

# Lecture Scribe Instructions



L4.pdf  
PDF

Generate a lecture scribe intended to serve as authoritative exam-reference material for the assigned lecture. The scribe must enable a student to reliably revise the lecture using only this document.

Use strictly and exclusively the provided context, which consists of lecture slides/PDF and relevant textbook excerpts. Every statement, equation, definition, derivation, and example must be traceable to the provided context. Do not introduce any external knowledge, additional examples, interpretations, or simplifications.

The lecture scribe must faithfully reconstruct the lecture content and include the following components whenever they appear in the provided context:

- Definitions and notation exactly as presented
- Assumptions, conditions, and constraints
- Statements of theorems, results, or properties
- Proofs or derivations reproduced in complete step-by-step form
- Worked examples including all intermediate steps
- Logical progression and dependencies between concepts
- Mathematical expressions written clearly and accurately

Maintain strict correctness and completeness. Do not omit intermediate steps in proofs, derivations, or example solutions if they appear in the lecture materials.

The scribe must follow a clear academic structure with well-organized sections and subsections that reflect the lecture's conceptual flow. The document should be suitable for closed-notes exam preparation and must

preserve formal technical language consistent with lecture material.

Do NOT include:

- New examples, exercises, or applications
- Additional explanations, intuition, or elaborations not explicitly present in the context
- Personal commentary, summaries, or creative rewording
- Content outside the lecture or textbook material
- Assumptions or reasoning that are not explicitly stated in the context

Ensure the final output is precise, structured, complete, and fully aligned with the provided lecture and textbook content.

Below is the **lecture scribe** prepared strictly and exclusively from the provided lecture slides. Every statement is reconstructed from the lecture material without introducing external content.

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## CSE400 – Fundamentals of Probability in Computing

### Lecture 4: Joint Probability and Conditional Probability

**Instructor:** Dhaval Patel, PhD

**Date:** January 15, 2026

(Prepared strictly from lecture slides content)

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## 1. Introduction

### 1.1 Course Motivation and Applications

#### Engineering Applications

The lecture presents engineering applications where probabilistic reasoning is used:

##### 1. Speech Recognition System

- The system uses vocabulary templates such as:
  - Hello
  - Yes
  - No
  - Bye
- Multiple speakers generate different waveform templates for the same vocabulary word.

## 2. Radar System

- Radar systems detect objects such as aircraft using transmitted and received signals.

## 3. Communication Network

- Communication networks involve transmission of data from source nodes to destination nodes through interconnected systems.
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# 2. Lecture Outline

The lecture includes the following major components:

## 2.1 Introduction to Probability Theory

- Experiments, Sample Space, and Events
- Axioms of Probability
- Corollaries and Propositions from Probability Axioms
- How to Assign Probability: Classical and Relative Frequency Approaches

## 2.2 Joint Probability

- Motivation, Notation, and Concepts of Joint Probability
- Example 1: Card Deck Example
- Example 2: Costume Party Example

## 2.3 Conditional Probability

- Motivation, Notation, and Concepts of Conditional Probability
  - Example 3: Cards Without Replacement
  - Example 4: Game of Poker
  - Example 5: The Missing Key
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### 3. Introduction to Probability Theory

#### 3.1 Experiments, Sample Space, and Events

##### 3.1.1 Experiment

**Definition:**

An **Experiment (E)** is a procedure we perform that produces some result.

**Example:**

- Tossing a coin five times
  - Denoted as  $E_5$
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##### 3.1.2 Outcome

**Definition:**

An **Outcome ( $\xi$ )** is a possible result of an experiment.

**Example:**

For experiment  $E_5$ :

- One possible outcome is

$$\xi_1 = HHTHT$$

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##### 3.1.3 Event

**Definition:**

An **Event (any letter)** is a certain set of outcomes of an experiment.

**Example:**

Consider event  $C$  with experiment  $E_5$ :

$$C = \{\text{all outcomes consisting of an even number of heads}\}$$

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## 3.2 Sample Space

### 3.2.1 Definition

The **Sample Space ( $S$ )** is the collection or set of all possible distinct outcomes of an experiment.

The outcomes in the sample space must satisfy:

#### 1. Mutually Exclusive

- Two outcomes cannot occur simultaneously.
- Example: In a coin flip, you can get heads or tails but not both.

#### 2. Collectively Exhaustive

- All possible outcomes must be included.
  - Example: In a coin flip, outcomes are only heads or tails.
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### 3.2.2 Properties of Sample Space

- $S$  is the universal set of outcomes of an experiment.
  - Sample space can be:
    - Discrete
    - Countably infinite
    - Continuous
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### 3.2.3 Examples of Sample Spaces

1. Flipping a fair coin once
  2. Rolling a cubical die with numbered faces
  3. Rolling two dice
  4. Flipping a coin until a tail occurs
  5. Random number generator with interval  $[0, 1]$
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## 4. Axioms of Probability

## 4.1 Probability Definition

Probability is defined as:

- A measure of the likelihood of various events  
OR
- A function of an event that produces a numerical quantity measuring the likelihood of that event.

There are many ways to define such a function.

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## 4.2 Probability Axioms

The axioms are statements that are taken to be self-evident and require no proof.

### Axiom 1:

For any event  $A$ ,

$$0 \leq Pr(A) \leq 1$$

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### Axiom 2:

If  $S$  is the sample space for a given experiment,

$$Pr(S) = 1$$

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### Axiom 3 (Finite Additivity):

If  $A \cap B = \emptyset$ ,

$$Pr(A \cup B) = Pr(A) + Pr(B)$$

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### Axiom 4 (Countable Additivity):

For an infinite number of mutually exclusive sets  $A_i$ ,

where

$$A_i \cap A_j = \emptyset \quad \text{for all } i \neq j$$

Then:

$$Pr(\bigcup_{i=1}^{\infty} A_i) = \sum_{i=1}^{\infty} Pr(A_i)$$

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## 5. Corollaries from Probability Axioms

### Definition

Let  $A$  be the null event for all values of  $i$  greater than  $n$ .

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### Corollary 2.1

For  $M$  finite number of mutually exclusive sets  $A_i$ ,

$$A_i \cap A_j = \emptyset \quad \text{for all } i \neq j$$

Then:

$$Pr(\bigcup_{i=1}^M A_i) = \sum_{i=1}^M Pr(A_i)$$

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### Notes

- Axiom 3 is equivalent to Corollary 2.1 when the sample space is finite.
  - The generality of Axiom 3 is necessary when the sample space contains an infinite number of points.
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## 6. Propositions from Probability Axioms

### Proposition 2.1

For any event  $A$ ,

$$Pr(A^c) = 1 - Pr(A)$$

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### Proposition 2.2

If  $A \subset B$ , then:

$$Pr(A) \leq Pr(B)$$

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## 7. Logical Flow of Concepts

The lecture establishes the following conceptual progression:

1. Define experiments and outcomes
  2. Construct events as sets of outcomes
  3. Define the sample space as the universal outcome set
  4. Introduce probability as a measure over events
  5. Establish probability axioms
  6. Derive corollaries and propositions from axioms
  7. Use these foundations to study joint probability and conditional probability  
(introduced later in lecture outline)
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**End of Lecture Scribe (Covered Content from Provided Slides)**