CDS/CAPF VIRAAT 2.0 2024

Algebra

Mathematics

Lecture - 03

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TOPICS *to be covered*

1. – Linear eqn
2. → Maximum & Minimum value of Algebric expression
3.

Linear equation :—

$ax + by + c = 0$

Linear expression

Linear eqn

a, b, c ⇒ Constant term

Linear eqn in two variable

Degree →

$ax^1 + by^1 + c$

↳ Degree - (1)

$ax^2 + bx + c$

Degree ⇒ '2'

↳ Quadratic eqn

$ax^3 + by + c$

↳ Degree ⇒ 3

↓

Cubic eqn

Linear eqn in two variables

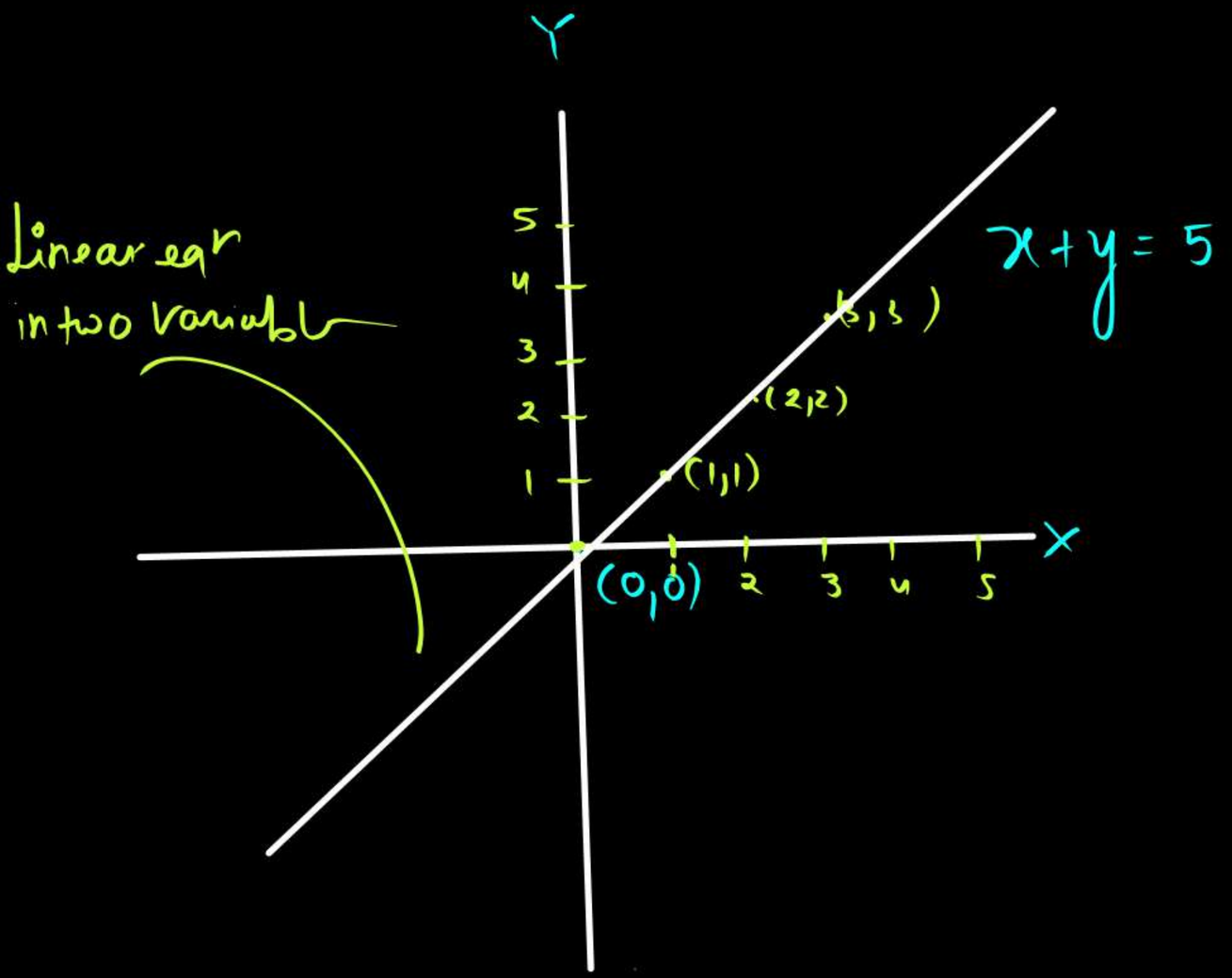
$x = y$

$x = 0$

$y = 0$

$x = 1$

$y = 1$



Linear eqn in two variable :—

General Form = $ax+bx+c=0$

⇒ Solution of Linear eqn in two variable

Case I ⇒ Non $||^{rl}$

Case II = $||^{rl}$

Case III = $||^{rl}$ and equal

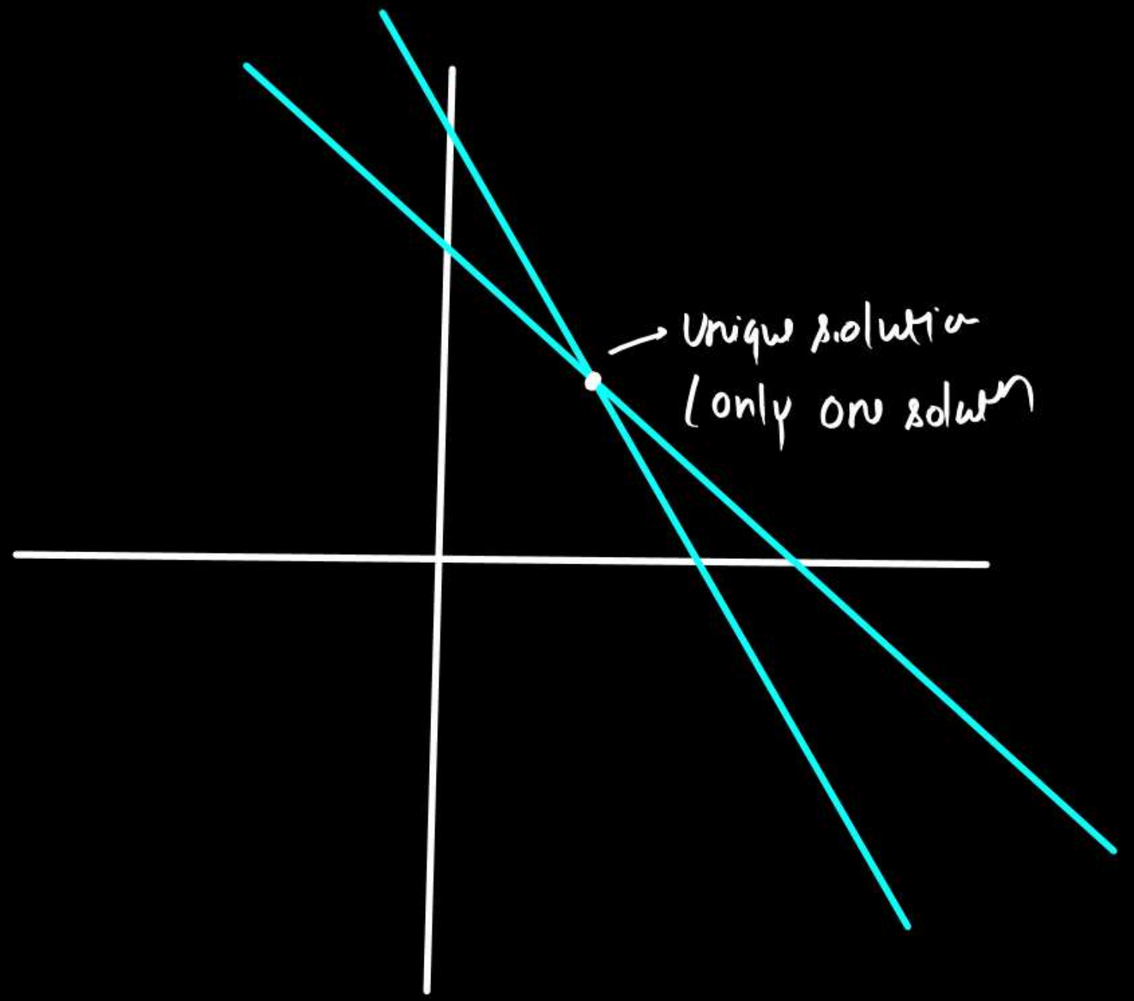
Case I ⇒ Non ||^{el}

$$a_1x + b_1y + c_1 = 0$$

$$a_2x + b_2y + c_2 = 0$$

⇒ Non ||^{el} ⟶ Unique [only one soln]

