Mathematical exercise: Calculus

May 4, 2020

1 Proof of the chain rule (20 points)

Prove that

$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$$

where

$$y = f(g(x))$$

Hint: Look at this video `https://www.youtube.com/watch?v=m0LZX19DyyI&t=185s`

2 Can you write out the chain rule for... (20 points)

$$u = f(x, y, z)$$

and

$$x = x(a, b), y = y(a, b), z = z(a, b)$$

3 Find the gradients $(\nabla_{\mathbf{x}} f(\mathbf{x}) = [\frac{\partial}{\partial x_i}, ...]^{\top})$ of the following function. (20 points)

$$y = f(\mathbf{x}) = 3x_1^2 + 5e^{x_2}$$

4 What is the gradient $\nabla f(\mathbf{x})$ of the following function (20 points)

$$f(x) = ||\mathbf{x}||_2$$

5 Find the gradient $\nabla f(\mathbf{x})$ of the following function. And then solve the equation to find the minimum value of f. (20 points)

$$f(\mathbf{x}) = (\mathbf{y} - \mathbf{x})^\top (\mathbf{y} - \mathbf{x})$$