Optimization
Project 1 – Linear Programming
Group 11
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In this project, we use linear programming to build a simple marketing budget allocation strategy.

Objective

To build a simple marketing budget allocation strategy, subject to constraints and budget limits so as to maximize the overall returns on investment.

1. We have a total budget of $10M, and received the following Return on Investment data from the first consulting firm:

Platform	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
ROI	3.1%	4.9%	2.4%	3.9%	1.6%	2.4%	4.6%	2.6%	3.3%	4.4%

2. In addition to this, we have the following three constraints which we need to consider to create the optimal budget allocation for the above 10 channels:
    - The amount invested in print and TV should be no more than the amount spent on Facebook and Email.
    - The total amount used in social media (Facebook, LinkedIn, Instagram, Snapchat, and Twitter) should be at least twice of SEO and AdWords.
    - For each platform, the amount invested should be no more than $3M.
3. We formulated the above constraints in Python as shown below:

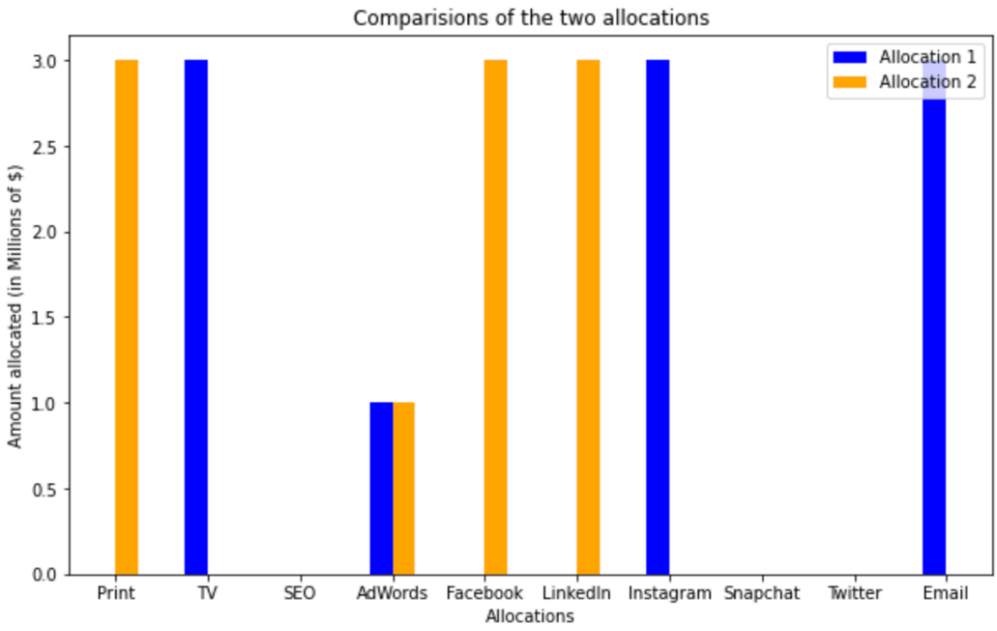
```
A = np.zeros((13,10)) # initialize constraint matrix
A[0,:] = [1,1,0,0,-1,0,0,0,0,-1] # amount invested (print and TV is no more than FB and Email) constraint
A[1,:] = [0,0,2,2,-1,-1,-1,-1,-1,0] # social media (at least twice of SEO and AdWords) constraint
A[2,:] = [1,0,0,0,0,0,0,0,0,0] # Print individual constraint
A[3,:] = [0,1,0,0,0,0,0,0,0,0] # TV individual constraint
A[4,:] = [0,0,1,0,0,0,0,0,0,0] # SEO individual constraint
A[5,:] = [0,0,0,1,0,0,0,0,0,0] # AdWords individual constraint
A[6,:] = [0,0,0,0,1,0,0,0,0,0] # Facebook individual constraint
A[7,:] = [0,0,0,0,0,1,0,0,0,0] # LinkedIn individual constraint
A[8,:] = [0,0,0,0,0,0,1,0,0,0] # Instagram individual constraint
A[9,:] = [0,0,0,0,0,0,0,1,0,0] # Snapchat individual constraint
A[10,:] = [0,0,0,0,0,0,0,0,1,0] # Twitter individual constraint
A[11,:] = [0,0,0,0,0,0,0,0,0,1] # Email individual constraint
A[12,:] = [1,1,1,1,1,1,1,1,1,1] # Budget constraint

b = np.array([0,0,3,3,3,3,3,3,3,3,3,10]) # limits on production, storage, and demand
sense = np.array(['<','<','<','<','<','<','<','<','<','<','<','<','<']) # all constraints are less than or equal
```