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$



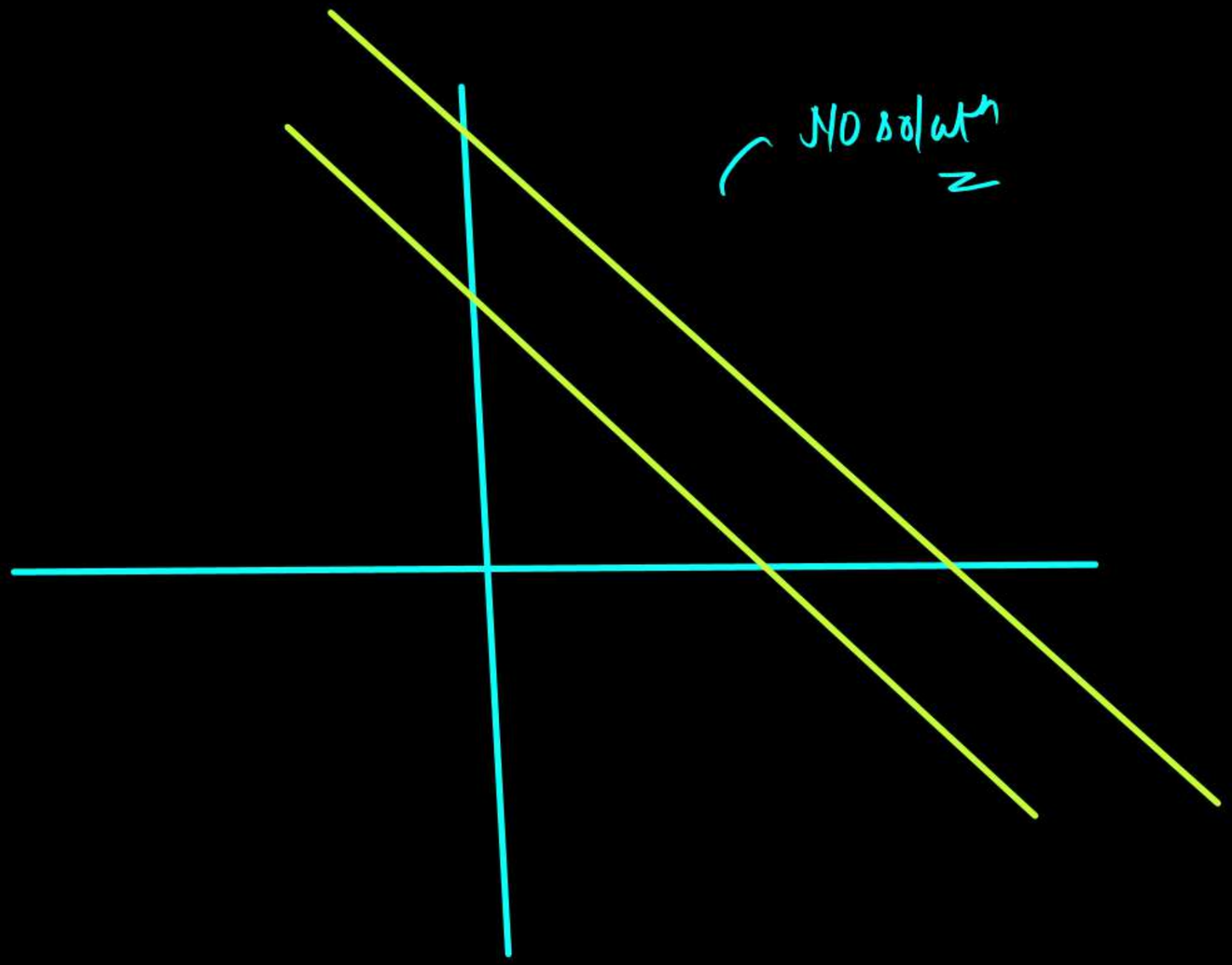
Case-II Lines are $\parallel^{rl}$

$$a_1x + b_1y + c_1 = 0$$

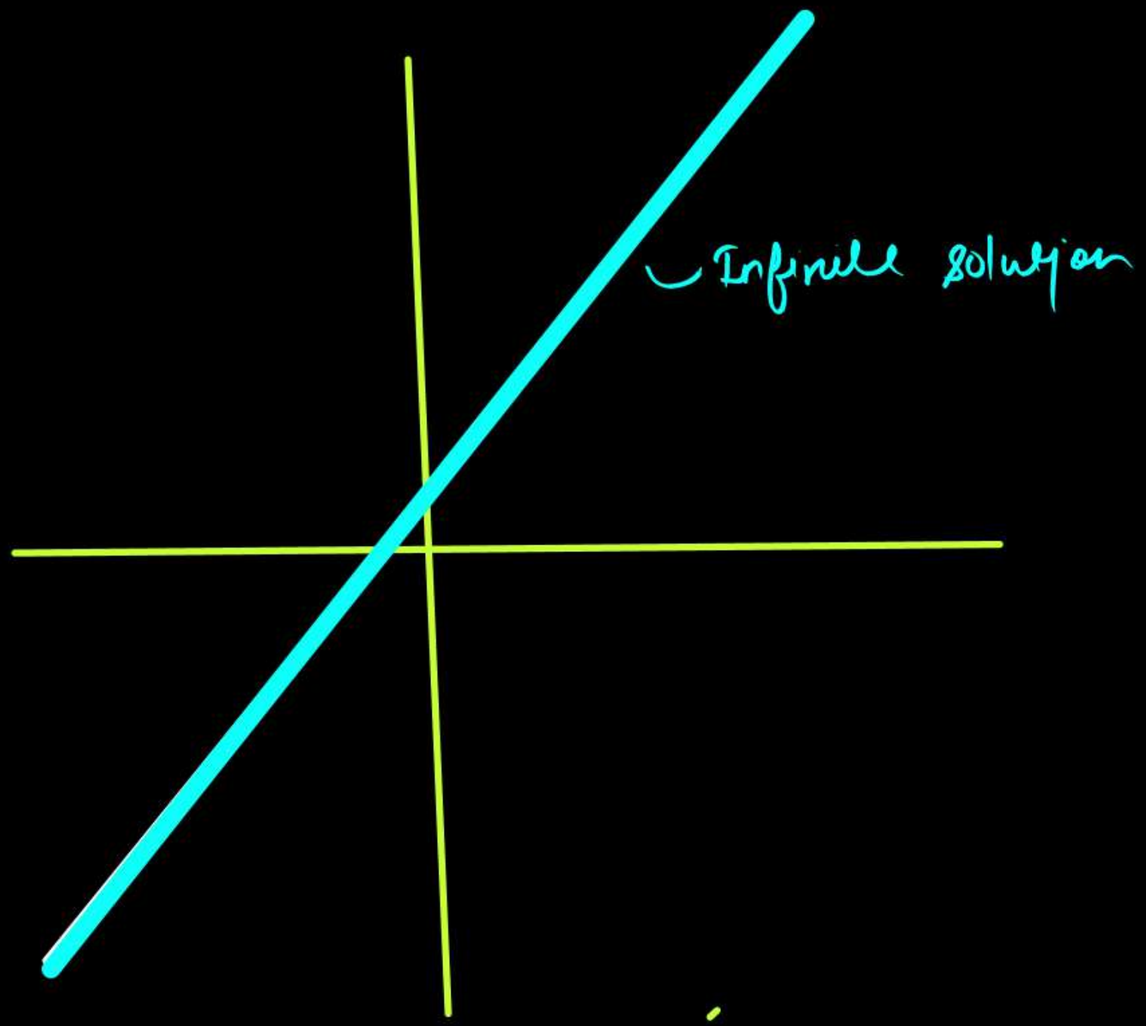
$$a_2x + b_2y + c_2 = 0$$

Line = $\parallel^{rl}$ $\Rightarrow$ No solution

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$



Case III 11st and same



→ Infinite solution

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$4x + 3y = 5$

$8x + 6y = 10$

$a_1x + b_1y + c_1 = 0$

$a_2x + b_2y + c_2 = 0$

Non $\parallel^{gl}$ $\longrightarrow$ Unique solutn
Only one soln

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$\parallel^{gl}$ $\longrightarrow$ No solution

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

$\parallel^{gl}$ & same $\longrightarrow$ Infinite soln

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

QUESTION

Assertion (A): The equations $2x - 3y = 5$ and $6y - 4x = 11$ cannot be solved graphically.
Reason (R): The equations given above represent parallel lines. [2007-I]

$-4x+6y = 11$

$\frac{a_1}{a_2} \quad \frac{b_1}{b_2} \quad \frac{c_1}{c_2}$

$\frac{2}{-4} \quad \frac{-3}{6} \quad \frac{5}{11}$

$-\frac{1}{2} = -\frac{1}{2} \neq \frac{5}{11}$

$\Downarrow$ lines $\parallel^{rd} \Rightarrow$ No solutn

(A) A and R are correct and R is correct explanation of A

(B) A and R are correct but R is not correct explanation of A

(C) A is correct but R is wrong

(D) A is wrong but R is correct

QUESTION

If a two-digit number is added to a number obtained by reversing the digits of the given number, then the sum is always divisible by which one of the following numbers?

