

《《《 CDS/CAPF 》》》

VIRRAAT 2.0

2024

Algebra

Mathematics

Lecture - 02

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TOPICS *to be covered*

- 1 Questions
- 2 Different type of que
- 3

If $x = \frac{\sqrt{5}+\sqrt{1}}{\sqrt{5}-\sqrt{1}}$ and $y = \frac{\sqrt{5}-\sqrt{1}}{\sqrt{5}+\sqrt{1}}$ then find the value of $\frac{x^2-xy+y^2}{x^2+xy+y^2}$?

$$x+y = \frac{2[5+1]}{5-1} = \frac{2 \times 6}{4} = 3$$

$$x+y = 3$$

$$xy = 1 = \frac{(3)^2 - 3 \times 1}{(3)^2 - 1}$$

$$= \frac{9-3}{9-1} = \frac{6}{8} = \frac{3}{4} = 0.75$$

$$\frac{x^2+y^2-xy}{x^2+y^2+xy}$$

$$\Rightarrow \frac{(x+y)^2 - 3xy}{(x+y)^2 - xy}$$

$$P = \frac{x+y}{x-y}$$

$$Q = \frac{x-y}{x+y}$$

$$P+Q = \frac{2[x^2+y^2]}{[x^2-y^2]}$$

If $a = \frac{\sqrt{3}+\sqrt{1}}{\sqrt{3}-\sqrt{1}}$ and $b = \frac{\sqrt{3}-\sqrt{1}}{\sqrt{3}+\sqrt{1}}$ then find the value of $\frac{a}{b} \times \frac{b}{a} \Rightarrow \frac{a^2+b^2}{ab}$

$$ab = 1$$

$$a+b = \frac{2[3+1]}{[3-1]} = 4$$

$$a+b = 4$$

$$\Rightarrow \frac{4^2 - 2 \times 1}{1}$$

$$\Rightarrow \frac{16-2}{1}$$

$$\Rightarrow \underline{14}$$

$$a^2+b^2 \begin{matrix} \swarrow (a+b) \\ \searrow ab \end{matrix}$$

$$\underline{a^2+b^2 = (a+b)^2 - 2ab}$$



If $a = \frac{\sqrt{3}+\sqrt{1}}{\sqrt{3}-\sqrt{1}}$ and $b = \frac{\sqrt{3}-\sqrt{1}}{\sqrt{3}+\sqrt{1}}$ then find the value of $\frac{a^2}{b} + \frac{b^2}{a}$?

$$\frac{a^3+b^3}{ab} = \frac{64-3 \times 1 \times 4}{1}$$

$$ab = 1$$

$$a \times b = \frac{\cancel{\sqrt{3}}+1}{\cancel{\sqrt{3}}} \times \frac{\cancel{\sqrt{3}}-1}{\cancel{\sqrt{3}+1}} = 1$$

$$= 64 - 12$$

$$= \underline{52}$$

$$a+b = \frac{2[3+1]}{2-1}$$

$$a^3+b^3 = \underline{(a+b)^3 - 3ab(a+b)}$$

$$a+b = 4$$

If $a = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$ and $b = \frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$ then find the value of $a^2 + b^2 - 3ab$?

$$ab = 1$$

$$a+b = \frac{2[3+2]}{3-2}$$

$$a+b = 2 \times 5 = \underline{10}$$

↓

$$a^2 + b^2 + 2ab - 3ab - 2ab$$

$$\underline{(a+b)^2 - 5ab}$$

$$\Rightarrow (10)^2 - 5 \times 1$$

$$\Rightarrow 100 - 5 \Rightarrow \underline{95}$$

If $P = \frac{X+Y}{X-Y}$ and $Q = \frac{X-Y}{X+Y}$ than what is the value of $P-Q$

$$P-Q = \frac{(x+y)}{(x-y)} - \frac{(x-y)}{(x+y)}$$

$$(a+b)^2 - (a-b)^2 = \underline{\underline{4ab}}$$

$$= \frac{(x+y)^2 - (x-y)^2}{(x+y)(x-y)}$$

$$= \frac{4xy}{x^2 - y^2}$$

$$P-Q = \frac{4xy}{x^2 - y^2}$$

If $\frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}-\sqrt{3}} - \frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}} = 2\sqrt{P}$ then find the value of P ?

