BT5110: Tutorial 3 — Functional Dependencies

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AY25/26 S1

Example

Example Relation, $R = \{A, B, C\}$ and $\Sigma = \{\{A\} \rightarrow \{B\}, \{B\} \rightarrow \{C\}\}$

The column in red decides the color of the row: The rows having same values in the red column, have same color.

Task: Check which other columns are following "same value if same color" for each deciding (red) column. The columns for which this holds, are implied by the red column.

A	B	C
a1	b1	c1
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a2	b2	c2
a2	b2	c2
a3	b3	c2
a3	b3	c2
a4	b1	c1
a4	b1	c1
a5	b2	c2

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$\{A\} \rightarrow \{A, B, C\}$

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$\{B\} \rightarrow \{B, C\}$

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$\{A\} \rightarrow \{A, B, C\}$

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$\{B\} \rightarrow \{B, C\}$

A	B	C
a1	b1	c1
a1	b1	c1
a2	b2	c2
a2	b2	c2
a3	b3	c2
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$\{A\} \rightarrow \{A, B, C\}$

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a3	b3	c2
a3	b3	c2
a4	b1	c1
a4	b1	c1
a5	b2	c2

$\{B\} \rightarrow \{B, C\}$

A	B	C
a1	b1	c1
a1	b1	c1
a2	b2	c2
a2	b2	c2
a3	b3	c2
a3	b3	c2
a4	b1	c1
a4	b1	c1
a5	b2	c2

$\{C\} \rightarrow \{C\}$

Rules for Functional Dependencies (Armstrong's Axioms)

W, X, Y, Z mentioned here are sets of attributes.

Sound & complete inference system

- **Reflexivity**: If $Y \subseteq X$, then $X \to Y$.
- **Augmentation**: If $X \to Y$, then $XZ \to YZ$ for any Z.
- **Transitivity**: If $X \to Y$ and $Y \to Z$, then $X \to Z$.

Common derived rules (from the three above)

- **Union / Additivity**: If $X \to Y$ and $X \to Z$, then $X \to Y \cup Z$.
- **Decomposition / Projectivity**: If $X \to Y \cup Z$, then $X \to Y$ and $X \to Z$.
- **Pseudotransitivity**: If $X \to Y$ and $W \cup Y \to Z$, then $W \cup X \to Z$.
- **Composition**: If $X \to Y$ and $Z \to W$, then $X \cup Z \to Y \cup W$.
- **Self-determination**: $X \to X$.

How we use them here

- *LHS reduction*: test if $X \setminus \{A\} \to Y$ holds via closure using the axioms.
- *Redundancy removal*: test if an FD $X \to Y$ is implied by the rest (i.e., $Y \subseteq X^+$ computed from the others).

Question 1.a.

Your company, Apasaja Private Limited, is commissioned by *Toko Kopi Luwak* to design the relational schema for coffee beans, drinks, and cafes. We are given

$$R = \{A, B, C, D, E, F, G, H\},$$

$\Sigma = \{\{A\} \rightarrow \{C, E\}, \{A, B\} \rightarrow \{D\}, \{F\} \rightarrow \{H\}, \{C, E\} \rightarrow \{A\}, \{B, C, E\} \rightarrow \{D\}, \{A, B, F\} \rightarrow \{D, G\}, \{B, C, E, F\} \rightarrow \{G\}\}$.

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Attribute mapping

Symbol	Meaning
A	`name` (bean brand)
B	`dname` (drink name)
C	`cultivar`
D	`price`

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A	`name` (bean brand)
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Attribute mapping

Symbol	Meaning
E	`region`
F	`bname` (branch)
G	`qty` (quantity sold)
H	`address`

Notes: A bean is identified by name or by (cultivar, region); a drink name is unique *per bean*; branches sell drinks, with address and quantity recorded.

Question 1.b. Candidate keys via closures

$\Sigma = \{\{A\} \rightarrow \{C, E\}, \{A, B\} \rightarrow \{D\}, \{F\} \rightarrow \{H\}, \{C, E\} \rightarrow \{A\}, \{B, C, E\} \rightarrow \{D\}, \{A, B, F\} \rightarrow \{D, G\}, \{B, C, E, F\} \rightarrow \{G\}\}$.

Observation: Since B and F never appear on the RHS of any FD, every key must contain B and F. We enumerate closures starting from supersets of $\{B, F\}$.

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Size 2

$\{B, F\}^+ = \{B, F, H\}$ (not a key)

Size 3 (supersets of $\{B, F\}$)

$\{A, B, F\}^+ =$
$\{A, B, C, D, E, F, G, H\}$

$\{B, C, F\}^+ = \{B, C, F, H\}$

$\{B, D, F\}^+ = \{B, D, F, H\}$

$\{B, E, F\}^+ = \{B, E, F, H\}$

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$\{B, D, F\}^+ = \{B, D, F, H\}$

$\{B, E, F\}^+ = \{B, E, F, H\}$

$\{B, F, G\}^+ = \{B, F, G, H\}$

$\{B, F, H\}^+ = \{B, F, H\}$

Size 4 (not containing $\{A, B, F\}$)

$\{B, C, D, F\}^+ = \{B, C, D, F, H\}$

$\{B, C, E, F\}^+ = \{A, B, C, D, E, F, G, H\}$

$\{B, C, F, G\}^+ = \{B, C, F, G, H\}$

$\{B, C, F, H\}^+ = \{B, C, F, H\}$

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Question 1.b. Candidate keys via closures

$\Sigma = \{\{A\} \to \{C, E\}, \{A, B\} \to \{D\}, \{F\} \to \{H\}, \{C, E\} \to \{A\}, \{B, C, E\} \to \{D\}, \{A, B, F\} \to \{D, G\}, \{B, C, E, F\} \to \{G\}\}$.