[2007-II]

A 7

B 9

C 10

D 11

$10x+y$

$10y+x$

$11x+11y$

$= 11(x+y)$

QUESTION

What is/are the solutions of the set of homogeneous equations $(4x + 2y = 0)$ and $(6x + 3y = 0)$? [2008–I]

$2x + y = 0$

$2x + y = 0$

$\frac{2}{4} \quad \frac{2}{3}$

$\frac{4}{6}$

$\frac{3}{2}$

$\frac{2}{3} = \frac{2}{3}$

(A) Only $x = 0, y = 0$

(B) Only $x = 0, y = 0$ and $x = 1, y = 2$

(C) An infinite number of solutions

(D) No solution

QUESTION

The cost of 4 books and 3 pencils is same as that of 8 books and 1 pencil. This cost will be same as that of which one of the following? [2008-I]

$4B + 3P = 8B + P$

$2P = 4B$

$P = 2B$

$4B + 3 \times 2B$

$4B + 6B$

$10B$

A 2 books and 6 pencils

B 5 books and 5 pencils

C 6 books and 2 pencils

D 12 books and 4 pencils

$6B + 2 \times 2B$

$6B + 4B = 10B$

QUESTION

The solution of the equations $\frac{3x-y+1}{3}=\frac{2x+y+2}{5}=\frac{3x+2y+1}{6}$ is given by which one of the following? [2008–II]

$15x-5y+5=6x+3y+6$

$9x-8y=1$

$\frac{3x-y+1}{3}=\frac{3x+2y+1}{6}$

$6x-2y+2=3x+2y+1$

$(3x-4y=-1)\times 3$

$9x-12y=-3$

$9x-8y=1$

$9x-12y=-3$

$4y=4$

$y=1$

$9x-8=1$

$9x=9$

$x=1$

A $x=2, y=1$

B $x=1, y=1$

C $x=-1, y=-1$

D $x=1, y=2$

QUESTION

What is the solution of the equations $x - y = 0.9$ and $11(x + y)^{-1} = 2$? [2009–I]

$x - y = 0.9$

$\frac{11}{(x+y)} = 2$

$x + y = 5.5$

$x - y = 0.9$

$2x = 6.4$

$x = 3.2$

A. $x = 3.2$ and $y = 2.3$

B. $x = 1$ and $y = 0.1$

C. $x = 2$ and $y = 1.1$

D. $x = 1.2$ and $y = 0.3$

QUESTION

What is the value of k for which the system of equations $x + 2y - 3 = 0$ and $5x + ky + 7 = 0$ has no solution? [2009–I]

$$\frac{1}{5} = \frac{2}{k}$$

$$k = 5 \times 2$$

$$k = 10$$

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$$

A $-\frac{3}{14}$

B $-\frac{14}{3}$

C $\frac{1}{10}$

D 10

QUESTION

Under what condition do the equations $kx - y = 2$ and $6x - 2y = 3$ have a unique solution?
[2010–II]

$\frac{a_1}{a_2} \neq \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$\frac{k}{6} \neq \frac{-1}{-2}$

$k \neq 3$

A. $k = 3$

B. $k \neq 3$

C. $k = 0$

D. $k \neq 0$

QUESTION

If $\frac{2}{x}+\frac{3}{y}=\frac{9}{xy}$ and $\frac{4}{x}+\frac{9}{y}=\frac{21}{xy}$, where $x \neq 0$ and $y \neq 0$, then what is the value of $x+y$? [2011-I]

A 2

B 3

C 4

D 8

$$\frac{2y+3x}{xy}=\frac{9}{xy}$$

$$(3x+2y=9)\times 2$$

$$9x+4y=21$$

$$6x+4y=18$$

$$3x=3$$

$$x=1$$

$$2y+3=9$$

$$xy=6$$

$$y=3$$

$$x+y=1+3=4$$

QUESTION

If (x, y) = (4, 1) is the solution of the pair of linear equations mx + y = 2x + ny = 5, then what is m + n equal to? [2011–II]