4. We used Gurobi to optimize this problem, which would give us the maximum return and the allocation to individual channels i.e. how should we distribute $10M among the ten channels so as to have the maximum return subject to given constraints.
5. Using returns on investment data obtained from the first consulting company, we can make a maximum profit of $ 456K.

6. And the allocation is as follows:
    - 3.0 M for TV
    - 1.0 M for Adwords
    - 3.0 M for Instagram
    - 3.0 M for Email, $0M for all other mediums
7. This allocation allocates $3M to the channels giving maximum ROIs individually and the remaining $1M to AdWords, the next highest in line.
8. Also, this allocation covers a diverse forms of channels, TV for video advertisements, Instagram being the most trending social media platform, Adwords a platform ensuring that whenever someone searches for a service similar to what we provide, our suggestion comes up and Email as specified in the problem statement document, provides a personal touch while reaching out to the customers.
9. To be cautious about the decision, we rerun the analysis with return on investment data from another consulting firm with all the constraints remaining the same.
10. The second consulting firm returns the estimates of the ROI data in the table below:

Platform	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
ROI	4.9%	2.3%	2.4%	3.9%	4.4%	4.6%	2.6%	1.9%	3.7%	2.6%

11. Using returns on investment data obtained from the second consulting company, we again got a maximum profit of $ 456K.
12. However, the allocation of $10M changed among the advertisement channels:
    - 3.0 M for Print
    - 1.0 M for Adwords
    - 3.0 M for Facebook
    - 3.0 M for LinkedIn, $0M for all other mediums
13. Below is a graph visualizing the two allocations obtained above:



14. Both the distributions ROIs give the same profit of 456K Dollars but for different allocations, The allocation based on first ROI implies to invest 3M Dollars in TV, Instagram and Email and 1M Dollars in Adwords whereas the allocation based on second ROI implies to invest 3M Dollars in Print, Facebook and LinkedIn and 1M Dollars in AdWords.

15. Both allocations differ majorly except for AdWords, both the allocations suggest investing in different channels for reaching out to people.

16. Also, in both cases, there is no allocation for SEO, Snapchat, and Twitter channels.
17. Assuming the first ROI data is correct, if we were to use the second allocation, how much lower would the objective be relative to the optimal objective - to answer this question, we substitute the optimal allocations obtained from the second ROI data, into the objective of the first ROI data to obtain the revenue for this case.
18. Using ROI data obtained from the First consulting company, and substituting the optimal allocation obtained from second allocation, the objective would be $252K, **$204K** lower than the optimal revenue/objective obtained by using first ROI objective and first optimal allocation.
19. Similarly, If we use the allocation obtained from second consulting company, and substitute the objective obtained from the first ROI, we get the revenue for that particular case to be $264K, **$192K** lower than the optimal revenue obtained by using second ROI objective and second optimal allocation.
20. Based on the above analysis, suggestion from our boss' experience of considering multiple ROI data is useful because, if we consider second ROI data to be actually matching the market trends, and we perform optimization of revenue using allocations obtained by using first company's ROI data, we would lose out on $192K and same for the other case i.e. if we consider First ROI data to be actually matching the market trends, and we perform optimization the revenue using allocations obtained by using second company's ROI data, we would be losing out on $204K.
21. To examine whether the third constraint above of each investment to be capped at $3M is useful or not, let us run a model without any of those constraints, on both, First ROI data and Second ROI data.

```
A = np.zeros((3,10)) # initialize constraint matrix
A[0,:] = [1,1,0,0,-1,0,0,0,0,-1] # amount invested (print and TV is no more than FB and Email) constraint
A[1,:] = [0,0,2,2,-1,-1,-1,-1,-1,0] # social media (at least twice of SEO and AdWords) constraint
A[2,:] = [1,1,1,1,1,1,1,1,1,1] # Budget constraint

b = np.array([0,0,10]) # limits on production, storage, and demand
sense = np.array(['<','<','<']) # all constraints are less than or equal constraints
```

