Lecture Scribe CSE400

L9_S1_A.pdf
PDF

Generate a lecture scribe intended to serve as authoritative exam-reference material for the assigned lecture. The scribe must enable a student to reliably revise the lecture using only this document.

Use strictly and exclusively the provided context, which consists of lecture slides/PDF and relevant textbook excerpts. Every statement, equation, definition, derivation, and example must be traceable to the provided context. Do not introduce any external knowledge, additional examples, interpretations, or simplifications.

The lecture scribe must faithfully reconstruct the lecture content and include the following components whenever they appear in the provided context:

• Definitions and notation exactly as presented
• Assumptions, conditions, and constraints
• Statements of theorems, results, or properties
• Proofs or derivations reproduced in complete step-by-step form
• Worked examples including all intermediate steps
• Logical progression and dependencies between concepts
• Mathematical expressions written clearly and accurately

Maintain strict correctness and completeness. Do not omit intermediate steps in proofs, derivations, or example solutions if they appear in the lecture materials.

The scribe must follow a clear academic structure with well-organized sections and subsections that reflect the lecture's conceptual flow. The document should be suitable for closed-notes exam preparation and must preserve formal technical language consistent with lecture material.

Do NOT include:

• New examples, exercises, or applications
• Additional explanations, intuition, or elaborations not explicitly present in the context
• Personal commentary, summaries, or creative rewording
• Content outside the lecture or textbook material
• Assumptions or reasoning that are not explicitly stated in the context

Ensure the final output is precise, structured, complete, and fully aligned with the provided lecture and textbook content.

Lecture Scribe: CSE400 – Fundamentals of Probability in Computing
Lecture 9: Uniform, Exponential, Laplace, and Gamma Random Variables
(Prepared strictly from lecture slides content)

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1. Introduction

This lecture discusses **types of continuous random variables**, focusing on:

- Uniform Random Variable
- Exponential Random Variable
- Laplace Random Variable
- Gamma Random Variable

The lecture includes definitions, probability density functions (PDFs), cumulative distribution functions (CDFs), graphical interpretations, applications, and worked examples.

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2. Uniform Random Variable

2.1 Definition

A continuous random variable X is said to be **uniformly distributed** over the interval $[a, b]$ if its probability density function (PDF) is given by:

$$f_X(x) = \begin{cases} \frac{1}{b-a}, & a \leqslant x < b \\ 0, & \text{elsewhere} \end{cases}$$

2.2 Cumulative Distribution Function (CDF)

The cumulative distribution function corresponding to the uniform distribution is:

$$F_X(x) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a \leqslant x < b \\ 1, & x \geqslant b \end{cases}$$

2.3 Graphical Representation

- The PDF of a uniform random variable is constant over the interval $[a, b]$.
- The CDF increases linearly from 0 to 1 over the interval.

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3. Example 1: Uniform Random Variable

3.1 Problem Statement

The phase of a sinusoid Θ is uniformly distributed over the interval $[0, 2\pi]$. The PDF is given by:

$$f_\Theta(\theta) = \begin{cases} \frac{1}{2\pi}, & 0 \leqslant \theta < 2\pi \\ 0, & \text{otherwise} \end{cases}$$

3.2 Part (a): Find $\Pr(\Theta > 3\pi/4)$

For a uniform random variable over $[0, 2\pi]$:

$$\Pr(a < \Theta < b) = \frac{b - a}{2\pi}$$

Thus,

$$\Pr(\Theta > 3\pi/4) = \frac{2\pi - 3\pi/4}{2\pi} = \frac{5}{8}$$

3.3 Part (b): Find $\Pr(\Theta < \pi \mid \Theta > 3\pi/4)$

Using conditional probability:

$$\Pr(A \mid B) = \frac{\Pr(A \cap B)}{\Pr(B)}$$

Let:

