BT5110: Tutorial 4 — Normal Forms

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Question

Your company, Apasaja Private Limited, is commissioned by *Toko Kopi Luwak* to design the relational schema for the management of their coffee beans, drinks, and cafes.

A coffee bean is fully identified by its unique brand name *or* by the pair (cultivar, region). A drink can be made using a particular coffee bean; the drink name is only unique *per* bean (e.g., "Espresso" with bean "The Waterfall" or with bean "La Bella"). We also record the drink price.

A branch (identified by branch name) may sell zero or more drinks; a drink may be sold by zero or more branches. Each branch records an address, and for each (branch, drink) pair we record the quantity sold.

We are given only an abstract schema:

$$R = \{A, B, C, D, E, F, G, H\}$$

$$\Sigma = \{\{A\} \rightarrow \{C, E\}, \{A, B\} \rightarrow \{D\}, \{F\} \rightarrow \{H\}, \{C, E\} \rightarrow \{A\},$$

$$\{\text{B,C,E}\} \rightarrow \{D\}, \{A, B, F\} \rightarrow \{D, G\}, \{B, C, E, F\} \rightarrow \{G\}\}.$$

Question

1. Normal Form

(a) Is R with Σ in 3NF?

(b) Is R with Σ in BCNF?

2. Normalisation

(a) Decompose (synthesise) R with Σ into a 3NF decomposition using the lecture algorithm.

(b) Is the result lossless?

(c) Is the result dependency preserving?

(d) Is the result in BCNF?

(e) Decompose R with Σ into a BCNF decomposition using the lecture algorithm.

(f) Is the result lossless?

(g) Is the result dependency preserving?

1(a). Is R with Σ in 3NF?

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Criteria for 3NF

If $X \rightarrow \{A\} \in \Sigma$:

- $X \rightarrow \{A\}$ is trivial or
- X is a superkey or
- A is a prime attribute

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- X is a superkey or
- A is a prime attribute

Known keys (from previous tutorial): $\{A, B, F\}$ and $\{B, C, E, F\}$.
Prime attributes: $\{A, B, C, E, F\}$.

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Consider $\{A, B\} \rightarrow \{D\}$:

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- Nontrivial.

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Consider $\{A, B\} \rightarrow \{D\}$:

- Nontrivial.
- $\{A, B\}$ is *not* a superkey.

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Consider $\{A, B\} \rightarrow \{D\}$:

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- $\{A, B\}$ is *not* a superkey.
- D is not a prime attribute.

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Known keys (from previous tutorial): $\{A, B, F\}$ and $\{B, C, E, F\}$.
Prime attributes: $\{A, B, C, E, F\}$.
Consider $\{A, B\} \rightarrow \{D\}$:

- Nontrivial.
- $\{A, B\}$ is *not* a superkey.
- D is not a prime attribute.

$\Rightarrow$ R with Σ **is not in 3NF.**

1(b). Is R with Σ in BCNF?

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Criteria for 3NF

If $X \to \{A\} \in \Sigma$:

- $X \to \{A\}$ is trivial or
- X is a superkey or
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1(b). Is R with Σ in BCNF?

Criteria for 3NF

If $X \rightarrow \{A\} \in \Sigma$:

- $X \rightarrow \{A\}$ is trivial or
- X is a superkey or
- A is a prime attribute

Criteria for BCNF

If $X \rightarrow \{A\} \in \Sigma$:

- $X \rightarrow \{A\}$ is trivial or
- X is a superkey

1(b). Is R with Σ in BCNF?

Criteria for 3NF

If $X \rightarrow \{A\} \in \Sigma$:

- $X \rightarrow \{A\}$ is trivial or
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Criteria for BCNF

If $X \rightarrow \{A\} \in \Sigma$:

- $X \rightarrow \{A\}$ is trivial or
- X is a superkey

Since not 3NF, cannot be BCNF.
A direct violation: $\{A\} \rightarrow \{C\}$ is nontrivial and

$$A^+ = \{A, C, E\} \subset R \quad \Rightarrow \quad \{A\} \text{ is not a superkey.}$$

So R with Σ **is not in BCNF**.

