

BT5110: Tutorial 3 — Functional Dependencies

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AY25/26 S1



Identifying Functional Dependencies

Example Relation, $R = \{A, B, C\}$ and $\Sigma = \{\{A\} \rightarrow \{B\}, \{B\} \rightarrow \{C\}\}$

The column in **red** decides the color of the row: The rows having same values in the **red** column, have same color.

Task: Check which other columns are following "same value if same color" for each deciding (**red**) column. The columns for which this holds, are implied by the **red** column.

A	B	C
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a2	b2	c2
a2	b2	c2
a3	b3	c2
a3	b3	c2
a4	b1	c1
a4	b1	c1
a5	b2	c2

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$\{A\} \rightarrow \{A, B, C\}$

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$\{A\} \rightarrow \{A, B, C\}$

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a4	b1	c1
a5	b2	c2

$\{B\} \rightarrow \{B, C\}$

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$\{A\} \rightarrow \{A, B, C\}$

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a2	b2	c2
a3	b3	c2
a3	b3	c2
a4	b1	c1
a4	b1	c1
a5	b2	c2

$\{B\} \rightarrow \{B, C\}$

A	B	C
a1	b1	c1
a1	b1	c1
a2	b2	c2
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a3	b3	c2
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$\{A\} \rightarrow \{A, B, C\}$

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a2	b2	c2
a2	b2	c2
a3	b3	c2
a3	b3	c2
a4	b1	c1
a4	b1	c1
a5	b2	c2

$\{B\} \rightarrow \{B, C\}$

A	B	C
a1	b1	c1
a1	b1	c1
a2	b2	c2
a2	b2	c2
a3	b3	c2
a3	b3	c2
a4	b1	c1
a4	b1	c1
a5	b2	c2

$\{C\} \rightarrow \{C\}$

Rules for Functional Dependencies (Armstrong's Axioms)

W, X, Y, Z mentioned here are **sets** of attributes.

Sound & complete inference system

- **Reflexivity:** If $Y \subseteq X$, then $X \rightarrow Y$.
- **Augmentation:** If $X \rightarrow Y$, then $X \cup Z \rightarrow Y \cup Z$ for any Z .
- **Transitivity:** If $X \rightarrow Y$ and $Y \rightarrow Z$, then $X \rightarrow Z$.

Common derived rules (from the three above)

- **Union / Additivity:** If $X \rightarrow Y$ and $X \rightarrow Z$, then $X \rightarrow Y \cup Z$.
- **Decomposition / Projectivity:** If $X \rightarrow Y \cup Z$, then $X \rightarrow Y$ and $X \rightarrow Z$.
- **Pseudotransitivity:** If $X \rightarrow Y$ and $W \cup Y \rightarrow Z$, then $W \cup X \rightarrow Z$.
- **Composition:** If $X \rightarrow Y$ and $Z \rightarrow W$, then $X \cup Z \rightarrow Y \cup W$.
- **Self-determination:** $X \rightarrow X$.

How we use them here

- *LHS reduction:* test if $X \setminus \{A\} \rightarrow Y$ holds via closure using the axioms.
- *Redundancy removal:* test if an FD $X \rightarrow Y$ is implied by the rest (i.e., $Y \subseteq X^+$ computed from the others).

Question 1.a.

Your company, Apasaja Private Limited, is commissioned by *Toko Kopi Luwak* to design the relational schema for coffee beans, drinks, and cafes. We are given

$$R = \{A, B, C, D, E, F, G, H\},$$

$$\Sigma = \{\{A\} \rightarrow \{C, E\}, \{A, B\} \rightarrow \{D\}, \{F\} \rightarrow \{H\}, \{C, E\} \rightarrow \{A\}, \{B, C, E\} \rightarrow \{D\}, \{A, B, F\} \rightarrow \{D, G\}, \{B, C, E, F\} \rightarrow \{G\}\}.$$

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Attribute mapping

Symbol	Meaning
A	name (bean brand)
B	dname (drink name)
C	cultivar
D	price

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Attribute mapping

Symbol	Meaning
E	region
F	bname (branch)
G	qty (quantity sold)
H	address

Notes: A bean is identified by **name** or by (**cultivar**, **region**); a drink name is unique *per bean*; branches sell drinks, with address and quantity recorded.

Question 1.b. Candidate keys via closures

$\Sigma = \{\{A\} \rightarrow \{C, E\}, \{A, B\} \rightarrow \{D\}, \{F\} \rightarrow \{H\}, \{C, E\} \rightarrow \{A\}, \{B, C, E\} \rightarrow \{D\}, \{A, B, F\} \rightarrow \{D, G\}, \{B, C, E, F\} \rightarrow \{G\}\}.$

Observation: Since **B** and **F** never appear on the RHS of any FD, **every key must contain B and F**. We enumerate closures starting from supersets of $\{B, F\}$.