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Larger supersets (that are not supersets of keys above)
All such closures remain proper subsets of R.

Conclusion

Candidate keys: $\{A, B, F\}$ and $\{B, C, E, F\}$.

Tedious by hand?
I made your life a bit easier: Here

Question 1.c. Prime attributes

Keys are $\{A, B, F\}$ and $\{B, C, E, F\}$. Prime attributes are those that appear in at least one key.

$$\text{Prime attributes} = \{A, B, F\} \cup \{B, C, E, F\} = \{A, B, C, E, F\}.$$

Question 2.a. Minimal cover of R with Σ

Start from Σ:

$\{A\} \rightarrow \{C, E\}$,
$\{\text{A,B}\} \rightarrow \{D\}$,
$\{\text{F}\} \rightarrow \{H\}$,
$\{\text{C,E}\} \rightarrow \{A\}$,
$\{\text{B,C,E}\} \rightarrow \{D\}$,
$\{\text{A,B,F}\} \rightarrow \{D, G\}$,
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Step 1: Split RHS

$\{A\} \rightarrow \{C\}$,
$\{\text{A}\} \rightarrow \{E\}$,
$\{\text{A,B}\} \rightarrow \{D\}$,
$\{\text{F}\} \rightarrow \{H\}$,
$\{\text{C,E}\} \rightarrow \{A\}$,
$\{\text{B,C,E}\} \rightarrow \{D\}$,
$\{\text{A,B,F}\} \rightarrow \{D\}$,
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Question 2.a. Minimal cover of R with Σ

Start from Σ:
$\{A\} \rightarrow \{C, E\}$,
{A,B} $\rightarrow \{D\}$,
{F} $\rightarrow \{H\}$,
{C,E} $\rightarrow \{A\}$,
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Step 1: Split RHS
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Step 2: Reduce LHS where possible
$\{A\} \rightarrow \{C\}$,
{A} $\rightarrow \{E\}$,
{A,B} $\rightarrow \{D\}$,
{F} $\rightarrow \{H\}$,
{C,E} $\rightarrow \{A\}$,
{B,C,E} $\rightarrow \{D\}$,
~~{A,B,F} $\rightarrow \{D\}$~~
because {A,B} $\rightarrow \{D\}$
{A,B,F} $\rightarrow \{G\}$,
{B,C,E,F} $\rightarrow \{G\}$.

Question 2.a. Minimal cover of R with Σ

Step 3: Remove redundancies

$\{A\} \rightarrow \{C\}$,
$\{\text{A}\} \rightarrow \{E\}$,
$\cancel{\{\text{A,B}\} \rightarrow \{D\}}$
because $\{\text{A}\} \rightarrow \{C, E\}$ and $\{B, C, E\} \rightarrow \{D\}$,
$\{\text{F}\} \rightarrow \{H\}$,
$\{\text{C,E}\} \rightarrow \{A\}$,
$\{\text{B,C,E}\} \rightarrow \{D\}$,
$\cancel{\{\text{A,B,F}\} \rightarrow \{G\}}$
because $\{\text{A}\} \rightarrow \{C, E\}$ and $\{B, C, E\} \rightarrow \{D\}$,
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Minimal Cover

$\{A\} \rightarrow \{C\}$,
$\{\text{A}\} \rightarrow \{E\}$,
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$\{\text{C,E}\} \rightarrow \{A\}$,
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Question 2.a. Minimal cover of R with Σ

Step 3: Remove redundancies

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because $\{A\} \rightarrow \{C, E\}$ and $\{B, C, E\} \rightarrow \{D\}$,
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(Alternative minimal covers exist, e.g., replace $\{B, C, E\} \rightarrow \{D\}$ by $\{A, B\} \rightarrow \{D\}$ and $\{B, C, E, F\} \rightarrow \{G\}$ by $\{A, B, F\} \rightarrow \{G\}$.)

Question 2.b. Canonical cover (merged by LHS)

One canonical cover (aka Compact Minimal Cover) (grouping by identical LHS) is:

$\{A\} \rightarrow \{C, E\}$,
$\{\text{F}\} \rightarrow \{H\}$,
$\{\text{C,E}\} \rightarrow \{A\}$,
$\{\text{B,C,E}\} \rightarrow \{D\}$,
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Other canonical covers are possible depending on which equivalent minimal cover you merge.

Questions?

Drop a mail at: pratik.karmakar@u.nus.edu