Probability Distribution function & Types of Distribution

• probobility distribution functions describe how the probabilities are distributed over the value of random variable.

There are three main type of distribution function

1] Probability mass functions (pmf) → used for discrete random variable.

2] Probability Density function (PDF) → Used for Continuous random variable.

3] Cumulative Distribution function (CDF)

1] pmf (Probability Mass function)

• pmf is used for countable things (discrete numbers)
• It gives exact probability of value happening.

eg → In dice roll

$$P(X=3) = \frac{1}{6}$$

pmf tell's What is the chance of getting particular number

2] Probability Density function (PDF)

- PDF is used for uncountable things (continous number)
- It gives shape of probability but not exact point chance.
- You get chance by taking area under curve.

eg → Height of people.

You can't say that height of being exactly 175.5cm (its 0)

But you can say

"Chance of height between 170 & 180 cm = area under PDF curve"

3] CDF (Cumulative Distribution function).

- CDF shows total chance from the starting point up to any value.
- It keeps adding probability till that point.

Example →

Dice roll → $P(X \leq 4) = P(1)+P(2)+P(3)+P(4) = \frac{4}{6}$.

Types of distribution

1) Bernoulli Distribution. → (PMF)
2) Binomial Distribution. → (PMF)
3) Normal / Gaussian Distribution → (PDF)
4) Poisson Distribution (PMF)
5) Log Normal Distribution (PDF)
6) Uniform Distribution (PMF)

1) Bernoulli distribution

The Bernoulli distribution is the simplest probability distribution. It represent the probability distribution of a random variable that has exactly two possible outcomes. success (with probability p) & failure (with probability $1-p=q$). It is used to model binary outcomes, such as a coin flip or yes/no question.

$$\boxed{\text{pmf} = p^{K} * (1-p)^{1-K}}$$

$$\text{pmf} \begin{cases} q = 1-p & \text{if } K=0 \\ p & \text{if } K=1 \end{cases}$$

2] Binomial Distribution

$$P(r) = {}^{n}C_{r} \, (p)^{r} \cdot (q)^{n-r}$$

n = no of trials

p = probability of success

$q = 1-p$ as $\boxed{p+q=1}$

r = result

mean = np

variance = npq

Standard Deviation = $\sqrt{npq}$

3] Poisson Distribution

$$P(r) = \frac{e^{-z} \cdot (z)^{r}}{r!}$$

$e = 2.718281828$

z = np

r = result

mean = variance = z = np

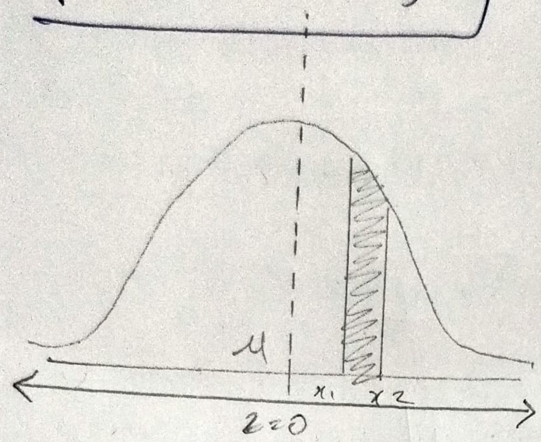
std. deviation = $\sqrt{np}$

4] Normal / Gaussian Distribution.

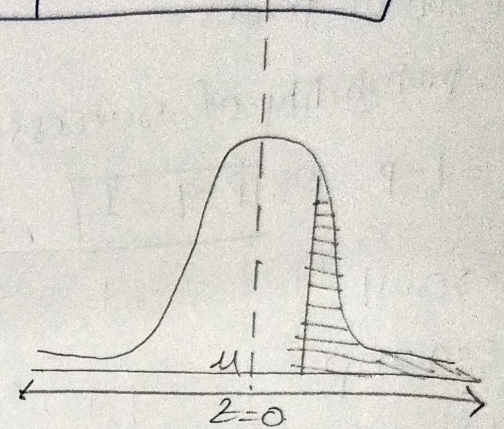
$$y = f(x) = \frac{1}{\sigma(\sqrt{2\pi})} e^{-1/2\left(\frac{x_i - \mu}{\sigma}\right)^2}$$