$$\frac{4 \times \sqrt{5} \times \sqrt{3}}{5-3}$$

$$\frac{2 \sqrt{15}}{2}$$

$$2\sqrt{15} = 2\sqrt{P}$$

$$P=15$$

If $\frac{7+\sqrt{5}}{7-\sqrt{5}} - \frac{7-\sqrt{5}}{7+\sqrt{5}} = \frac{7\sqrt{5}}{11}a + b$ then find the value of $\frac{a+b}{a-b}$? $\Rightarrow \frac{1+0}{1-0} = 2$ ✓

$$\frac{4 \times 7 \times \sqrt{5}}{49-5}$$

$$\frac{7\sqrt{5}}{11} = \frac{7\sqrt{5}}{11}a + b$$

$$\frac{4 \times 7 \times \sqrt{5}}{44}$$

$$\frac{7\sqrt{5}}{11} + 0 = \frac{7\sqrt{5}}{11}a + b$$

$$a=1$$

$$b=0$$



If $x = 2 + \sqrt{3}$ and $y = 2 - \sqrt{3}$, then find the value of $\frac{x^2+y^2}{x^3+y^3}$?

$$x+y = 2+\sqrt{3}+2-\sqrt{3}$$

$$x+y = 4$$

$$\begin{aligned} xy &= (2+\sqrt{3})(2-\sqrt{3}) = 1 \\ &= 4-3 = 1 \end{aligned}$$

$$\Rightarrow \frac{4^2 - 2 \times 1}{4^3 - 3 \times 1 \times 4}$$

$$\Rightarrow \frac{16-2}{64-12}$$

$$= \frac{14}{52} = \frac{7}{26}$$

$$= 7/26$$

$$x^2+y^2 = (x+y)^2 - 2xy$$

$$x^3+y^3 = (x+y)^3 - 3xy(x+y)$$

$$(a+b)(a-b) = a^2 - b^2$$



If $x = \left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)$ and $y = \left(\sqrt{3} + \frac{1}{\sqrt{3}}\right)$, then find the value of $\frac{x^2}{y} \times \frac{y^2}{x}$?

$$x+y = \sqrt{3} - \frac{1}{\sqrt{3}} + \sqrt{3} + \frac{1}{\sqrt{3}} = 2\sqrt{3}$$

$$\frac{x^3+y^3}{xy} \Rightarrow \frac{8\sqrt{3} \times 3}{8} = 3\sqrt{3}$$

$$xy = (3) - \left(\frac{1}{3}\right)$$

$$xy = \frac{9-1}{3} = \frac{8}{3}$$

$$xy = \left(\sqrt{3} - \frac{1}{\sqrt{3}}\right)\left(\sqrt{3} + \frac{1}{\sqrt{3}}\right) = (\sqrt{3})^2 - \left(\frac{1}{\sqrt{3}}\right)^2 = 3 - \frac{1}{3} = \frac{8}{3}$$

$$\begin{aligned} x^3+y^3 &= (x+y)^3 - 3xy(x+y) \\ &= (2\sqrt{3})^3 - 3 \times \frac{8}{3} \times 2\sqrt{3} \\ &= 24\sqrt{3} - 16\sqrt{3} \\ &= 8\sqrt{3} \end{aligned}$$

If $x + \frac{1}{x} = 4$, find the value of $\frac{x^4 + \frac{1}{x^2}}{x^2 - 2x + 1}$?

$$\Rightarrow \frac{x \left[x^3 + \frac{1}{x^3} \right]}{x \left[x - 2 + \frac{1}{x} \right]} \Rightarrow \frac{\left(x^3 + \frac{1}{x^3} \right)}{\left(x + \frac{1}{x} - 2 \right)}$$

$$\Rightarrow \underline{\underline{26}}$$

$$\begin{aligned} x^3 + \frac{1}{x^3} &= 4^3 - 3 \times 4 \\ &= 64 - 12 = 52 \end{aligned}$$

$$= \frac{52}{4 - 2}$$

$$= \frac{\cancel{52}}{\cancel{2}} = \underline{\underline{26}}$$



If $x + \frac{1}{x} = 6$, find the value of $\frac{x^4 + \frac{1}{x^2}}{x^2 - 2x + 1}$?

