Milestone 2 Scribe Notes
Group-13 Networks
Binary Exponential Backoff Algorithm Probabilistic Modeling.

Scribe Question 1: Project System and Objective

In this project we are developing a probabilistic model for the analysis of the Binary Exponential Backoff Algorithm. It controls the behavior of different wireless stations and works on reducing the number of collisions.

The model is used to evaluate:

- Probability of idle slots
- Probability of successful transmissions
- Probability of collisions
- Network throughput

Uncertainty sources:

1. Random backoff counter choice
2. Random number of transmitting stations
3. Random channel state transitions
4. Random contention window (CW) values

Scribe Question 2: The Major Random Variables and Uncertainty Modeling

1. Backoff Counter Random Variable

In this case R is the selected counter of backoff.

$$R \in \{0, 1, 2, \ldots, CW - 1\}$$

Probability Mass Function

$$P(R = r) = \frac{1}{CW}, \quad r = 0, 1, \ldots, CW - 1 \tag{1}$$

Cumulative Distribution Function

$$F_R(r) = P(R \leq r) = \frac{r + 1}{CW} \tag{2}$$

2. Transmission Attempt Indicator Random Variable

$$A_i(t) = \begin{cases} 1 & \text{station transmits} \\ 0 & \text{otherwise} \end{cases}$$

Probability Mass Function

$$P(A_i(t) = 1) = \tau \tag{3}$$

$$P(A_i(t) = 0) = 1 - \tau \tag{4}$$

Cumulative Distribution Function

$$F_A(a) = \begin{cases} 0 & a < 0 \\ 1 - \tau & 0 \leq a < 1 \\ 1 & a \geq 1 \end{cases}$$

3. Number of Transmitting Stations

$$K(t) = \sum_{i=1}^{N} A_i(t) \tag{5}$$

$$N = No.of stations$$

Probability Mass Function

$$P(K = k) = \binom{N}{k} \tau^k (1 - \tau)^{N-k} \tag{6}$$

Cumulative Distribution Function

$$F_K(k) = \sum_{j=0}^{k} \binom{N}{j} \tau^j (1-\tau)^{N-j} \tag{7}$$

4. Channel State Random Variable

$$X(t) = \begin{cases} I & K(t) = 0 \\ S & K(t) = 1 \\ C & K(t) \geq 2 \end{cases}$$

Channel State Probabilities

$$P_I = (1-\tau)^N \tag{8}$$

$$P_S = N\tau(1-\tau)^{N-1} \tag{9}$$

$$P_C = 1 - P_I - P_S \tag{10}$$

Scribe Question 3: Probabilistic Reasoning and Markov Chain Model

Attempt Probability Model

Expected backoff counter:

$$E[R] = \frac{CW - 1}{2} \tag{11}$$

Expected waiting time:

$$E[T] = \frac{CW}{2} \tag{12}$$

Transmission attempt probability:

$$\tau = \frac{2}{CW} \tag{13}$$

Three State Markov Model

A discrete Markov chain is being used to model the states in the channel:

- Idle State (I)
- Success State (S)
- Collision State (C)

Idle State

Condition:

$$K(t) = 0 \tag{14}$$

Probability:

$$P_I = (1 - \tau)^N \tag{15}$$

Success State

Condition:

$$K(t) = 1 \tag{16}$$

Probability:

$$P_S = N\tau(1 - \tau)^{N-1} \tag{17}$$

Collision State

Condition:

$$K(t) \geq 2 \tag{18}$$

Probability:

$$P_C = 1 - P_I - P_S \tag{19}$$

State Transition Probabilities

Idle Transitions

$$P_{II} = (1 - \tau)^N \tag{20}$$

$$P_{IS} = N\tau(1 - \tau)^{N-1} \tag{21}$$

$$P_{IC} = 1 - P_{II} - P_{IS} \tag{22}$$

Success Transitions

$$P_{SS} = \frac{1}{CW} \tag{23}$$

$$P_{SI} = 1 - \frac{1}{CW} \tag{24}$$

Steady-State Distribution

$$\pi = \pi P \tag{25}$$

$$\pi_I + \pi_S + \pi_C = 1 \tag{26}$$

Scribe Question 4: Model Implementation Alignment

For the simulations the Markov model of BEBA is directly used. All the probabilistic elements defined in theory are explicitly used to implement at the slot level to maintain consistency between experiments and the analysis.