2(a). 3NF synthesis (decomposition)

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Canonical cover: $\{ \{A\} \rightarrow \{C, E\}, \{F\} \rightarrow \{H\}, \{C, E\} \rightarrow \{A\}, \{B, C, E\} \rightarrow \{D\}, \{B, C, E, F\} \rightarrow \{G\} \}$.

2(a). 3NF synthesis (decomposition)

Canonical cover: $\{\ \{A\} \rightarrow \{C, E\},\ \{F\} \rightarrow \{H\},\ \{C, E\} \rightarrow \{A\},\ \{B, C, E\} \rightarrow \{D\},\ \{B, C, E, F\} \rightarrow \{G\}\ \}$.
Candidate keys: $\{A, B, F\}, \{B, C, E, F\}$

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Candidate keys: $\{A, B, F\}, \{B, C, E, F\}$

Fragments:

$R_1 = \{A, C, E\}, \quad R_2 = \{F, H\},$

$\mathrm{R}_3 = \{A, C, E\}, \quad R_4 = \{B, C, D, E\},$

$\mathrm{R}_5 = \{B, C, E, F, G\}$

2(a). 3NF synthesis (decomposition)

Canonical cover: $\{ \{A\} \rightarrow \{C, E\}, \{F\} \rightarrow \{H\}, \{C, E\} \rightarrow \{A\}, \{B, C, E\} \rightarrow \{D\}, \{B, C, E, F\} \rightarrow \{G\} \}$.

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$R_1 = \{A, C, E\}$, $R_2 = \{F, H\}$,
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Remove subsumed/duplicates: R_1 and R_3 are same. So, we remove R_3.

A key fragment is not needed (the key $\{B, C, E, F\} \subseteq \{B, C, E, F, G\}$ is covered).

2(b). Is the result dependency preserving?

Dependency preserving: Yes, guaranteed by the algorithm (each FD is placed in some fragment, and cover equivalence is preserved).

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Dependency preserving: Yes, guaranteed by the algorithm (each FD is placed in some fragment, and cover equivalence is preserved).
Also:
Lossless: Yes, guaranteed by the 3NF synthesis algorithm.

2(c). Is the result in BCNF?

[1] Dependency projection algorithm: Algorithm 1

2(c). Is the result in BCNF?

$R_1 = \{A, C, E\}$
$\Sigma_1 = \{\{A\} \rightarrow \{C\}, \{A\} \rightarrow \{E\}, \{C, E\} \rightarrow \{A\}\}$
Candidate keys:
$\{A\}, \{C, E\}$

[1] Dependency projection algorithm: Algorithm 1

2(c). Is the result in BCNF?

$R_1 = \{A, C, E\}$
$\Sigma_1 = \{\{A\} \rightarrow \{C\}, \{A\} \rightarrow \{E\}, \{C, E\} \rightarrow \{A\}\}$
Candidate keys: $\{A\}, \{C, E\}$

$R_2 = \{F, H\}$
$\Sigma_2 = \{\{F\} \rightarrow \{H\}\}$
Candidate keys: $\{F\}$

[1] Dependency projection algorithm: Algorithm 1

2(c). Is the result in BCNF?

$R_1 = \{A, C, E\}$
$\Sigma_1 = \{\{A\} \rightarrow \{C\}, \{A\} \rightarrow \{E\}, \{C, E\} \rightarrow \{A\}\}$
Candidate keys: $\{A\}, \{C, E\}$

$R_2 = \{F, H\}$
$\Sigma_2 = \{\{F\} \rightarrow \{H\}\}$
Candidate keys: $\{F\}$

$R_4 = \{B, C, D, E\}$
$\Sigma_4 = \{\{B, C, E\} \rightarrow \{D\}\}$
Candidate keys: {B,C,E}

[1] Dependency projection algorithm: Algorithm 1

2(c). Is the result in BCNF?

