

1. a) &amp; b)

$$\begin{aligned} f(x) &= (x-1)^2 + (x+1) \\ &= x^2 + x^2 - 2x^2 - 2x + x + 1 \\ &= x^3 - x^2 - x + 1 \end{aligned}$$

$$\frac{d}{dx} f(x) = 3x^2 - 2x - 1$$

$$f'(x) = 3x^2 - 2x - 1$$

$$f''(x) = 6x - 2$$

$$6x - 2 = 0$$

$$\Rightarrow 6x = 2$$

$$\Rightarrow \boxed{x = \frac{1}{3}} \quad \text{~~0.33~~}$$

$$f''(x) = 6x - 2 \geq 0 \text{ for } x \geq \frac{1}{3}$$

So  $f(x)$  is convex for  $[\frac{1}{3}, \infty]$

a)  $f(x)$  is not convex for all  $x \in \mathbb{R}$   
Real numbers as it is  
convex for <sup>only</sup>  $x \geq \frac{1}{3}$ .

b)  $f(x)$  is convex for  $x \in [\frac{1}{3}, \infty]$   
as it is convex for  $x \in [\frac{1}{3}, \infty]$