# CS 785 A

## ASSIGNMENT - 1

NAME - Pratik Mishra
ROLL - 14493

Ques 1.

Note: This question seemed to be a little ambiguous, specially the statement that "..... to him the repair was actually only worth Rs. 2000/-."
I have interpreted this as ~~xxxx~~ follows:

- Even though the repair costs Rs 5000/-, the value that he recieves upon the repair of the car is only Rs 2000/-.

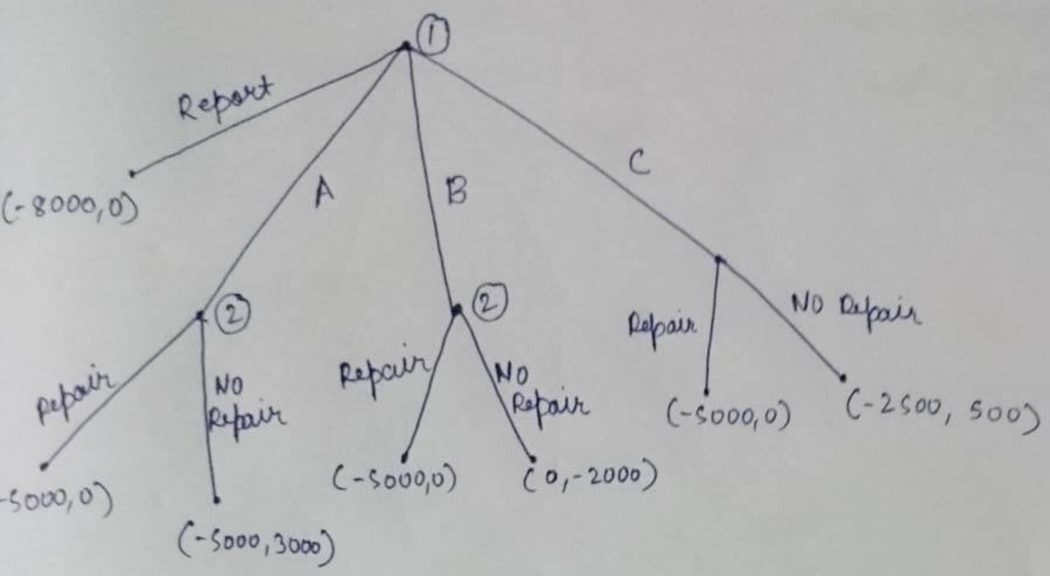
Possible actions for Ram:

1) report the accident and the insurance company pays for the repair (abbreviation: Report)
2) A (refers to option (a) in the question).
3) B (refers to option (b) in the question).
4) C (refers to option (c) in the question).

Possible actions for Rahim:

1) get car repaired (Repair)
2) not get the car repaired (NO Repair)

(a) Game tree for the game : [Player 1: Ram, Player 2: Rahim]




(b) <u>Normal form of the game:</u>

Strategy sets for Player 1: { (Report), (A), (B), (C) }

~~Strategy set for Player 2: {(Repair, Repair, Repair)/,~~

<u>Denote Repair by R.</u>

<u>Denote NO Repair by N.</u>

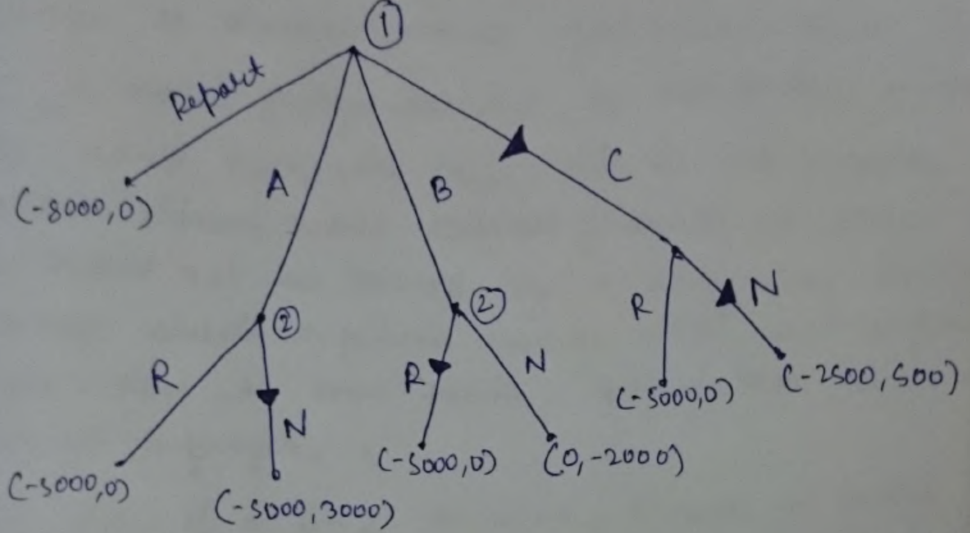
Strategy set for Player 2: { (R,R,R), (R,R,N), (R,N,R), (R,N,N), (N,R,R), (N,R,N), (N,N,R), (N,N,N) }