As a discrete random variable backoff counters are produced over the interval $\{0, 1, \ldots, CW - 1\}$ for every node. So we can assume and approximate the independence of the transmission attempts for bernouli trials with parameter.

$$\tau = \frac{2}{CW}.$$

The transmit probability for each node at every time slot is given by τ. The number of transmitting nodes are

$$K(t) = \sum_{i=1}^{N} A_i(t),$$

which decides the channel state. When $K = 0$ the slot is considered to be Idle, Success when $K = 1$, and Collision when $K \geq 2$.

Over a large number of slots the empirical probabilities starts approaching the steady state probabilities π_I, π_S, and π_C predicted by the Markov chain.

An important implementation used is the exponential growth of contention window:

$$CW_i = 2^i CW_{\min}. \tag{27}$$

Since BEBA alters CW any collision or success, it changes the probability τ and further changes the Markov transition probabilities.

Throughput is calculated using the steady state probabilities:

$$S = \frac{\pi_S \cdot L}{\pi_I \sigma + \pi_S T_S + \pi_C T_C}, \tag{28}$$

L is the size of payload and σ, T_S, and T_C denote slot durations. Therefore this implementation in the long run approximates the steady state solution in Markov process.

Scribe Question 5: Cross-Milestone Consistency and Refinement

This project has developed from identification of uncertainties to making of a structured Markov representation of BEBA. In the earlier stages we focused on defining the random variables and in this milestone, the all those components are integrated together to form a coherent framework.

The steady-state equation:

$$\pi = \pi P,$$

with normalization condition

$$\pi_I + \pi_S + \pi_C = 1,$$

P is the transition matrix for the Markov chain and $\pi = [\pi_I, \pi_S, \pi_C]$ represents the stationary distribution.

The steady state success probability is:

$$\pi_S = N\tau(1-\tau)^{N-1}.$$

The analysis can get complex when the number of competing nodes N increases. Hence the sensitivity of the system can be calculated using partial differentiation:

$$\frac{\partial \pi_S}{\partial N} = \tau(1-\tau)^{N-1} + N\tau(1-\tau)^{N-1}\ln(1-\tau). \tag{29}$$

Since $\ln(1-\tau) < 0$, an increase in N reduces π_S after a threshold which leads to the rapid growth of collision probability:

$$\pi_C = 1 - (1-\tau)^N - N\tau(1-\tau)^{N-1}.$$

This shows that stability of the system is highly sensitive to network density.

The exponential backoff implementation:

$$CW_i = 2^i CW_{\min}$$

acts as a stabilizing factor by reducing τ after collisions, thus it helps in lowering the probability of collision.

In the upcoming milestones, the analysis will be more refined as it will include explicit backoff stages which will lead to a more detailed analysis of delay, fairness and integration with the Markov framework.

Scribe Question 6: Open Issues and Role Attribution

The open probabilistic questions are:

1. What is the dependence of the steady state distribution on the number of competing nodes?
2. Does the probability of collision tend to one, when the network is high density?
3. Sensitivity of throughput to the growth of the contention window?
4. Is it possible to extend the Markov model to detailed backoff stages without losing its tractability?

These open ended questions form the second stage of the project, which involves further analytic and numerical investigation. research will be carried out.

Role allocation:

- Modeling Team: Obtaining the steady state equations out of the extended Markov state space.
- Simulation Team: authenticating the behavior at the steady state and the transition behaviors.
- Analysis Team: Process stability limits and throughput limits through different network densities.