$\frac{x^3 + \frac{1}{x^3}}{x - 2 + \frac{1}{x}} = \frac{x^3 + \frac{1}{x^3}}{(x + \frac{1}{x}) - 2}$

$$\begin{aligned} x^3 + \frac{1}{x^3} &= 6^3 - 3 \times 6 \\ &= 216 - 18 \\ &= \underline{198} \end{aligned}$$

$$= \frac{198}{6 - 2}$$

$$= \frac{198}{4} = \frac{99}{2}$$

$$= \frac{99}{2} = \underline{\underline{49.5}}$$



$$a + \frac{1}{a} = ?$$

If $a^4 + \frac{1}{a^4} = 1$, then find the value of $a^{49} + \frac{1}{a^{49}}$?

$$\Rightarrow \left(a + \frac{1}{a}\right)$$

$$\Rightarrow \frac{a \times a^{48}}{a \times a^{48}} + \frac{1}{a \times a^{48}}$$

$$\Rightarrow \frac{a \times [a^{12}]^4}{a \times [a^{12}]^4} + \frac{1}{a \times [a^{12}]^4}$$

$$\Rightarrow \frac{a \times 1 + \frac{1}{a \times 1}}{a \times 1} = a + \frac{1}{a}$$

$$x + \frac{1}{x} = 1$$

$$[x^3 = -1]$$

$$\boxed{\begin{aligned} x + \frac{1}{x} &= 1 \\ \underline{\underline{x^3 &= -1}} \end{aligned}}$$

$$\underline{\underline{a^4 = x}}$$

$$[a^4]^3 = -1$$

$$\underline{\underline{a^{12} = -1}}$$

$$a^4 + \frac{1}{a^4} + 2 = 1 + 2$$

$$\left(a^2 + \frac{1}{a^2}\right)^2 = 3$$

$$a^2 + \frac{1}{a^2} = \sqrt{3}$$

$$a^2 + \frac{1}{a^2} + 2 = \sqrt{3} + 2$$

$$\left(a + \frac{1}{a}\right)^2 = [\sqrt{3} + 2]$$

$$a + \frac{1}{a} = [\sqrt{3} + 2]^{\frac{1}{2}}$$

Answer



Maths by pramod yadav

If $\underbrace{(x \pm a)^2} + \underbrace{(y \pm b)^2} + \underbrace{(z \pm c)^2} = 0$

From the above equation we can say:

$$\Rightarrow x - a = 0 \Rightarrow x = a \text{ and } \Rightarrow x + a = 0 \Rightarrow x = -a$$

$$\Rightarrow y - b = 0 \Rightarrow y = b \text{ and } \Rightarrow y + b = 0 \Rightarrow y = -b$$

$$\Rightarrow z - c = 0 \Rightarrow z = c \text{ and } \Rightarrow z + c = 0 \Rightarrow z = -c$$



$$2a - 2b + 2c$$

If $a^2 + b^2 + c^2 = 2(a - b + c) - 3$, find the value of $5a + 2b + 7c$?

$$a^2 + b^2 + c^2 = 2a - 2b + 2c - 3$$

$$a = \frac{2}{2} = 1$$

$$b = -\frac{2}{2} = -1$$

$$c = \frac{2}{2} = 1$$

$$a^2 - 2a + b^2 + 2b + c^2 - 2c + 3 = 0$$

$$\underbrace{a^2 - 2a + 1^2} + \underbrace{b^2 + 2b + 1^2} + \underbrace{c^2 - 2c + 1^2} = 0$$

$$(a-1)^2 + (b+1)^2 + (c-1)^2 = 0$$

$$a-1=0$$

$$b+1=0$$

$$c-1=0$$

$$a=1$$

$$b=-1$$

$$\underline{c=1}$$

$$5a + 2b + 7c$$

$$\Rightarrow 5 \times 1 + 2 \times -1 + 7 \times 1$$

$$\Rightarrow 5 - 2 + 7$$

$$\Rightarrow 3 + 7 = 10$$



If $a^2 + b^2 + c^2 + 9 = \underline{4b} - \underline{8a} + \underline{6c} - 20$, find the value of $2a - 4b + c$?