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$\Sigma_1 = \{\{A\} \rightarrow \{C\}, \{A\} \rightarrow \{E\}, \{C, E\} \rightarrow \{A\}\}$
Candidate keys: $\{A\}, \{C, E\}$

$R_2 = \{F, H\}$
$\Sigma_2 = \{\{F\} \rightarrow \{H\}\}$
Candidate keys: $\{F\}$

$R_4 = \{B, C, D, E\}$
$\Sigma_4 = \{\{B, C, E\} \rightarrow \{D\}\}$
Candidate keys: {B,C,E}

$R_5 = \{B, C, E, F, G\}$
$\Sigma_5 = \{\{B, C, E, F\} \rightarrow \{G\}\}$
Candidate keys: {B,C,E,F}

[1] Dependency projection algorithm: Algorithm 1

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Candidate keys: $\{A\}, \{C, E\}$

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$\Sigma_2 = \{\{F\} \rightarrow \{H\}\}$
Candidate keys: $\{F\}$

$R_4 = \{B, C, D, E\}$
$\Sigma_4 = \{\{B, C, E\} \rightarrow \{D\}\}$
Candidate keys: {B,C,E}

$R_5 = \{B, C, E, F, G\}$
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$\Sigma_1, \Sigma_2, \Sigma_4$ and Σ_5 are projected dependencies. We obtain these by projecting Σ on R_1, R_2, R_4 and R_5 respectively.[1]

[1] Dependency projection algorithm: Algorithm 1

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$\Sigma_2 = \{\{F\} \rightarrow \{H\}\}$
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$R_4 = \{B, C, D, E\}$
$\Sigma_4 = \{\{B, C, E\} \rightarrow \{D\}\}$
Candidate keys: {B,C,E}

$R_5 = \{B, C, E, F, G\}$
$\Sigma_5 = \{\{B, C, E, F\} \rightarrow \{G\}\}$
Candidate keys: {B,C,E,F}

$\Sigma_1, \Sigma_2, \Sigma_4$ and Σ_5 are projected dependencies. We obtain these by projecting Σ on R_1, R_2, R_4 and R_5 respectively.[1]

R_1 with Σ_1, R_2 with Σ_2, R_4 with Σ_4, and R_5 with Σ_5 satisfy the BCNF criteria individually.

[1] Dependency projection algorithm: Algorithm 1

2(c). Is the result in BCNF?

$R_1 = \{A, C, E\}$
$\Sigma_1 = \{\{A\} \rightarrow \{C\}, \{A\} \rightarrow \{E\}, \{C, E\} \rightarrow \{A\}\}$
Candidate keys: $\{A\}, \{C, E\}$

$R_2 = \{F, H\}$
$\Sigma_2 = \{\{F\} \rightarrow \{H\}\}$
Candidate keys: $\{F\}$

$R_4 = \{B, C, D, E\}$
$\Sigma_4 = \{\{B, C, E\} \rightarrow \{D\}\}$
Candidate keys: {B,C,E}

$R_5 = \{B, C, E, F, G\}$
$\Sigma_5 = \{\{B, C, E, F\} \rightarrow \{G\}\}$
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$\Sigma_1, \Sigma_2, \Sigma_4$ and Σ_5 are projected dependencies. We obtain these by projecting Σ on R_1, R_2, R_4 and R_5 respectively.[1]

R_1 with Σ_1, R_2 with Σ_2, R_4 with Σ_4, and R_5 with Σ_5 satisfy the BCNF criteria individually. Thus the result is in BCNF. (This is not a general case though. There is no guarantee that synthesising a 3NF will yield a BCNF too.)