| | (R,R,R) | (R,R,N) | (R,N,R) | (R,N,N) | (N,R,R) | (N,R,N) | (N,N,R) | (N,N,N) |
|---|---|---|---|---|---|---|---|---|
| Report | -8000,0 | -8000,0 | -8000,0 | -8000,0 | -8000,0 | -8000,0 | -8000,0 | -8000,0 |
| A | -5000,0 | -5000,0 | -5000,0 | -5000,0 | -5000, 3000 | -5000, 3000 | -5000, 3000 | -5000, 3000 |
| B | -5000,0 | -5000,0 | 0,-2000 | 0,-2000 | -5000,0 | -5000,0 | 0,-2000 | 0,-2000 |
| C | -5000,0 | -2500,500 | -5000,0 | -2500,500 | -5000,0 | -2500,500 | -5000,0 | -2500, 500 |

(The best responses are underlined to calculate Nash equilibria).

(d)

Note: For every subtree of the game tree, arrows starting from the root of that subtree form the optimal path in that subtree.




Thus, via backward induction, we get the optimal strategies for Ram Rahim.

For Ram:

The optimal strategy is (C).

For Rahim:

The optimal strategy is (N, R, N).

(c) The game tree has the following equilibria:

1) {B, (R,R,R)}
2) {C, (R,R,N)}
3) {A, (N,R,R)}
4) {B, (N,R,R)}
5) {C, (N,R,N)}

Ques 2

(a) Justification of Payoffs for Player 2 :

A unanimous decision by the supreme court judges is always worse for Nixon than if 2 (or more) judges decided to not order Nixon to hand over the tapes. This is so because it allows Nixon more ~~correct~~ grounds to argue that he should not be bound by a conflicting decision. Also it helps create a public image that the supreme ~~court~~ court, to some extent, believe that such a request is ~~not~~ improper.

In case of a 6-2 decision, Nixon is better off defying than complying because that way he does not have to hand over proof of his involvement in illegal activities and he also has a justification for defying as 2 judges of the supreme court feel the same.

In case of an 8-0 decision, Nixon is better off complying than defying because by a unanimous decision, the supreme court have already made it clear that they believe Nixon is in the wrong. By defying, he would make their suspicions even more certain whereas by complying, there is still a chance that the prosecutor does not find the evidence ~~to~~ of his wrongdoings.

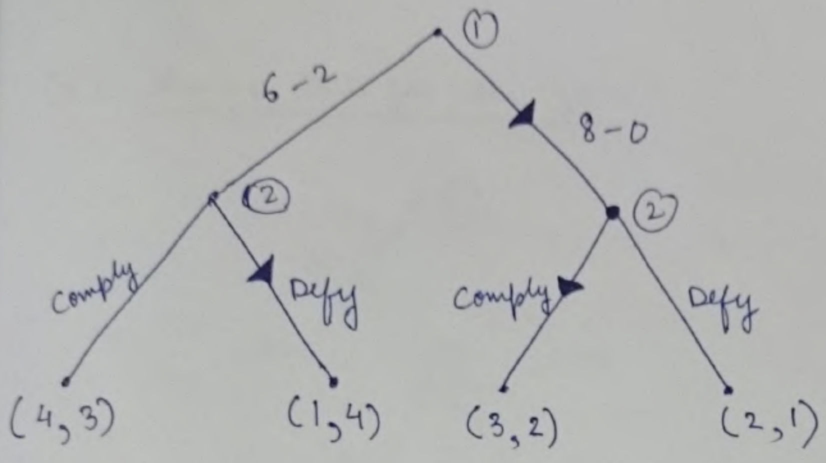
## Justification of Payoffs for Player 1 :

In general, Nixon complying would suggest that he is confident that there is no evidence of his involvement in illegal activities which indicates towards his innosense. Also, Nixon complying is always better for the judges as it allows the prosecution to get to the truth, which is what the supreme court wants. Thus, Nixon complying is always better than Nixon defying for the 2 judges.