$$a = \frac{-8}{2} = a = -4 \quad c = \frac{6}{2} = 3$$

$$b = \frac{4}{1} = b = 2 \quad c = 3$$

$$a^2 + 8a + b^2 - 4b + c^2 - 6c + 9 + 20 = 0$$

$$a^2 + 8a + 16 + b^2 - 4b + 4 + c^2 - 6c + 9 = 0$$

$$(a+4)^2 + (b-2)^2 + (c-3)^2 = 0$$

$$a = -4 \quad b = 2 \quad c = 3$$

$$\Rightarrow 2a - 4b + c$$

$$\Rightarrow 2(-4) - 4(2) + 3$$

$$= -8 - 8 + 3$$

$$= -16 + 3$$

$$= \underline{\underline{-13}}$$



If $a^2 + b^2 + c^2 = 4b - 8a + 4c - 20$, find the value of $a - b + c$?

$$a = -\frac{8}{2} = -4$$

$$\Rightarrow -4 - \cancel{2} + \cancel{2}$$

$$b = \frac{4}{2} = 2$$

$$\Rightarrow -4$$

$$c = \frac{4}{2} = 2$$



If $a^2 + b^2 + c^2 = 4(a - b + c) - 12$, find the value of $a + b + c$?

$$= 4a - 4b + 4c$$

$$a = \frac{4}{2} = 2$$

$$b = -\frac{4}{2} = -2$$

$$c = \frac{4}{2} = 2$$

$$a + b + c$$

$$\cancel{2} - \cancel{2} + 2 = \underline{\underline{2}}$$

$$(a+b+c)^2 = a^2 + b^2 + c^2 + 2(ab+bc+ca)$$

$$a^2 + b^2 + c^2 - ab - bc - ca \Rightarrow \frac{1}{2} [2(a^2 + b^2 + c^2) - 2(ab + bc + ca)]$$

$$\Rightarrow \frac{1}{2} [2a^2 + 2b^2 + 2c^2 - 2a\underline{b} - 2\underline{b}c - 2ca]$$

$$\Rightarrow \frac{1}{2} [\underbrace{a^2 + b^2 - 2ab}_{(a-b)^2} + \underbrace{b^2 + c^2 - 2bc}_{(b-c)^2} + \underbrace{c^2 + a^2 - 2ca}_{(c-a)^2}]$$

$$a^2 + b^2 + c^2 - ab - bc - ca \Rightarrow \frac{1}{2} [(a-b)^2 + (b-c)^2 + (c-a)^2]$$

$$a^2 + b^2 + c^2 - ab - bc - ca$$

On multiplying by 2 in numerator and denominator

$$= \frac{1}{2} [2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca]$$

$$= \frac{1}{2} [a^2 + b^2 - 2ab + b^2 + c^2 - 2bc + c^2 + a^2 - 2ca]$$

$$a^2 + b^2 + c^2 - ab - bc - ca = \frac{1}{2} [(a - b)^2 + (b - c)^2 + (c - a)^2]$$

If $a = 876$, $b = 874$ and $c = 875$, Find the value of $a^2 + b^2 + c^2 - ab - bc - ca$?