[1] Dependency projection algorithm: Algorithm 1

Dependency Projection Algorithm

Algorithm 1 Dependency Projection Algorithm[2]

Require: Σ: Set of functional dependencies
Require: R': Set of attributes to project on
Ensure: Σ': Projected set of functional dependencies

```
Initialize Σ' ← ∅
for each (lhs → rhs) in Σ do
    if lhs is a subset of R' then
        y ← rhs ∩ R'
        if y is not empty then
            Σ' ← Σ' ∪ {{lhs → y}}
        end if
    end if
end for
return Σ'
```

[2] Link to code→ Cell 1: project_dependency

Dependency Projection Example

$R = \{A, B, C, D, E\}$

$\Sigma =$
$\{ \{ \text{A} \} \rightarrow \{A, B, C\},$
$\{ \text{A, B} \} \rightarrow \{A\},$
$\{ \text{B, C} \} \rightarrow \{A, D\},$
$\{ \text{B} \} \rightarrow \{A, B\},$
$\{ \text{C} \} \rightarrow \{D\}\}$

$R' = \{A, B, C\}$

Dependency Projection Example

$$\Sigma' = \emptyset$$

$R = \{A, B, C, D, E\}$

$\Sigma =$
{ { A } $\rightarrow \{A, B, C\}$,
{ A, B } $\rightarrow \{A\}$,
{ B, C } $\rightarrow \{A, D\}$,
{ B } $\rightarrow \{A, B\}$,
{ C } $\rightarrow \{D\}$}

$R' = \{A, B, C\}$

Dependency Projection Example

$\Sigma' = \emptyset$

$\{A\} \rightarrow \{A, B, C\}$
$lhs = \{A\}$ **is a subset of** R'
$rhs = \{A, B, C\}$
y = rhs $\cap R' = \{A, B, C\} \cap \{A, B, C\} = \{A, B, C\} \neq \emptyset$
$\Sigma' = \{\{A\} \rightarrow \{A, B, C\}\}$

$R = \{A, B, C, D, E\}$

$\Sigma =$
$\{ \{ \text{A} \} \rightarrow \{A, B, C\},$
$\{ \text{A, B} \} \rightarrow \{A\},$
$\{ \text{B, C} \} \rightarrow \{A, D\},$
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$\{ \text{A, B} \} \rightarrow \{A\},$
$\{ \text{B, C} \} \rightarrow \{A, D\},$
$\{ \text{B} \} \rightarrow \{A, B\},$
$\{ \text{C} \} \rightarrow \{D\}\}$

$R' = \{A, B, C\}$

$\Sigma' = \emptyset$

$\{A\} \rightarrow \{A, B, C\}$
$lhs = \{A\}$ **is a subset of** R'
$rhs = \{A, B, C\}$
$\mathbf{y = rhs} \cap R' = \{A, B, C\} \cap \{A, B, C\} = \{A, B, C\} \neq \emptyset$
$\Sigma' = \{\{A\} \rightarrow \{A, B, C\}\}$

$\{A, B\} \rightarrow \{A\}$
$lhs = \{A, B\}$ **is a subset of** R'
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$\mathbf{y = rhs} \cap R' = \{A\} \cap \{A, B, C\} = \{A\} \neq \emptyset$
$\Sigma' = \{\{A\} \rightarrow \{A, B, C\}, \{A, B\} \rightarrow \{A\}\}$

Dependency Projection Example

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$\{ \text{A, B} \} \rightarrow \{A\},$
$\{ \text{B, C} \} \rightarrow \{A, D\},$
$\{ \text{B} \} \rightarrow \{A, B\},$
$\{ \text{C} \} \rightarrow \{D\}\}$

$R' = \{A, B, C\}$

$\Sigma' = \emptyset$

$\{A\} \rightarrow \{A, B, C\}$
$lhs = \{A\}$ is a subset of R'
$rhs = \{A, B, C\}$
y = rhs $\cap R' = \{A, B, C\} \cap \{A, B, C\} = \{A, B, C\} \neq \emptyset$
$\Sigma' = \{\{A\} \rightarrow \{A, B, C\}\}$