$x=4, y=1$

$mx+y=5$

$2x+ny=5$

$2\times4+y=5$

$y=5-8$

$y=-3$

$4m+1=5$

$4m=4$

$m=1$

$m+n$

$1-3=-2$

A −2

B −1

C 2

D 1

QUESTION

The sum of two numbers is 10 and their product is 20. What is the sum of their reciprocals? [2011–II]

$x + y = 10$

$xy = 20$

$\frac{x+y}{xy} = \frac{10}{20}$

$\frac{x}{xy} + \frac{y}{xy} = \frac{1}{2}$

$\frac{1}{x} + \frac{1}{y} = \frac{1}{2}$

A $\frac{1}{10}$

B $\frac{1}{2}$

C 1

D 2

QUESTION

The system of equations $x + 2y = 3$ and $3x + 6y = 9$ has [2011-II]

$\frac{1}{3}$ $\frac{2}{6}$ $\frac{3}{9}$

$\frac{1}{3} = \frac{1}{3} = \frac{1}{3}$ → equal & all

In

A Unique solution

B No solution

C Infinitely many solutions

D Finite number of solutions

QUESTION

If $\frac{3}{x+y}+\frac{2}{x-y}=2$ and $\frac{9}{x+y}-\frac{4}{x-y}=1$, then what is the value of $\frac{x}{y}$? [2013-I]

$x+y=3$

$x-y=2$

(A) $\frac{3}{2}$

(B) 5

(C) $\frac{2}{3}$

(D) $\frac{1}{5}$

$x+y=3$

$x-y=2$

$x=\frac{5}{2}$

$y=\frac{1}{2}$

$\frac{x}{y}=\frac{5}{\frac{1}{2}}\Rightarrow 5$

QUESTION

The sum of two numbers is 20 and their product is 75. What is the sum of their reciprocals? [2013-I]

$\frac{20}{75}$ $\frac{4}{15}$

A $\frac{1}{15}$

B $\frac{1}{5}$

C $\frac{4}{15}$

D $\frac{7}{15}$

QUESTION

If the sum of a number and its reciprocal is $\frac{10}{3}$, then the numbers are [2013-I]

$x + \frac{1}{x} = \frac{10}{3} \Rightarrow 3 + \frac{1}{3}$

$3, \frac{1}{3}$

A $3, \frac{1}{3}$

B $3, -\frac{1}{3}$

C $-3, \frac{1}{3}$

D $-3, -\frac{1}{3}$

QUESTION

The sum of the squares of two numbers is 97 and the squares of their difference is 25. The product of the two numbers is [2013-I]

A 45

B 36

C 54

D 63

$x^2+y^2=97$ 1

$(x-y)^2=25$

$x^2+y^2-2xy=25$

$97-2xy=25$ 1

$2xy = 97 - 25$ 2

$2xy=72$

$xy=36$

QUESTION

The system of equations $3x + y - 4 = 0$ and $6x + 2y - 8 = 0$ has [2013-I]

A. A unique solution $x = 1, y = 1$

B. A unique solution $x = 0, y = 4$

C. No solution

D. Infinité solution

QUESTION

The number of pairs (x, y) where x, y are integers satisfying the equation 21x + 48y = 5 is [2015–II]

$21x + 48y = 5$

$21x = 5 - 48y$

$x = \frac{5 - 48y}{21}$

$\Rightarrow$ Any integer solution

A Zero

B One

C Two

D Infinity

QUESTION

The value of k, for which the system of equation $3x - ky - 20 = 0$ and $6x - 10y + 40 = 0$ has no solution, is [2016-I]

$$\frac{3}{6} = \frac{+k}{+10}$$

$$\frac{1}{2} = \frac{k}{10}$$

$$5$$

$$k = 5$$

A 10

B 6

C 5

D 3

Maximum and Minimum Value of Algebraic equation

ax^2+bx+c

$\Rightarrow a\left[x^2+\frac{b}{a}x+\frac{c}{a}\right]$

$\Rightarrow a\left[x^2+2\times\frac{b}{2a}\times x+\frac{c}{a}+\frac{b^2}{4a^2}-\frac{b^2}{4a^2}\right]$

$\Rightarrow a\left[\left(x+\frac{b}{2a}\right)^2+\frac{c}{a}-\frac{b^2}{4a^2}\right]$