2

$$a^2 + b^2 + c^2 - ab - bc - ca = \frac{1}{2} [(a-b)^2 + (b-c)^2 + (c-a)^2]$$

$$= \frac{1}{2} [2^2 + (-1)^2 + (-1)^2]$$

$$= \frac{1}{2} [4 + 1 + 1]$$

$$= \frac{6}{2} = 3$$



If $a = 91.5$, $b = 100$ and $c = 97.3$, Find the value of $a^2 + b^2 + c^2 - ab - bc - ca$?

$$= \frac{1}{2} [(a-b)^2 + (b-c)^2 + (c-a)^2]$$

$$= \frac{1}{2} [(8.5)^2 + (2.7)^2 + (5.8)^2]$$

$$= \frac{1}{2} \times 113.18$$

$$= \underline{\underline{56.59}}$$

$$\begin{array}{r} 111 \\ 72.25 \\ 7.29 \\ 33.64 \\ \hline 113.18 \end{array}$$

$$\begin{array}{l} 55^2 = 3025 \\ 56^2 = 3136 \\ 57^2 = 3249 \\ 58^2 = 3364 \end{array}$$

$$a^2 + b^2 + c^2 - ab - bc - ca = \frac{1}{2}[(a-b)^2 + (b-c)^2 + (c-a)^2]$$

$$\text{Hence, } a^3 + b^3 + c^3 - 3abc = \frac{1}{2}(a+b+c)[(a-b)^2 + (b-c)^2 + (c-a)^2]$$

$$\text{Hence, } a^3 + b^3 + c^3 - 3abc = (a+b+c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$= \Rightarrow (a+b+c) \left[(a+b+c)^2 - 3(ab+bc+ca) \right]$$

//



If $a = 120, b = 125$ and $c = 129$, Find the value of $a^3 + b^3 + c^3 - 3abc$?

$$= \frac{1}{2}(a+b+c) \left[(a-b)^2 + (b-c)^2 + (c-a)^2 \right]$$

$$= 187 [25 + 16 + 81]$$

$$\Rightarrow 187 [122]$$

$$= 187 \times 122$$

$$\Rightarrow 187 [100 + 20 + 2]$$

$$\begin{array}{r} 120 \\ 125 \\ 129 \\ \hline 374 \\ \hline \end{array} \quad 187$$

✓

$$\begin{array}{r} 18700 \\ 3740 \\ 374 \\ \hline 22814 \end{array}$$



If $a + b + c = 28$ and $ab + bc + ca = 42$, Find the value of $a^3 + b^3 + c^3 - 3abc$?

$$= (a+b+c) \left((a+b+c)^2 - 3(ab+bc+ca) \right)$$

$$\Rightarrow 28 \left[(28)^2 - 3 \times 42 \right]$$

$$\Rightarrow 28 \left[784 - 126 \right]$$

$$\Rightarrow 28 \left[658 \right]$$

$$\Rightarrow 28 \times 658$$

$$\Rightarrow (30-2) \times 658$$

$$\begin{array}{r} 2 \\ 19740 \\ 1316 \\ \hline 18424 \end{array}$$

We know that:

$$a^3 + b^3 + c^3 - 3abc = \frac{1}{2}(a + b + c)[(a - b)^2 + (b - c)^2 + (c - a)^2]$$

$$\text{If } a + b + c = 0$$

$$\underline{a^3 + b^3 + c^3 - 3abc} = \frac{1}{2} \times 0 \times [(a - b)^2 + (b - c)^2 + (c - a)^2] = 0$$

$$\text{then, } \underline{a^3 + b^3 + c^3} = \underline{3abc}$$

Hence, if $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$

or

$$\text{If } a^3 + b^3 + c^3 = 3abc, \text{ then } a + b + c = 0$$



If $(4x - 1)^3 + (6x - 3)^3 + (2x - 4)^3 = 18(4x - 1)(x - 2)(2x - 1)$

a
=

Find the value of x?
=

$$= 3 \times \underset{3 \times 2}{6} (4x - 1) (x - 2) (2x - 1)$$

$$4x - 1 + 6x - 3 + 2x - 4 = 0$$

$$12x - 8 = 0$$

$$a^3 + b^3 + c^3 = 3abc$$

$$a + b + c = 0$$

$$= 3 (4x - 1) (2x - 4) (6x - 3)$$

$$12x = 8$$

$$x = \frac{8}{12} = \frac{2}{3}$$

If $x^4 + y^4 = -z^4$, then find the value of $\frac{x^{12} + y^{12} + z^{12}}{x^4 y^4 z^4}$?