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$lhs = \{A, B\}$ is a subset of R'
$rhs = \{A\}$
y = rhs $\cap R' = \{A\} \cap \{A, B, C\} = \{A\} \neq \emptyset$
$\Sigma' = \{\{A\} \rightarrow \{A, B, C\}, \{A, B\} \rightarrow \{A\}\}$

$\{B, C\} \rightarrow \{A, D\}$
$lhs = \{B, C\}$ is a subset of R'
$rhs = \{A, D\}$
y = rhs $\cap R' = \{A, D\} \cap \{A, B, C\} = \{A\} \neq \emptyset$
$\Sigma' = \{\{A\} \rightarrow \{A, B, C\}, \{A, B\} \rightarrow \{A\}, \{B, C\} \rightarrow \{A\}\}$

Dependency Projection Example

$R = \{A, B, C, D, E\}$

$\Sigma =$
$\{ \{ A \} \rightarrow \{A, B, C\},$
$\{ A, B \} \rightarrow \{A\},$
$\{ B, C \} \rightarrow \{A, D\},$
$\{ B \} \rightarrow \{A, B\},$
$\{ C \} \rightarrow \{D\}\}$

$R' = \{A, B, C\}$

$\Sigma' = \emptyset$

$\{A\} \rightarrow \{A, B, C\}$
$lhs = \{A\}$ is a subset of R'
$rhs = \{A, B, C\}$
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$\Sigma' = \{\{A\} \rightarrow \{A, B, C\}\}$

$\{A, B\} \rightarrow \{A\}$
$lhs = \{A, B\}$ is a subset of R'
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$\Sigma' = \{\{A\} \rightarrow \{A, B, C\}, \{A, B\} \rightarrow \{A\}\}$

$\{B, C\} \rightarrow \{A, D\}$
$lhs = \{B, C\}$ is a subset of R'
$rhs = \{A, D\}$
y = rhs $\cap R' = \{A, D\} \cap \{A, B, C\} = \{A\} \neq \emptyset$
$\Sigma' = \{\{A\} \rightarrow \{A, B, C\}, \{A, B\} \rightarrow \{A\}, \{B, C\} \rightarrow \{A\}\}$

$\{B\} \rightarrow \{A, B\}$
$lhs = \{B\}$ is a subset of R'
$rhs = \{A, B\}$
y = rhs $\cap R' = \{A, B\} \cap \{A, B, C\} = \{A, B\} \neq \emptyset$
$\Sigma' = \{\{A\} \rightarrow \{A, B, C\}, \{A, B\} \rightarrow \{A\}, \{B, C\} \rightarrow \{A\}, \{B\} \rightarrow \{A, B\}\}$

Dependency Projection Example

$R = \{A, B, C, D, E\}$

$\Sigma =$
$\{ \{ A \} \rightarrow \{A, B, C\},$
$\{ A, B \} \rightarrow \{A\},$
$\{ B, C \} \rightarrow \{A, D\},$
$\{ B \} \rightarrow \{A, B\},$
$\{ C \} \rightarrow \{D\}\}$

$R' = \{A, B, C\}$

$\Sigma' = \emptyset$

$\{A\} \rightarrow \{A, B, C\}$
$lhs = \{A\}$ is a subset of R'
$rhs = \{A, B, C\}$
y = rhs $\cap R' = \{A, B, C\} \cap \{A, B, C\} = \{A, B, C\} \neq \emptyset$
$\Sigma' = \{\{A\} \rightarrow \{A, B, C\}\}$

$\{A, B\} \rightarrow \{A\}$
$lhs = \{A, B\}$ is a subset of R'
$rhs = \{A\}$
y = rhs $\cap R' = \{A\} \cap \{A, B, C\} = \{A\} \neq \emptyset$
$\Sigma' = \{\{A\} \rightarrow \{A, B, C\}, \{A, B\} \rightarrow \{A\}\}$

