithm 3 Split Function

```
split(π, f, ϵ)
idx = 0
for each currentCell in π do
    sortedCell = SORT(currentCell) using f as the comparison key
    currentDegree = f(sortedCell[0])
    for each vertex in sortedCell do
        if (f(vertex) − currentDegree) ≤ ϵ then
            Add vertex to cell π_s[idx]
        else
            currentDegree = f(vertex)
            idx = idx + 1
            Add vertex to cell π_s[idx]
        end if
    end for
    idx = idx + 1
end for
return(π_s)
```

Results: Distribution of Block Size, Flickr Graph EP $= 10$



Results: Distribution of Block Size, Flickr EP = 2