$$x^4 + y^4 + z^4 = 0$$

$$(x^4)^3 + (y^4)^3 + (z^4)^3 = 3x^4 y^4 z^4$$

$$\underbrace{x^{12} + y^{12} + z^{12}} = \underbrace{3x^4 y^4 z^4}$$

$$\underline{a + b + c = 0}$$

$$\underline{a^3 + b^3 + c^3 = 3abc}$$

$$\Rightarrow \frac{3x^4 y^4 z^4}{x^4 y^4 z^4}$$

$$\Rightarrow 3$$

If $(4x)^{\frac{1}{2}} + (9y)^{\frac{1}{2}} = -z^{\frac{1}{2}}$, Find the value of $\frac{(4x)^{\frac{3}{2}} + (9y)^{\frac{3}{2}} + z^{\frac{3}{2}}}{\sqrt{xyz}}$?

$$\Rightarrow \frac{18\sqrt{xyz}}{\sqrt{xyz}} = \underline{18}$$

$$\begin{matrix} (4x)^{\frac{1}{2}} + (9y)^{\frac{1}{2}} + z^{\frac{1}{2}} = 0 \\ \uparrow \quad \quad \uparrow \quad \quad \uparrow \\ a \quad \quad b \quad \quad c = 0 \end{matrix}$$

$$a^3 + b^3 + c^3 = 3xyz$$

$$\left[(4x)^{\frac{1}{2}}\right]^3 + \left[(9y)^{\frac{1}{2}}\right]^3 + \left[z^{\frac{1}{2}}\right]^3 = 3 \times (4x)^{\frac{1}{2}} \times (9y)^{\frac{1}{2}} + z^{\frac{1}{2}}$$

$$\underbrace{(4x)^{\frac{3}{2}} + (9y)^{\frac{3}{2}} + z^{\frac{3}{2}}}$$

$$\begin{aligned} &= 3 \times 2\sqrt{x} \times 3\sqrt{y} \times \sqrt{z} \\ &= \underline{18\sqrt{xyz}} \end{aligned}$$



If $x^{\frac{2}{3}} + y^{\frac{2}{3}} + z^{\frac{2}{3}} = 0$, then Find the value of $\frac{(x^2+y^2+z^2)^3}{3y^2z^2 \cdot 3x^2y^2z^2}$?

$$a+b+c=0$$

$$a^3+b^3+c^3 = 3abc$$

$$\left[x^{\frac{2}{3}}\right]^3 + \left[y^{\frac{2}{3}}\right]^3 + \left[z^{\frac{2}{3}}\right]^3 = 3x^{\frac{2}{3}}y^{\frac{2}{3}}z^{\frac{2}{3}}$$

$$x^2+y^2+z^2 = 3x^{\frac{2}{3}}y^{\frac{2}{3}}z^{\frac{2}{3}}$$

$$= \frac{\left(3x^{\frac{2}{3}}y^{\frac{2}{3}}z^{\frac{2}{3}}\right)^3}{3x^2y^2z^2}$$

$$= \frac{27 \times \cancel{x^2} \cancel{y^2} \cancel{z^2}}{3x^2y^2z^2}$$

$$= 9$$

If $x^a \cdot x^b \cdot x^c = \underline{1}$, then find the value of $\frac{(a^3+b^3+c^3)^2}{(abc)^2}$?

$$x^{a+b+c} = x^0$$

$$a+b+c = 0$$

$$a^3+b^3+c^3 = \underline{3abc}$$

$$\Rightarrow \frac{(3abc)^2}{(abc)^2}$$

$$= \frac{9 \times \cancel{(abc)^2}}{\cancel{(abc)^2}}$$

$$= \underline{\underline{9}}$$

Find the value of $\frac{(x-2y)^3 + (2y-3z)^3 + (3z-x)^3}{24(x-2y)(2y-3z)(3z-x)}$?

$$\cancel{x-2y} + \cancel{2y-3z} + \cancel{3z-x} = 0$$

$$\Rightarrow \frac{\cancel{3(x-2y)(2y-3z)(3z-x)}}{\cancel{24(x-2y)(2y-3z)(3z-x)}} = \frac{1}{8}$$