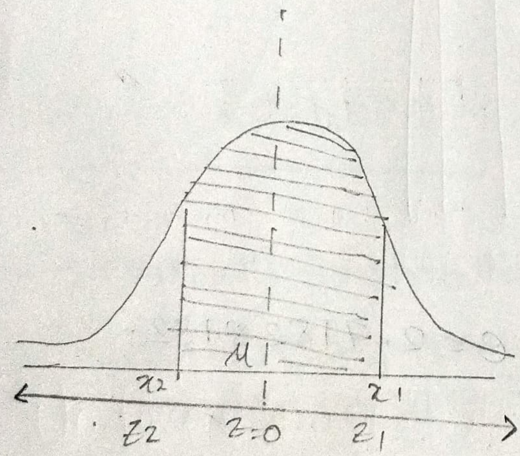
$$\boxed{z_1 = \frac{x_1 - \mu}{\sigma}} \qquad \boxed{z_2 = \frac{x_2 - \mu}{\sigma}}$$

$\boxed{A(z_2) - A(z_1)}$

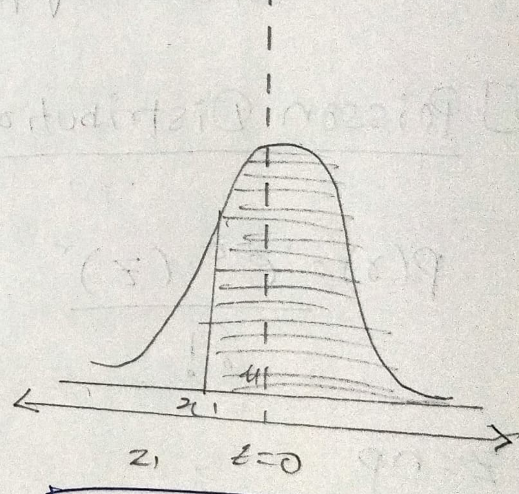


$\boxed{0.5 - A(z_1)}$





$\boxed{A(z_1) + A(z_2)}$



$\boxed{0.5 + A(z_1)}$

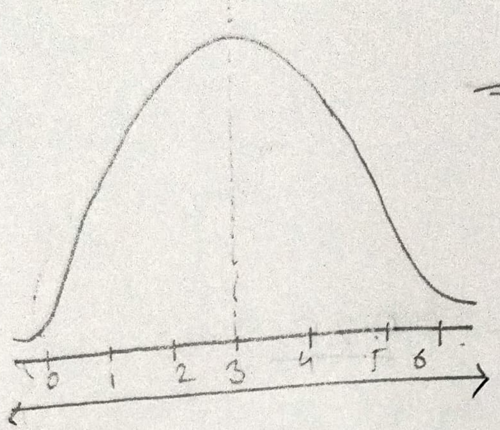
Mean of Normal / Gaussian.

$$\boxed{\mu = \sum_{i=1}^{n} x_i}$$

Variance

$$\sigma^2 = \sum_{i=1}^{n} \frac{(x_i - \mu)^2}{n} \qquad \sigma = \sqrt{\text{variance}}.$$