In the case when Nixon complies with the decision, 6-2 decision is better than 8-0 decision for the 2 judges because a 6-2 decision would show their support towards their president and Nixon complying indicates his innosense, thus, beaing on the side of a "not-guilty" president is better than not being on his side.

In the case when Nixon defies, 8-0 decision is better than 6-2 decision for the 2 judges because Nixon defying indicates his guilt and therefore the 2 judges would prefer to be seen as seeking the truth rather than abetting the president in his crime.

(b) Game tree for the game:



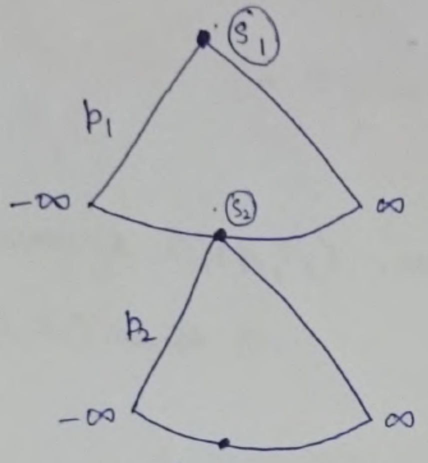

Thus, by backward induction, we see that the outcome would be that Player 1 chooses 8-0 decision and Player 2 complies with the decision.

(d) In the real ~~earth~~ world, the supreme court produced a unanimous 8-0 decision and Nixon complied with the decision.
As a result, the prosecution found proof implicating Nixon in illegal activities and fearing certain impeachment, Nixon resigned the presidency on August 9, 1974.

(c) Yes, Nixon could have had a better pay off if he had co-operated with 2 judges. To be specific, if Nixon could have convinced the 2 judges to produce a 6-2 judgement by guaranteeing that he would comply, he could have gotten a better pay off.

Ques 3

(a) Game tree for the game:




$(10p_1 - 2p_1^2 - p_1p_2 , 10p_2 - p_1p_2 - 2p_2^2)$

For Backward induction, assume $S_1$ chooses $p_1$

$\therefore$ If $S_2$ chooses $p_2$,

$\pi_2(p_2, a_2) = p_2 a_2 = p_2(10 - p_1 - 2p_2) = 10p_2 - p_1p_2 - 2p_2^2$

In order to maximize $\pi_2(p_2, a_2)$, we must equate $\frac{d}{dp_2}\pi_2(p_2, a_2)$ with zero.

$$\frac{d}{dp_2}\pi_2(p_2, a_2) = 10 - p_1 - 4p_2$$

$$\therefore 10 - p_1 - 2p_2 = 0 \Rightarrow p_2 = \frac{10-p_1}{4}$$

Thus, best response for $S_2$ is $p_2 = \frac{10-p_1}{4}$

continuing with backward induction, we know that if $s_1$ plays $p_1$ then $s_2$ plays $\frac{10-p_1}{4}$.

$\therefore \pi_1(p_1, a_1) = p_1 a_1 = p_1\left(10 - 2p_1 - \left(\frac{10-p_1}{4}\right)\right)$

$= 10p_1 - 2p_1^2 - \frac{10}{4}p_1 + \frac{p_1^2}{4}$

$\therefore$ to maximize $\pi_1(p_1, a_1)$, we need to equate $\frac{d}{dp_1}\pi(p_1, a_1)$ with 0.