If $x = (2a - 3b)(c - 2d)$, $y = (c - 2a)(3b - 2d)$, $z = (3b - c)(2a - 2d)$,

then find the value of $\frac{2(x^3 + y^3 + z^3)}{3xyz}$?

$$x = \cancel{2ac} - \cancel{4ad} - \cancel{3bc} + \cancel{6bd}$$

$$y = \cancel{3bc} - \cancel{2dc} - \cancel{6ab} + \cancel{4ad}$$

$$z = \cancel{6ab} - \cancel{6bd} - \cancel{2ac} + \cancel{2dc}$$

$$x + y + z = 0$$

$$x^3 + y^3 + z^3 = 3xyz$$

$$\begin{aligned} &= \cancel{2 \times 3xyz} \\ &\quad \hline &\quad \cancel{3xyz} \\ &= 2 \end{aligned}$$

If $\frac{1}{\sqrt[3]{36} - \sqrt[3]{6} + 1} = A\sqrt[3]{36} + B\sqrt[3]{6} + C$, find the value of $A + B + C$?

$$\frac{1}{\sqrt[3]{36} - \sqrt[3]{6} + 1}$$

$$0 \times \sqrt[3]{36} + \frac{\sqrt[3]{6}}{7} + \frac{1}{7} = A\sqrt[3]{36} + B\sqrt[3]{6} + C$$

$$a^3 + b^3 = (a+b)(a^2 + ab + b^2)$$

$$= (a+b)^3 - 3ab(a+b)$$

$$A = 0$$

$$B = \frac{1}{7}$$

$$C = \frac{1}{7}$$

$$A + B + C = 0 + \frac{1}{7} + \frac{1}{7}$$

$$A + B + C = \frac{2}{7}$$

$$\frac{(6^{1/3} + 1)}{\{(6^{1/3})^2 - 1 \times (6^{1/3}) + (1)^2\}} (6^{1/3} + 1)$$

$$\frac{6^{1/3} + 1}{(6^{1/3})^3 + (1)^3} \Rightarrow \frac{6^{1/3} + 1}{6 + 1}$$

$$\Rightarrow \frac{\sqrt[3]{6} + 1}{7}$$

If $\frac{p}{q} + \frac{q}{p} = -1$ ($p, q \neq 0$) then find the value of $\frac{7}{8}(p^3 - q^3)$?

$$\frac{p^2 + q^2}{pq} = -1$$

$$= \frac{7}{8} (p^3 - q^3)$$

$$p^2 + q^2 = -pq$$

$$= \frac{7}{8} \times 0 \Rightarrow 0$$

$$(p - q)(p^2 + q^2 + pq) = 0$$

$$(p^3 - q^3) = 0$$

If $x = 1 + \sqrt{2} + \sqrt{3}$ then find the value of $x + \frac{1}{x-1}$?

$$x-1 = \sqrt{2} + \sqrt{3}$$

$$\frac{1}{x-1} = \frac{1}{\sqrt{2} + \sqrt{3}} \times \frac{(\sqrt{3} - \sqrt{2})}{(\sqrt{3} - \sqrt{2})}$$

$$\frac{1}{x-1} = \sqrt{3} - \sqrt{2}$$

$$\Rightarrow 1 + \cancel{\sqrt{2}} + \sqrt{3} + \sqrt{3} - \cancel{\sqrt{2}}$$

$$= (1 + 2\sqrt{3})$$



If $x = \sqrt[4]{1 + \sqrt{2} + \sqrt{3}}$ then find the value of $x^4 + \frac{1}{x^4 - 1}$?

$$x^4 = 1 + \sqrt{2} + \sqrt{3}$$

$$= 1 + \cancel{\sqrt{2}} + \sqrt{3} + \sqrt{3} - \cancel{\sqrt{2}}$$

$$= \underline{1 + 2\sqrt{3}}$$

$$x^4 - 1 = \sqrt{3} + \sqrt{2}$$

$$\frac{1}{x^4 - 1} = \underline{\sqrt{3} - \sqrt{2}}$$



Homework

Maximum and minimum value – Algebraic expression

Condition of linear equation



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