$\Rightarrow a\left[\left(x+\frac{b}{2a}\right)^2+\frac{1}{a}\left[\frac{4ac-b^2}{4a}\right]\right] = a\left(x+\frac{b}{2a}\right)^2+\left(\frac{4ac-b^2}{4a}\right)$

x

$\downarrow$

x^2

Max $\Rightarrow \infty$

Min $\Rightarrow -\infty$

Max $\Rightarrow \pm\infty$

Min $\Rightarrow$ '0'

$x=-\frac{b}{2a}$

$\frac{d}{dx} x^n = nx^{n-1}$

Minima

$2x^2 + 5x + 3$

$2 \times 2x + 5 = 0$

$4x + 5 = 0$

$4x = -5$

$x = -\frac{5}{4}$

$= 2 \times \left(-\frac{5}{4}\right)^2 + 5x - \frac{5}{4} + 3$

$= \frac{2 \times 25}{16} - \frac{25}{4} + 3$

$= \frac{25}{8} - \frac{25}{4} + 3$

$= -\frac{25}{8} + 3$

$= \frac{-25+24}{8}$

$= -\frac{1}{8}$

	A·M	G·M
a, b	$\frac{a+b}{2}$	$\sqrt{ab}$
a, b, c	$\frac{a+b+c}{3}$	$\sqrt[3]{abc}$
a, b, c, d	$\frac{a+b+c+d}{4}$	$\sqrt[4]{abcd}$

$$A \cdot M \geq G \cdot M$$

QUESTION

If $15 - (3 - a)^2$ is maximum, then find the value of a?

यदि $15 - (3 - a)^2$ का मान अधिकतम हो, तो a का मान ज्ञात कीजिए?

$(3-a)^2$ → minimum

$15 - (3-a)^2$ → maximum

Ilomy

$3 - a = 0$

$a = 3$

A 0

B −3

C 12

D 3

QUESTION

Find the ratio of a and b? if the value of $5 - (3a - b)^2$ is maximum.
a तथा b का अनुपात ज्ञात कीजिये? यदि $5 - (3a - b)^2$ का मान अधिकतम है।

$3a - b = 0$

$3a = b$

$\frac{a}{b} = \frac{1}{3}$

$a : b = 1 : 3$

A 1 : 3

B 3 : 1

C 3 : 5

D 5 : 3

QUESTION

Find the value of x? if the value of $(\sqrt{5}+x-\sqrt{3})(\sqrt{5}+\sqrt{3}-x)$ is maximum?

x का मान ज्ञात कीजिये? यदि $(\sqrt{5}+x-\sqrt{3})(\sqrt{5}+\sqrt{3}-x)$ का मान अधिकतम है।

$$(\sqrt{5}+(x-\sqrt{3}))\ (\sqrt{5}-(x-\sqrt{3}))$$

$$\Rightarrow \quad 5-(x-\sqrt{3})^2$$

$x-\sqrt{3}=0$

$x=\sqrt{3}$

A 0

B $\sqrt{5}$

C $-\sqrt{3}$

D $\sqrt{3}$

QUESTION

Find the minimum value of $4x^2 - 2x + 5$?

$4x^2 - 2x + 5$ का न्यूनतम मान ज्ञात कीजिये?

A $\frac{19}{4}$

B $\frac{4}{19}$

C $-\frac{19}{4}$

D $-\frac{4}{19}$

$4 \times 2x - 2 = 0$

$4 \cdot 8x = x$

$x = \frac{1}{4}$

$= 4 \times \frac{1}{16} - \frac{2}{4} + 5$

$= \frac{-1}{4} + 5$

$= \frac{-1+20}{4}$

$= \frac{19}{4}$

QUESTION

Find the minimum value of $4x^2 + 4x + 9$?

$4x^2 + 4x + 9$ का न्यूनतम मान ज्ञात कीजिये?

$4 \times 2x + 4 = 0$

$8x = -4$

$x = \frac{-4}{2}$

$x = \frac{-1}{2}$

$= \frac{4}{4} - \frac{4^2}{4} + 9$

$= 1 - 2 + 9$

$= -1 + 9 = +8$

A −8

B 8

C −9

D 9

QUESTION

Find the maximum value of $20 - x^2 + 2x$?

$20 - x^2 + 2x$ का अधिकतम मान ज्ञात कीजिये?