$\frac{d}{dp_1}\pi(p_1, a_1) = 10 - 4p_1 - \frac{10}{4} + \frac{p_1}{2}$

$10 - 4p_1 - \frac{10}{4} + \frac{p_1}{2} = 0 \Rightarrow \frac{7p_1}{2} = \frac{30}{4} = \frac{15}{2}$

$\therefore p_1 = 15/7$

$\therefore p_2 = \frac{10-p_1}{4} = \frac{55}{28}$

$\therefore$ Revenue for $s_1 = \pi_1(p_1, a_1) = p_1(10 - 2p_1 - p_2)$

$= \frac{15}{7}\left(10 - \frac{30}{7} - \frac{55}{28}\right)$

$= \frac{15}{7} \times \frac{(280 - 120 - 55)}{28}$

$= \frac{15}{7} \times \frac{105}{28} = \frac{225}{28}$

Revenue for $S_2 = \pi_2(p_2, a_2) = p_2(10 - p_1 - 2p_2)$

$$= \frac{55}{28}\left(10 - \frac{15}{7} - \frac{55}{14}\right)$$

$$= \frac{55}{28} \times \frac{(140 - 30 - 55)}{14}$$

$$= \frac{55}{28} \times \frac{55}{14} = \frac{3025}{392}$$

(b) $\pi_1(p_1, a_1) = p_1 a_1$

$\pi_2(p_2, a_2) = p_2 a_2$

Under the new scenario,

$a_1 \geq 0$ and $a_2 \geq 0$

Thus, $p_1 \geq 0$ and $p_2 \geq 0$

because, if $p_i < 0$ then $\pi_i(p_i, a_i) < 0$ which is worse than $\pi_i(0, a_i) = 0$.

If $p_i \geq 5$,

then, $2p_i + p_{3-i} \geq 10$

$\therefore a_i = 0$

Thus, $\pi_i(p_i, a_i) = 0$.

Hence there is no advantage for $S_i$ to have $p_i > 5$.

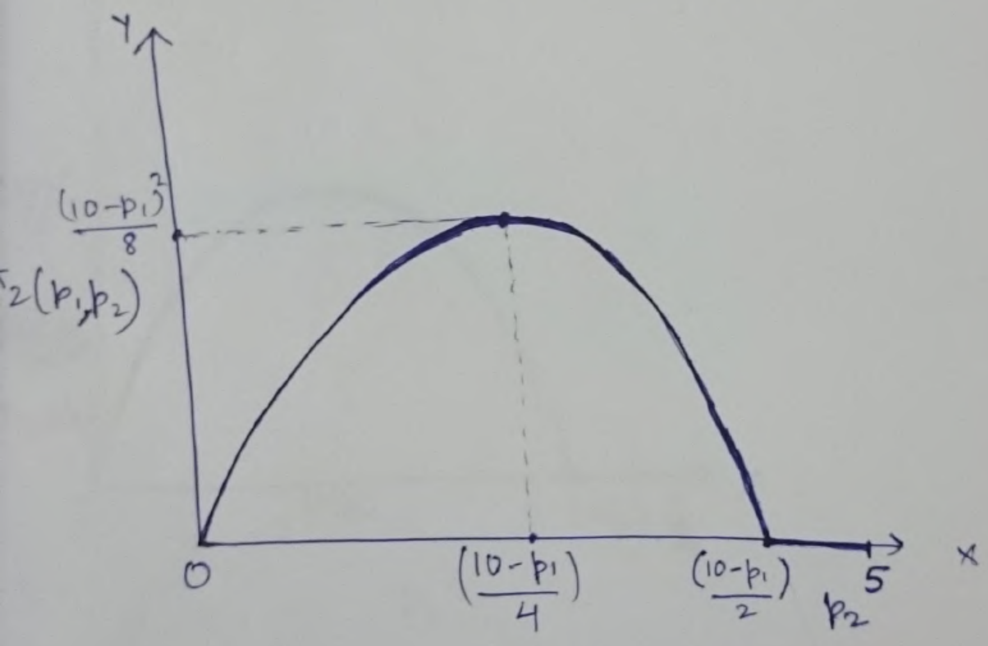
Thus, realistically, $0 \leq p_1, p_2 \leq 5$.

(c) $$\pi_1(p_1,p_2) = \begin{cases} p_1(10-2p_1-p_2) & \text{if } 2p_1+p_2<10 \\ 0 & \text{otherwise} \end{cases}$$

$$\pi_2(p_1,p_2) = \begin{cases} p_2(10-p_1-2p_2) & \text{if } p_1+2p_2<10 \\ 0 & \text{otherwise} \end{cases}$$

(d)

Let $0 \le p_1 \le 5$ be fixed.