22. Removing the third constraint of capping each investment at $3M, and using ROI data from the first company, we can make a maximum profit of 465.0 K Dollars, with the following allocations:
    - 5.0 M for TV
    - 5.0 M for Email, $0 for all other channels
23. Similarly, using the second ROI data, we again get a maximum profit of $465 K, but with the following allocations:
    - 5.0 M for Print
    - 5.0 M for Facebook, $0 for all other channels
24. As we can see above, if we remove the third constraint based on boss' experience, we get a higher return of $465K, $9K dollars higher than the objective with the 3rd constraint in both cases of using first ROI data and second ROI data as well.

25. However, this particular allocation takes a hit with respect to the diversity of investment channels being chosen. In the previous case, there were 4 allocations in each case where as in this case there are only 2, with $10 M being split among two channels which provide the highest ROI according to each ROI data. Thus, in order to cover more channels for advertisement, **the third constraint is useful.**
26. To determine by how much could each advertising medium's ROI increase or decrease and still result in the same optimal allocation, we perform sensitivity analysis and get the following results for each advertising mediums ROI (Considering First Company's ROI data)
    - Print: negative infinity to 0.049
    - TV: 0.039 to 0.062
    - SEO: negative infinity to 0.039
    - AdWords: 0.033 to 0.046
    - Facebook: negative infinity to 0.029
    - LinkedIn: negative infinity to 0.039
    - Instagram: 0.039 to positive infinity
    - Snapchat: negative infinity to 0.039
    - Twitter: negative infinity to 0.039
    - Email: 0.029 to positive infinity
27. However, to verify these values, we considered 0 as lowest ROI and replaced -inf with 0 and 100% as highest ROI and replaced +inf with 1 and got the below upper and lower bounds:

Channel	Lower Bound	Upper Bound
Print	0.000	0.049
TV	0.039	0.062
SEO	0.000	0.039
AdWords	0.033	0.046
Facebook	0.000	0.029
LinkedIn	0.000	0.039
Instagram	0.039	1.000
Snapchat	0.000	0.039
Twitter	0.000	0.039
Email	0.029	1.000

28. We verified that by using these upper and lower bound ROIs the optimal allocation stays the same as the one we found in point 6.
    - 3.0 M for TV
    - 1.0 M for AdWords
    - 3.0 M for Instagram
    - 3.0 M for Email, $0M for all other mediums
29. However, the objective value i.e. the total revenue as profit increases to $6.232M using the upper bounds as the ROIs and reduces to $354K upon using the lower bound as ROIs.