$-2x + 2 = 0$

$f(x) = -2$

$x = 1$

$20 - 1 + 2$

$= 20 + 1$

$= 21$

A −20

B 20

C −21

D 21

QUESTION

Find minimum value of $x + \frac{1}{x}$? If x is a positive real number?

$x + \frac{1}{x}$ का न्यूनतम मान ज्ञात कीजिये? यदि x एक धनात्मक वास्तविक संख्या है?

$x = 1$

$= 1 + 1 = 2$

A 1

B 2

C $-\infty$

D ∞

QUESTION

If a, b, c, d are positive numbers and a + b + c + d = 1, then find the maximum value of abcd?

यदि a, b, c, d धनात्मक संख्याएँ हो तथा a + b + c + d = 1 हो, तो abcd का अधिकतम मान ज्ञात कीजिये?

A $\left(\frac{1}{4}\right)^4$

B $\left(\frac{1}{4}\right)^{\frac{1}{2}}$

C $\frac{1}{4}$

D 1

$a=b=c=d$

$\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}$

$\frac{a+b+c+d}{4} \geq \sqrt[4]{abcd}$

$\left(\frac{1}{4}\right)^4 \geq \left(\sqrt[4]{abcd}\right)^4$

$abcd = \left(\frac{1}{4}\right)^4$

$\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4}$

$\left(\frac{1}{4}\right)^4$

QUESTION

If a + b + c = 15, then find the maximum value of abc?
यदि a + b + c = 15 हो, तो abc का अधिकतम मान ज्ञात कीजिये?

A 5

B 125

C 15

D 25

$a = b = c = 5$

$\frac{a+b+c}{3} \geq \sqrt[3]{abc}$

$\frac{15}{3} \geq (\sqrt[3]{abc})$

$5 \geq \sqrt[3]{abc}$

$(5)^3 \geq abc$

$abc = 5^3$

$= 125$

$5 \times 5 \times 5$

$= 5^3 = 125$

QUESTION

If a + b = 20, then find the maximum value of (a + 3)(b + 5)?

यदि a + b = 20 हो, तो (a + 3)(b + 5) का अधिकतम मान ज्ञात कीजिये?

14 14 20+8 14×14

$(a+3)+(b+5) = 28$

$\frac{(a+3)+(b+5)}{2} \geq \sqrt{(a+3)(b+5)}$

14

$28 \geq \sqrt{(a+3)(b+1)}$

196

$a+b = 20$

$(a+3)+(b+5) = 20+8$

$= 28$

A 196

B 195

C 192

D 100

QUESTION

If a + b + c + d = 1, then find the maximum value of (a + 1)(b + 1)(c + 1)(d + 1)?

यदि a + b + c + d = 1 हो, तो (a + 1)(b + 1)(c + 1)(d + 1) का अधिकतम मान ज्ञात कीजिये?

$\frac{5}{4} \times \frac{5}{4} \times \frac{5}{4} \times \frac{5}{4}$

$(a+1) + (b+1) + (c+1) + (d+1) = 1 + 4 = 5$

$\frac{5}{4} + \frac{5}{4} + \frac{5}{4} + \frac{5}{4}$

$$\frac{(a+1)+(b+1)+(c+1)+(d+1)}{4} \geq \sqrt[4]{(a+1)(b+1)(c+1)(d+1)}$$

$$\frac{5}{4} \geq \sqrt[4]{(a+1)(b+1)(c+1)(d+1)}$$

$$= \left(\frac{5}{4}\right)^4 = \frac{625}{256}$$

A. 1

B. 16

C. $\frac{1}{256}$

D. $\frac{625}{256}$

QUESTION

If $2x + 3y = 15$, then find the maximum value of x^2y^3?

यदि $2x + 3y = 15$ हो, तो x^2y^3 का अधिकतम मान ज्ञात कीजिये?

$x \times x \times y \times y \times y$

$= 3^2 \times 3^3$

$= 3^5 = 243$

$3 + 3 + 3 + 3 + 3$

$x + x + y + y + y = 15$

$$\frac{x+x+y+y+y}{5} \geq \sqrt[5]{x \times x \times y \times y \times y}$$

$$\frac{15}{5}^3 \geq \sqrt[5]{x^2y^3}$$

$$3 \geq \sqrt[5]{x^2y^3}$$

$x^2y^3 = 3^5$

$= 243$

A 243

B 81

C 27

D 15

Homework

Qn - Eqn


JAI
HIND