Deep Learning - Ex 3

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1.

P (Disease found) = 0.005

P (Tested positive | Disease found) = 0.99

P ( Test positive - disease not found ) = 0.05

P ( Test positive ) = (0.99) x (0.005) + (0.05) x (0.995)

= 0.0547

Bayes Rule :

$$P(A|B) = \frac{P(B|A)\, P(A)}{P(B)}$$

$$= \frac{P(TP \mid DF)\, P(DF)}{P(TP)}$$

$$= \frac{(0.99)\,(0.005)}{(0.0547)}$$

$$\boxed{P(DF \mid TP) = 0.0904}$$ or 9.04% probability of disease if tested +ve.

2. Expectation :

$$E[X] = \sum_{i=0}^{n} x_i\, P(X = x_i)$$

$$= 0 \cdot \frac{1}{8} + 1 \cdot \frac{3}{8} + 2 \cdot \frac{3}{\cancel{8}\,4} + 3 \cdot \frac{1}{8}$$

$$= \frac{6}{8} + \frac{3}{4}$$

$$= \frac{48}{32}$$

$$\boxed{E[X] = 3/2}$$

3. $X \rightarrow$ wheat production
$Y \rightarrow$ flour price

$\Rightarrow$ using ordinary least squares method to find weights

$$W = (X^T X)^{-1} X^T Y$$

Sample mean $\bar{x} = 28.6$
$\bar{y} = 35.4$

$$w_1 = \frac{\sum_x xy - n\bar{x}\bar{y}}{\sum_x x^2 - n\bar{x}^2} = \frac{9734 - (10)(28.6)(35.4)}{8468 - (10)(28.6)^2}$$

$$\boxed{w_1 = -1.3537}$$

$$\boxed{w_0} = \bar{y} - w_1\bar{x} = 35.4 + (1.3537)(28.6) = \boxed{74.116}$$

The regression line is $\boxed{\hat{y} = 74.116 - 1.3537x}$

To estimate variance

$$\hat{\sigma}^2 = \frac{\sum_{i=1}^{n} e_i^2}{n} \quad (e = \text{errors})$$

* First compute the errors from the regression line!

$$\boxed{\hat{y}_i = 74.116 - 1.3537\, x_i}$$

find $e_i = y_i - \hat{y}_i$

The variance, $\dfrac{207.92}{10} = \boxed{20.79 = \sigma^2}$

* log likelihood is given by,

$$L(w, \sigma^2) = -\left(\frac{n}{2}\log(2\pi\sigma^2) + \sum_{i=1}^{n} \frac{(X^T w - y)^2}{2\sigma^2}\right)$$

$\boxed{n = 10}$

$$\boxed{\text{log likelihood} = -29.3623}$$