BT5110: Tutorial 3 — Functional Dependencies

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AY25/26 S1

Example

Example Relation, $R = \{A, B, C\}$ and $\Sigma = \{\{A\} \rightarrow \{B\}, \{B\} \rightarrow \{C\}\}$

The column in red decides the color of the row: The rows having same values in the red column, have same color.

Task: Check which other columns are following "same value if same color" for each deciding (red) column. The columns for which this holds, are implied by the red column.

A	B	C
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a2	b2	c2
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a3	b3	c2
a3	b3	c2
a4	b1	c1
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a5	b2	c2

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$\{A\} \rightarrow \{A, B, C\}$

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$\{B\} \rightarrow \{B, C\}$

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$\{B\} \rightarrow \{B, C\}$

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a3	b3	c2
a3	b3	c2
a4	b1	c1
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$\{C\} \rightarrow \{C\}$

Question 1.a.

Your company, Apasaja Private Limited, is commissioned by *Toko Kopi Luwak* to design the relational schema for coffee beans, drinks, and cafes. We are given

$$R = \{A, B, C, D, E, F, G, H\},$$

$\Sigma = \{\{A\} \rightarrow \{C, E\}, \{A, B\} \rightarrow \{D\}, \{F\} \rightarrow \{H\}, \{C, E\} \rightarrow \{A\}, \{B, C, E\} \rightarrow \{D\}, \{A, B, F\} \rightarrow \{D, G\}, \{B, C, E, F\} \rightarrow \{G\}\}$.

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Attribute mapping

Symbol	Meaning
A	`name` (bean brand)
B	`dname` (drink name)
C	`cultivar`
D	`price`

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Attribute mapping

Symbol	Meaning
E	`region`
F	`bname` (branch)
G	`qty` (quantity sold)
H	`address`

Notes: A bean is identified by name or by (cultivar, region); a drink name is unique *per bean*; branches sell drinks, with address and quantity recorded.

Question 1.b. Candidate keys via closures

Observation: Since B and F never appear on the RHS of any FD, every key must contain B and F. We enumerate closures starting from supersets of $\{B, F\}$.

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Size 2
$\{B, F\}^+ = \{B, F, H\}$ (not a key)

Size 3 (supersets of $\{B, F\}$)
$\{A, B, F\}^+ =$
$\{A, B, C, D, E, F, G, H\}$
$\{B, C, F\}^+ = \{B, C, F, H\}$
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$\{B, F, G\}^+ = \{B, F, G, H\}$
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Size 4 (not containing $\{A, B, F\}$)

$\{B, C, D, F\}^+ = \{B, C, D, F, H\}$
$\{B, C, E, F\}^+ = \{A, B, C, D, E, F, G, H\}$
$\{B, C, F, G\}^+ = \{B, C, F, G, H\}$
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Larger supersets (that are not supersets of keys above)
All such closures remain proper subsets of R.

Conclusion
Candidate keys: $\{A, B, F\}$ and $\{B, C, E, F\}$.

Tedious by hand?
Take a look into this repository: Here

Question 1.c. Prime attributes

Keys are $\{A, B, F\}$ and $\{B, C, E, F\}$. Prime attributes are those that appear in at least one key.

$$\text{Prime attributes} = \{A, B, F\} \cup \{B, C, E, F\} = \{A, B, C, E, F\}.$$

Question 2.a. Minimal cover of R with Σ

Start from Σ:
$\{A\} \to \{C, E\}$,
$\{\text{A,B}\} \to \{D\}$,
$\{\text{F}\} \to \{H\}$,
$\{\text{C,E}\} \to \{A\}$,
$\{\text{B,C,E}\} \to \{D\}$,
$\{\text{A,B,F}\} \to \{D, G\}$,
$\{\text{B,C,E,F}\} \to \{G\}$.

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Step 1: Split RHS
$\{A\} \rightarrow \{C\}$,
$\{\text{A}\} \rightarrow \{E\}$,
$\{\text{A,B}\} \rightarrow \{D\}$,
$\{\text{F}\} \rightarrow \{H\}$,
$\{\text{C,E}\} \rightarrow \{A\}$,
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$\{\text{A,B,F}\} \rightarrow \{D\}$,
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Step 2: Reduce LHS where possible
$\{A\} \rightarrow \{C\}$,
$\{\text{A}\} \rightarrow \{E\}$,
$\{\text{A,B}\} \rightarrow \{D\}$,
$\{\text{F}\} \rightarrow \{H\}$,
$\{\text{C,E}\} \rightarrow \{A\}$,
$\{\text{B,C,E}\} \rightarrow \{D\}$,
$\cancel{\{\text{A,B,F}\} \rightarrow \{D\}}$
because $\{\text{A,B}\} \rightarrow \{D\}$
$\{\text{A,B,F}\} \rightarrow \{G\}$,
$\{\text{B,C,E,F}\} \rightarrow \{G\}$.

Question 2.a. Minimal cover of R with Σ

Step 3: Remove redundancies

$\{A\} \rightarrow \{C\}$,
$\{\mathrm{A}\} \rightarrow \{E\}$,
$\cancel{\{\mathrm{A,B}\} \rightarrow \{D\}}$
because $\{\mathrm{A}\} \rightarrow \{C, E\}$ and $\{B, C, E\} \rightarrow \{D\}$,
$\{\mathrm{F}\} \rightarrow \{H\}$,
$\{\mathrm{C,E}\} \rightarrow \{A\}$,
$\{\mathrm{B,C,E}\} \rightarrow \{D\}$,
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because $\{\mathrm{A}\} \rightarrow \{C, E\}$ and $\{B, C, E\} \rightarrow \{D\}$,
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Minimal Cover

$\{A\} \rightarrow \{C\}$,
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$\{\mathrm{C,E}\} \rightarrow \{A\}$,
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Question 2.a. Minimal cover of R with Σ

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Minimal Cover

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(Alternative minimal covers exist, e.g., replace $\{B, C, E\} \rightarrow \{D\}$ by $\{A, B\} \rightarrow \{D\}$ and $\{B, C, E, F\} \rightarrow \{G\}$ by $\{A, B, F\} \rightarrow \{G\}$.)

Question 2.b. Canonical cover (merged by LHS)

One canonical cover (grouping by identical LHS) is:
$\{A\} \rightarrow \{C, E\}$,
$\{\mathrm{F}\} \rightarrow \{H\}$,
$\{\mathrm{C,E}\} \rightarrow \{A\}$,
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Other canonical covers are possible depending on which equivalent minimal cover you merge.

Questions?
Drop a mail at: pratik.karmakar@u.nus.edu