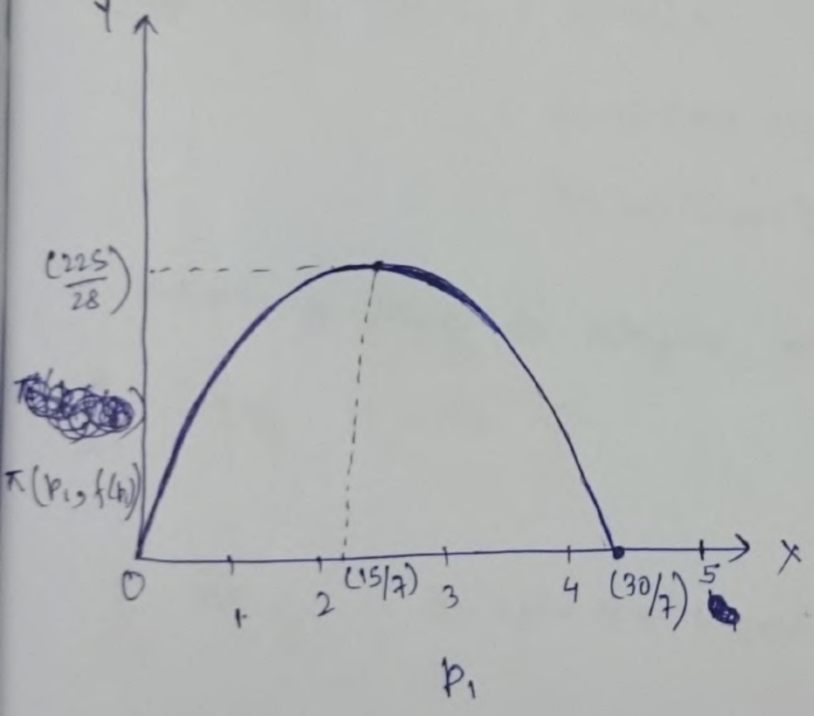

(e) $p_2 = f(p_1) = \frac{10-p_1}{4}$ [From the graph in part (d)]

(f)

$$\pi_1(p_1, f(p_1))$$

$$= \pi_1\left(p_1, \frac{10-p_1}{4}\right)$$

$$\therefore \pi_1\left(p_1, \frac{10-p_1}{4}\right) = \begin{cases} p_1\left(10 - \frac{7p_1+10}{4}\right) & \text{if } p_1 < \frac{30}{7} \\ 0 & \text{otherwise} \end{cases}$$




From the graph, we see that it is maximum at $p_1 = 15/7$.

Ques 4.

$a = 4e - 4re - e^2$

Payoff for government $= ra$
Payoff for citizens $= (1-r)a$

Let the government play 'r', i.e, let tax rate be r (fixed).

Payoff for citizens $= (1-r)a$

$= (1-r)(4e - 4re - e^2)$

$= 4e - e^2 + 4r^2e + e^2r - 8re$

To maximize payoff to citizens, we must equate $\frac{d}{de}$ Payoff with 0.

$\therefore \frac{d}{de}$ Payoff $= 4 - 2e + 4r^2 + 2er - 8r$

$\therefore 4 - 2e + 4r^2 + 2er - 8r = 0$

$\Rightarrow 4 + 4r^2 - 8r + 2er - 2e = 0$

$\Rightarrow e = \frac{2r^2 - 4r + 2}{1-r} = \frac{2(r^2 - 2r + 1)}{(1-r)} = 2(1-r)$

Hence best response for citizens is

$e = 2(1-r)$

Payoff for Government $= ra = r(4e - 4re - e^2)$

If $e = 2(1-r)$

Payoff for Government

$= r(8(1-r) - 8r(1-r) - 4(1-r)^2)$

$= r(1-r)(8 - 8r - 4 + 4r)$

$= r(1-r)(4 - 4r) = 4r(1-r)^2$

To maximize Payoff, we set $\frac{d}{dr}$ Payoff equal to zero.