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Size 2

$\{B, F\}^+ = \{B, F, H\}$ (not a key)

Size 3 (supersets of $\{B, F\}$)

$\{A, B, F\}^+ =$

$\{A, B, C, D, E, F, G, H\}$

$\{B, C, F\}^+ = \{B, C, F, H\}$

$\{B, D, F\}^+ = \{B, D, F, H\}$

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$\{B, E, F\}^+ = \{B, E, F, H\}$

$\{B, F, G\}^+ = \{B, F, G, H\}$

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Size 4 (not containing $\{A, B, F\}$)

$\{B, C, D, F\}^+ = \{B, C, D, F, H\}$

$\{B, C, E, F\}^+ =$

$\{A, B, C, D, E, F, G, H\}$

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Larger supersets (that are not supersets of keys above)
All such closures remain proper subsets of **R**.

Conclusion

Candidate keys: $\{A, B, F\}$ and $\{B, C, E, F\}$.

Tedious by hand?

I made your life a bit easier: [Here](#)

Question 1.c. Prime attributes

Keys are $\{A, B, F\}$ and $\{B, C, E, F\}$. Prime attributes are those that appear in at least one key.

$$\text{Prime attributes} = \{A, B, F\} \cup \{B, C, E, F\} = \{A, B, C, E, F\}.$$

Question 2.a. Minimal cover of R with Σ

Start from Σ :

$$\{A\} \rightarrow \{C, E\},$$

$$\{A, B\} \rightarrow \{D\},$$

$$\{F\} \rightarrow \{H\},$$

$$\{C, E\} \rightarrow \{A\},$$

$$\{B, C, E\} \rightarrow \{D\},$$

$$\{A, B, F\} \rightarrow \{D, G\},$$

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Question 2.a. Minimal cover of R with Σ **Start from Σ :**
$$\begin{aligned}\{A\} &\rightarrow \{C, E\}, \\ \{A, B\} &\rightarrow \{D\}, \\ \{F\} &\rightarrow \{H\}, \\ \{C, E\} &\rightarrow \{A\}, \\ \{B, C, E\} &\rightarrow \{D\}, \\ \{A, B, F\} &\rightarrow \{D, G\}, \\ \{B, C, E, F\} &\rightarrow \{G\}.\end{aligned}$$
Step 1: Split RHS
$$\begin{aligned}\{A\} &\rightarrow \{C\}, \\ \{A\} &\rightarrow \{E\}, \\ \{A, B\} &\rightarrow \{D\}, \\ \{F\} &\rightarrow \{H\}, \\ \{C, E\} &\rightarrow \{A\}, \\ \{B, C, E\} &\rightarrow \{D\}, \\ \{A, B, F\} &\rightarrow \{D\}, \\ \{A, B, F\} &\rightarrow \{G\}, \\ \{B, C, E, F\} &\rightarrow \{G\}.\end{aligned}$$

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\{B, C, E, F\} &\rightarrow \{G\}.
\end{aligned}$$
Step 2: Reduce LHS where possible

$$\begin{aligned}
\{A\} &\rightarrow \{C\}, \\
\{A\} &\rightarrow \{E\}, \\
\{A, B\} &\rightarrow \{D\}, \\
\{F\} &\rightarrow \{H\}, \\
\{C, E\} &\rightarrow \{A\}, \\
\{B, C, E\} &\rightarrow \{D\}, \\
~~\{A, B, F\} &\rightarrow \{D\}~~ \\
&\text{because } \{A, B\} \rightarrow \{D\} \\
\{A, B, F\} &\rightarrow \{G\}, \\
\{B, C, E, F\} &\rightarrow \{G\}.
\end{aligned}$$

Question 2.a. Minimal cover of R with Σ

Step 3: Remove redundancies

$$\{A\} \rightarrow \{C\},$$

$$\{A\} \rightarrow \{E\},$$

~~$$\{A, B\} \rightarrow \{D\}$$~~

because $\{A\} \rightarrow \{C, E\}$ and $\{B, C, E\} \rightarrow \{D\}$,

$$\{F\} \rightarrow \{H\},$$

$$\{C, E\} \rightarrow \{A\},$$

$$\{B, C, E\} \rightarrow \{D\},$$

~~$$\{A, B, F\} \rightarrow \{G\}$$~~

because $\{A\} \rightarrow \{C, E\}$ and $\{B, C, E, F\} \rightarrow \{G\}$,

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Minimal Cover

$$\{A\} \rightarrow \{C\},$$

$$\{A\} \rightarrow \{E\},$$

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$$\{B, C, E, F\} \rightarrow \{G\}.$$

(Alternative minimal covers exist, e.g., replace $\{B, C, E\} \rightarrow \{D\}$ by $\{A, B\} \rightarrow \{D\}$ and $\{B, C, E, F\} \rightarrow \{G\}$ by $\{A, B, F\} \rightarrow \{G\}$.)

Question 2.b. Canonical cover (merged by LHS)

One canonical cover (aka **Compact Minimal Cover**) (grouping by identical LHS) is:

$$\{A\} \rightarrow \{C, E\},$$

$$\{F\} \rightarrow \{H\},$$

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Other canonical covers are possible depending on which equivalent minimal cover you merge.

Questions?

Drop a mail at: pratik.karmakar@u.nus.edu