30. As we have the opportunity to invest half of the monthly returns, and ROIs for each month varying as shown in the below table(in %), we run our analysis for each month to optimize the allocation subject to constraints and a different investment value at the start of each month:

	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email
January	4.0	3.6	2.4	3.9	3.0	3.5	3.6	2.25	3.5	3.5
February	4.0	3.9	2.7	3.8	4.3	3.2	2.7	1.80	3.7	3.5
March	3.5	2.9	3.1	3.8	2.4	4.1	3.7	2.60	4.2	2.5
April	3.8	3.1	2.4	4.4	2.4	3.8	3.7	2.50	3.6	2.9
May	3.5	3.2	1.9	3.4	2.7	2.7	3.9	2.20	4.5	3.9
June	4.0	3.2	2.7	3.4	3.4	3.0	4.5	2.10	3.8	4.1
July	3.9	3.6	2.0	4.4	3.9	3.7	4.3	1.80	4.0	3.8
August	4.2	3.3	2.8	4.2	2.0	3.7	3.6	1.50	4.4	4.3
September	4.1	2.8	2.5	4.2	2.9	3.7	2.8	2.50	4.0	3.4
October	3.0	3.0	3.1	4.6	3.1	3.3	3.2	2.30	2.5	3.2
November	4.8	3.3	2.7	4.1	2.9	3.6	4.2	3.00	3.1	4.1
December	4.8	4.0	1.9	3.7	4.2	3.6	2.6	2.90	3.6	3.7

31. Running our analysis subject to the above constraints and original three constraints still in place, we get the following allocation for all the 12 months (in $M) :

	Print	TV	SEO	AdWords	Facebook	LinkedIn	Instagram	Snapchat	Twitter	Email	total investment	optimal
January	3.0	0.0	0.0	1.333333	0.0	0.0	2.666667	0.0	0.0	3.0	10.0	0.373
February	3.0	0.0	0.0	2.3955	3.0	0.0	0.0	0.0	1.791	0.0	10.1865	0.406296
March	0.0	0.0	0.0	3.0	0.0	3.0	1.203148	0.0	3.0	0.0	10.203148	0.407516
April	0.0	0.0	0.0	3.0	0.0	3.0	3.0	0.0	1.203758	0.0	10.203758	0.400335
May	1.200168	0.0	0.0	0.0	0.0	0.0	3.0	0.0	3.0	3.0	10.200168	0.411006
June	3.0	0.0	0.0	0.0	0.0	0.0	3.0	0.0	1.205503	3.0	10.205503	0.423809
July	0.0	0.0	0.0	3.0	1.211905	0.0	3.0	0.0	3.0	0.0	10.211905	0.428264
August	2.714132	0.0	0.0	1.5	0.0	0.0	0.0	0.0	3.0	3.0	10.214132	0.437994
September	0.609498	0.0	0.0	3.0	0.0	3.0	0.0	0.0	3.0	0.609498	10.218997	0.402712
October	0.0	0.0	0.0	3.0	0.0	3.0	3.0	0.0	0.0	1.201356	10.201356	0.371443
November	3.0	0.0	0.0	1.185722	0.0	0.0	3.0	0.0	0.0	3.0	10.185722	0.441615
December	3.0	2.110404	0.0	0.0	3.0	0.0	0.0	0.0	0.0	2.110404	10.220807	0.432501

32. The total investment column shows how much capital was available for investment for that particular month, which is $10M +50% of the optimal return from the previous month.
33. As per the above allocations it is not a stable allocation because it is clear that there is a monthly change in spend which is more than $1M. For example, the Facebook channel in January has a 0 allocation but in the February month it has an allocation of $3M.
34. This is clearly a difference of more than $1M. Likewise the other platforms have a variations more than $1M among consecutive months, making this budget unstable.
35. To combat the unstable budget, in the new model, a constraint needs to be added which makes sure that in each platform the monthly change in spend is no more than $1M. This is similar to the constraint that no investment should exceed $3M.

CONCLUSION:

We have successfully come up with effective allocation strategies of the advertisement budget for marketing purposes. Subject to constraints given, we can maximise the returns as well as market using a diverse set of platforms, ensuring we are not just sticking to one platform which has the highest Return on investment. Choosing just one platform with highest ROI might help us in the short run by maximising the revenue, but in the long run it will hamper us in terms of customer reach and eventually on profitability as well.