$\{B, C\} \rightarrow \{A, D\}$
$lhs = \{B, C\}$ is a subset of R'
$rhs = \{A, D\}$
y = rhs $\cap R' = \{A, D\} \cap \{A, B, C\} = \{A\} \neq \emptyset$
$\Sigma' = \{\{A\} \rightarrow \{A, B, C\}, \{A, B\} \rightarrow \{A\}, \{B, C\} \rightarrow \{A\}\}$

$\{B\} \rightarrow \{A, B\}$
$lhs = \{B\}$ is a subset of R'
$rhs = \{A, B\}$
y = rhs $\cap R' = \{A, B\} \cap \{A, B, C\} = \{A, B\} \neq \emptyset$
$\Sigma' = \{\{A\} \rightarrow \{A, B, C\}, \{A, B\} \rightarrow \{A\}, \{B, C\} \rightarrow \{A\}, \{B\} \rightarrow \{A, B\}\}$

$\{C\} \rightarrow \{D\}$
$lhs = \{C\}$ is a subset of R'
$rhs = \{D\}$
y = rhs $\cap R' = \{D\} \cap \{A, B, C\} = \emptyset$

Dependency Projection Example

$R = \{A, B, C, D, E\}$

$\Sigma =$
$\{ \{ A \} \rightarrow \{A, B, C\},$
$\{ A, B \} \rightarrow \{A\},$
$\{ B, C \} \rightarrow \{A, D\},$
$\{ B \} \rightarrow \{A, B\},$
$\{ C \} \rightarrow \{D\}\}$

$R' = \{A, B, C\}$

$\Sigma' = \emptyset$

$\{A\} \rightarrow \{A, B, C\}$
$lhs = \{A\}$ is a subset of R'
$rhs = \{A, B, C\}$
y = rhs $\cap R' = \{A, B, C\} \cap \{A, B, C\} = \{A, B, C\} \neq \emptyset$
$\Sigma' = \{\{A\} \rightarrow \{A, B, C\}\}$

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$lhs = \{A, B\}$ is a subset of R'
$rhs = \{A\}$
y = rhs $\cap R' = \{A\} \cap \{A, B, C\} = \{A\} \neq \emptyset$
$\Sigma' = \{\{A\} \rightarrow \{A, B, C\}, \{A, B\} \rightarrow \{A\}\}$

$\{B, C\} \rightarrow \{A, D\}$
$lhs = \{B, C\}$ is a subset of R'
$rhs = \{A, D\}$
y = rhs $\cap R' = \{A, D\} \cap \{A, B, C\} = \{A\} \neq \emptyset$
$\Sigma' = \{\{A\} \rightarrow \{A, B, C\}, \{A, B\} \rightarrow \{A\}, \{B, C\} \rightarrow \{A\}\}$

$\{B\} \rightarrow \{A, B\}$
$lhs = \{B\}$ is a subset of R'
$rhs = \{A, B\}$
y = rhs $\cap R' = \{A, B\} \cap \{A, B, C\} = \{A, B\} \neq \emptyset$
$\Sigma' = \{\{A\} \rightarrow \{A, B, C\}, \{A, B\} \rightarrow \{A\}, \{B, C\} \rightarrow \{A\}, \{B\} \rightarrow \{A, B\}\}$

$\{C\} \rightarrow \{D\}$
$lhs = \{C\}$ is a subset of R'
$rhs = \{D\}$
y = rhs $\cap R' = \{D\} \cap \{A, B, C\} = \emptyset$

$\Sigma' = \{\{A\} \rightarrow \{A, B, C\}, \{A, B\} \rightarrow \{A\}, \{B, C\} \rightarrow \{A\}, \{B\} \rightarrow \{A, B\}\}$