CSE400 – Fundamentals of Probability in Computing

Lecture 4: Joint Probability and Conditional Probability

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1. Introduction

1.1 Course Motivation and Applications

Engineering Applications

The lecture presents engineering applications where probabilistic reasoning is used:

1. Speech Recognition System
   - The system uses vocabulary templates such as:
     - Hello
     - Yes
     - No
     - Bye
2. Radar System
   - Radar systems detect objects such as aircraft using transmitted and received signals.
3. Communication Network
   - Communication networks involve transmission of data from source nodes to destination nodes through interconnected systems.

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2. Lecture Outline

The lecture includes the following major components:

2.1 Introduction to Probability Theory

- Experiments, Sample Space, and Events
- Axioms of Probability
- Corollaries and Propositions from Probability Axioms
- How to Assign Probability: Classical and Relative Frequency Approaches

2.2 Joint Probability

- Motivation, Notation, and Concepts of Joint Probability
- Example 1: Card Deck Example
- Example 2: Costume Party Example

2.3 Conditional Probability

- Motivation, Notation, and Concepts of Conditional Probability
- Example 3: Cards Without Replacement
- Example 4: Game of Poker
- Example 5: The Missing Key

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3. Introduction to Probability Theory

3.1 Experiments, Sample Space, and Events

3.1.1 Experiment

Definition:
An **Experiment (E)** is a procedure we perform that produces some result.

Example:

- Tossing a coin five times
- Denoted as E_5

3.1.2 Outcome

Definition:
An **Outcome (ξ)** is a possible result of an experiment.

Example:
For experiment E_5:

- One possible outcome is

$$\xi_1 = HHTHT$$

3.1.3 Event

Definition:
An **Event (any letter)** is a certain set of outcomes of an experiment.

Example:
Consider event C with experiment E_5:

$$C = \{alloutcomesconsistingofanevennumberofheads\}$$

3.2 Sample Space

3.2.1 Definition

The **Sample Space (S)** is the collection or set of all possible distinct outcomes of an experiment.

The outcomes in the sample space must satisfy:

1. **Mutually Exclusive**
   - Two outcomes cannot occur simultaneously.
   - Example: In a coin flip, you can get heads or tails but not both.
2. **Collectively Exhaustive**
   - All possible outcomes must be included.
   - Example: In a coin flip, outcomes are only heads or tails.

3.2.2 Properties of Sample Space

- S is the universal set of outcomes of an experiment.
- Sample space can be:
  - Discrete
  - Countably infinite
  - Continuous

3.2.3 Examples of Sample Spaces

1. Flipping a fair coin once
2. Rolling a cubical die with numbered faces
3. Rolling two dice
4. Flipping a coin until a tail occurs
5. Random number generator with interval $[0, 1]$

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4. Axioms of Probability

4.1 Probability Definition

Probability is defined as:

- A measure of the likelihood of various events

  OR

- A function of an event that produces a numerical quantity measuring the likelihood of that event.

There are many ways to define such a function.

4.2 Probability Axioms

The axioms are statements that are taken to be self-evident and require no proof.

Axiom 1:
For any event A,

$$0 \leq Pr(A) \leq 1$$

Axiom 2:
If S is the sample space for a given experiment,

$$Pr(S) = 1$$

Axiom 3 (Finite Additivity):
If $A \cap B = \emptyset$,

$$Pr(A \cup B) = Pr(A) + Pr(B)$$

Axiom 4 (Countable Additivity):
For an infinite number of mutually exclusive sets A_i,
where

$$A_i \cap A_j = \emptyset \quad foralli \neq j$$

Then:

$$Pr\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} Pr(A_i)$$

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5. Corollaries from Probability Axioms

Definition

Let A be the null event for all values of i greater than n.

Corollary 2.1

For M finite number of mutually exclusive sets A_i,

$$A_i \cap A_j = \emptyset \quad foralli \neq j$$

Then:

$$Pr\left(\bigcup_{i=1}^{M} A_i\right) = \sum_{i=1}^{M} Pr(A_i)$$

Notes

- Axiom 3 is equivalent to Corollary 2.1 when the sample space is finite.
- The generality of Axiom 3 is necessary when the sample space contains an infinite number of points.

6. Propositions from Probability Axioms

Proposition 2.1

For any event A,

$$Pr(A^c) = 1 - Pr(A)$$

Proposition 2.2

If $A \subset B$, then:

$$Pr(A) \leq Pr(B)$$

7. Logical Flow of Concepts

The lecture establishes the following conceptual progression:

1. Define experiments and outcomes
2. Construct events as sets of outcomes
3. Define the sample space as the universal outcome set

4. Introduce probability as a measure over events
5. Establish probability axioms
6. Derive corollaries and propositions from axioms
7. Use these foundations to study joint probability and conditional probability (introduced later in lecture outline)