$\frac{d}{dr}$ Payoff $= 4(1-r)^2 - 8r(1-r)$

$\therefore \quad 4(1-r)^2 - 8r(1-r) = 0$

$\Rightarrow \quad 4(1-r) - 8r = 0$

$\Rightarrow \quad 1 - r - 2r = 0$

$\Rightarrow \quad r = \frac{1}{3}$

Thus, the tax rate that maximizes government's payoff is $r = 1/3$.

Ques 5.

(a) $\pi_i(b_1, \ldots, b_n) = \begin{cases} v_i - h_i & \text{if } b_i > h_i \\ 0 & \text{otherwise} \end{cases}$

where $h_i = \max(b_1, b_2, \ldots b_{i-1}, b_{i+1}, b_{i+2}, \ldots, b_n)$

(b) Let the best possible strategy for $i^{th}$ player be $best_i$.

claim: $best_i = v_i$.

Proof:

Case I : $h_i > v_i$

In this case, if $b_i > h_i$

then payoff for $i^{th}$ player is $v_i - h_i < 0$.

If $b_i < h_i$

then payoff for $i^{th}$ player is 0.

Since $v_i < h_i$

$\therefore$ in this case, $best_i = v_i$

Case II : $h_i \leq v_i$

- If $b_i = v_i$,

then payoff for $i^{th}$ player $= v_i - h_i \geq 0$

If $b_i > v_i$

then payoff for $i^{th}$ player $= v_i - h_i$

If ~~bes~~ $v_i > b_i > h_i$

then payoff for ith player $= v_i - h_i$

If $b_i < h_i$

then payoff for ith player $= 0$

Hence, in this case also, $best_i = v_i$.

Ques 6

(a) $\pi_i(b_i, h_i) = \begin{cases} v_i - (h_i+1) & \text{if } b_i > h_i \\ 0 & \text{otherwise} \end{cases}$

(assuming increments of 1 in the bidding process)

(b)

(i) Let $b_i < (v_i - 1)$

~~Case I:~~ If $h_i \geq v_i - 1$

Payoff for ith player with strategy $(v_i - 1) = 0$

Payoff for ith player with strategy $(b_i) = 0$

If $b_i \leq h_i < v_i - 1$

Payoff with $(v_i - 1) = v_i - (h_i + 1) > 0$

Payoff with $(b_i) = 0$

If $h_i < b_i$

Payoff with $(v_i - 1) = v_i - (h_i + 1)$

Payoff with $(b_i) = v_i - (h_i + 1)$

Hence $v_i - 1 \succcurlyeq b_i$.

If $h_i \geq v_i$

Payoff with $(v_i) = 0$

Payoff with $(b_i) = 0$

If $h_i = v_{i-1}$

Payoff with $(v_i) = 0$

Payoff with $(b_i) = 0$

If $b_i \leq h_i < v_i - 1$

Payoff with $(v_i) = v_i - (h_i + 1) > 0$

Payoff with $(b_i) = 0$

If $h_i < b_i$

Payoff with $(v_i) = v_i - (h_i + 1)$

Payoff with $(b_i) = v_i - (h_i + 1)$

Hence $v_i \succcurlyeq b_i$

ii) Let $b_i > v_i$

If $\underline{h_i \geq b_i}$

Payoff with $(v_i) = 0$
Payoff with $(v_i - 1) = 0$
Payoff with $(b_i) = 0$

If $\underline{v_i \leq h_i < b_i}$

Payoff with $(v_i) = 0$
Payoff with $(v_i - 1) = 0$
Payoff with $(b_i)$ ~~[illegible]~~
$= v_i - (h_i + 1) < 0$

If $\underline{h_i < v_i}$

Payoff with $(v_i) = v_i - (h_i + 1)$
Payoff with $(v_i - 1) = v_i - (h_i + 1)$
Payoff with $(b_i) = v_i - (h_i + 1)$

Hence, $v_i \geq b_i$ and $v_i - 1 \geq b_i$

(c) If $\underline{v_i \leq h_i}$

Payoff with $(v_i) = 0$
Payoff with $(v_i - 1) = 0$

If $\underline{h_i < v_i}$
Payoff with $(v_i) = v_i - (h_i + 1)$
Payoff with $(v_i - 1) = v_i - (h_i + 1)$

Thus, both strategies give equal payoff in every case and hence neither weakly dominates the other.