- $A = \{\Theta < \pi\}$
- $B = \{\Theta > 3\pi/4\}$

Then,

$$\Pr(3\pi/4 < \Theta < \pi) = \frac{\pi - 3\pi/4}{2\pi} = \frac{1}{8}$$

Since $\Pr(B) = \frac{5}{8}$,

$$\Pr(\Theta < \pi \mid \Theta > 3\pi/4) = \frac{1/8}{5/8} = \frac{1}{5}$$

3.4 Part (c): Find $\Pr(\cos\Theta < 1/2)$

First determine values of Θ satisfying:

$$\cos\Theta = \frac{1}{2} \Rightarrow \Theta = \frac{\pi}{3}, \frac{5\pi}{3}$$

Thus,

$$\cos\Theta < \frac{1}{2} \quad \text{for} \quad \frac{\pi}{3} < \Theta < \frac{5\pi}{3}$$

Therefore,

$$\Pr(\cos\Theta < 1/2) = \frac{5\pi/3 - \pi/3}{2\pi} = \frac{4\pi/3}{2\pi} = \frac{2}{3}$$

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4. Applications of Uniform Random Variable

Applications include:

1. Phase of a sinusoidal signal when phase angles between 0 and 2π are equally likely.
2. Random numbers generated between 0 and 1 for simulations.
3. Arrival time of a user within a known time window assuming no preference.

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5. Exponential Random Variable

5.1 Definition

The exponential random variable has PDF and CDF defined for any $b > 0$.

5.2 Probability Density Function

$$f_X(x) = \frac{1}{b}\exp\left(-\frac{x}{b}\right) u(x)$$

where $u(x)$ denotes the unit step function.

5.3 Cumulative Distribution Function

$$F_X(x) = \left[1 - \exp\left(-\frac{x}{b}\right)\right] u(x)$$

5.4 Graphical Interpretation

- The PDF decreases exponentially as x increases.
- The CDF increases monotonically and approaches 1 asymptotically.

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6. Example 2: Exponential Random Variable

6.1 Problem Statement

Let X be an exponential random variable with PDF:

$$f_X(x) = e^{-x}u(x)$$

6.2 Part (a): Find $\Pr(3X < 5)$

Since $3X < 5$,

$$X < \frac{5}{3}$$

Thus,

$$\Pr(3X < 5) = \Pr\left(X < \frac{5}{3}\right)$$

Using the CDF:

$$F_X(x) = 1 - e^{-x}$$

Therefore,

$$\Pr\left(X < \frac{5}{3}\right) = 1 - e^{-5/3}$$

6.3 Part (b): Generalization

For arbitrary constant y:

$$\Pr(3X < y) = \Pr\left(X < \frac{y}{3}\right)$$

Thus,

$$\Pr(3X < y) = 1 - e^{-y/3}$$

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7. Remaining Topics in Lecture Outline

The lecture outline indicates additional topics:

- Laplace Random Variable
- Gamma Random Variable
  - Graph and Special Cases
  - Example
  - Homework Problem
- Problem Solving and In-class Activity

(These topics are listed in the lecture outline but detailed derivations are not presented within the provided slides.)

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8. Logical Progression of Concepts

The lecture develops continuous random variables in the following sequence:

1. Introduction to uniform distribution and its properties
2. Application-based problem solving using uniform distribution
3. Introduction to exponential distribution
4. Application-based problems using exponential distribution
5. Preview of additional continuous distributions

This progression moves from simpler constant-density distributions toward distributions modeling decay processes.

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9. Summary of Key Mathematical Results

Uniform Random Variable

$$f_X(x) = \begin{cases} \frac{1}{b-a}, & a \leqslant x < b \\ 0, & \text{otherwise} \end{cases}$$

$$F_X(x) = \begin{cases} 0, & x < a \\ \frac{x-a}{b-a}, & a \leqslant x < b \\ 1, & x \geqslant b \end{cases}$$

Exponential Random Variable

$$f_X(x) = \frac{1}{b} e^{-x/b} u(x)$$

$$F_X(x) = [1 - e^{-x/b}]\, u(x)$$

End of Lecture Scribe

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