5) Standard Normal distribution & Z score



$\Longrightarrow \mu = 0$

$\sigma = 1$

$x = \{1, 2, 3, 4, 5\}$

$$z\text{-score} = \frac{x_i - \mu}{\sigma}$$

$$y = \{-2, -1, 0, 1, 2\}$$

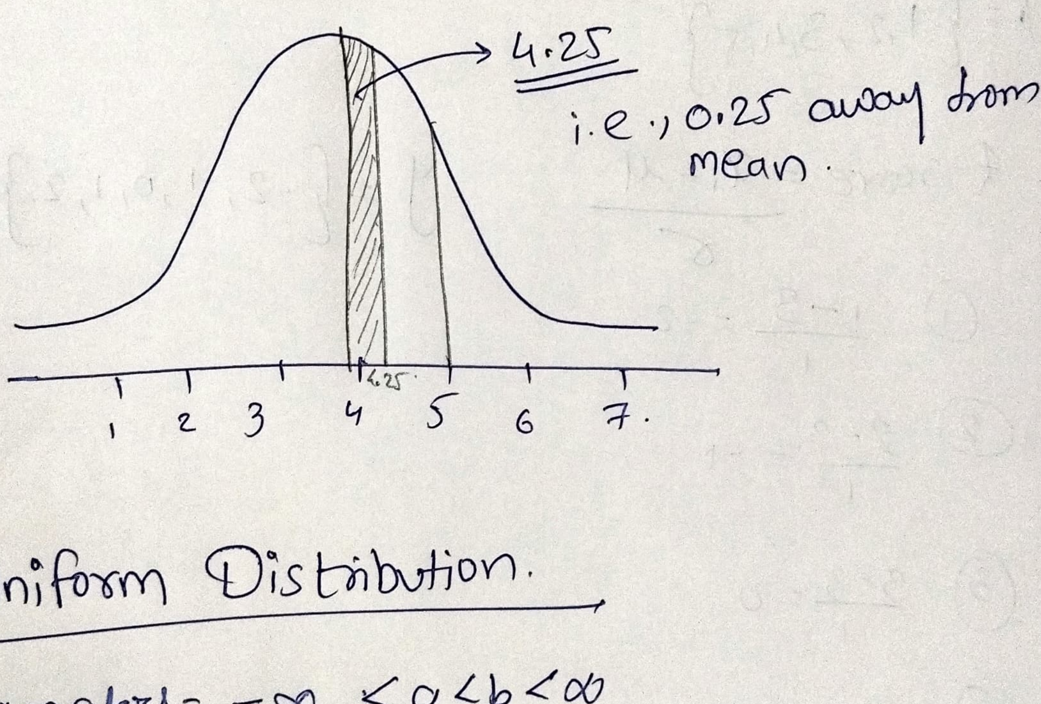
1. $\frac{1-3}{1} = -2$
2. $\frac{2-3}{1} = -1$
3. $\frac{3-3}{1} = 0$
4. $\frac{4-3}{1} = 1$
5. $\frac{5-3}{1} = 2$

Q] How many standard deviation 4.25 is away from the mean?

→ $x_i = 4.25$

$$Z\text{-score} = \frac{x_i - \mu}{\sigma}$$

$$= \frac{4.25 - 4}{1} = \underline{0.25}$$



6] Uniform Distribution.

parameters = $-\infty < a < b < \infty$

$$\text{Pdf} = \begin{cases} \frac{1}{b-a} & x \in [a, b] \\ 0 & \text{otherwise.} \end{cases}$$

$$\text{cdf} = \begin{cases} 0 & \text{for } x < a \\ \frac{x-a}{b-a} & \text{for } x \in [a, b) \\ 1 & \text{for } x > b \end{cases}$$

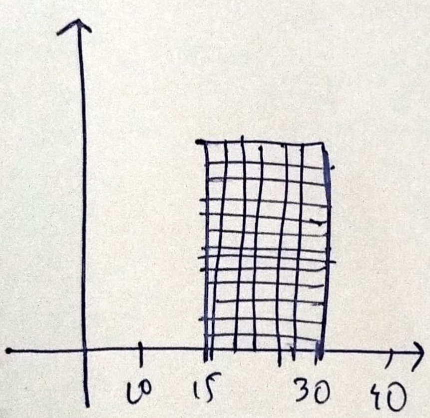
Mean $= \frac{1}{2}(a+b)$

Median $= \frac{1}{2}(a+b)$

Variance $= \frac{1}{12}(b-a)^2$

Q] eg, The no of candies sold daily at shop is uniformly distributed with a maximum of 40 candies & minimum of 10 candies.

i] probability of daily sales to fall between 15 & 30?



$x_1 = 15$ $\quad a = 10$

$x_2 = 30$ $\quad b = 40.$

$$Pr(15 \leq x \leq 30) = \frac{x_2 - x_1}{b-a} = \frac{30-15}{40-10} = \frac{\cancel{15}}{\cancel{30}_2} = \underline{0.5}$$