Lecture Scribe CSE400

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Presentation

Generate a lecture scribe intended to serve as exam reference material for a reading-based, closed-notes examination.

Use only the provided context, which consists of the lecture slides/PDF and the relevant textbook content.

The scribe must faithfully reconstruct what was taught in the lecture so that a student can rely on it later for revision.

Include definitions, notation, assumptions, theorem or result statements, proofs or derivations, and worked examples only if they appear in the provided context.

Do not introduce new examples, explanations, intuition, or material that is not present in the context.

Maintain strict faithfulness to the lecture's structure, scope, and logical flow.

Organize the scribe clearly with appropriate sections and headings so it can be used directly for exam preparation. make sure this is the only prompt no imporvisation would be done after this

Lecture Scribe — CSE400

Lecture 4: Joint Probability and Conditional Probability

Instructor: Dhaval Patel, PhD

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1. Introduction to Probability Theory

1.1 Experiments, Sample Space, and Events

Definitions

- **Experiment (E):** A procedure we perform that produces some result.
  Example: Tossing a coin five times.
- **Outcome (ω):** A possible result of an experiment.
  Example: One outcome of five coin tosses: `HHTHT`.
- **Event (A, B, C, ...):** A set of outcomes of an experiment.
  Example: Event C: all outcomes consisting of an even number of heads.
- **Sample Space (S):** The set of all possible distinct outcomes of an experiment.
  Outcomes in S are:
  - **Mutually Exclusive:** Only one can occur at a time.
  - **Collectively Exhaustive:** No other outcome is possible.

Sample space can be:

- Discrete
- Countably infinite
- Continuous

Examples

- Flipping a fair coin once
- Rolling a cubical die
- Rolling two dice
- Flipping a coin until tails occurs
- Random number generator on $[0, 1)$

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2. Axioms of Probability

Probability is a function that assigns a numerical measure to events.

Axioms

1. For any event A:
$$0 \leqslant Pr(A) \leqslant 1$$
2. If S is the sample space:
$$Pr(S) = 1$$
3. If $A \cap B = \varnothing$:
$$Pr(A \cup B) = Pr(A) + Pr(B)$$

For infinite mutually exclusive events A_i:

$$Pr\left(\bigcup_{i=1}^{\infty} A_i\right) = \sum_{i=1}^{\infty} Pr(A_i)$$

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3. Corollary from Axioms

Corollary 2.1 (Finite case)

For a finite number of mutually exclusive events A_i:

$$Pr\left(\bigcup_{i=1}^{n} A_i\right) = \sum_{i=1}^{n} Pr(A_i)$$

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4. Propositions from Axioms

- **Proposition 2.1**

$$Pr(A^c) = 1 - Pr(A)$$

- **Proposition 2.2**
  If $A \subset B$, then

$$Pr(A) \leqslant Pr(B)$$

- **Proposition 2.3**

$$Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$$

- **Proposition 2.4 (Inclusion–Exclusion Principle)**

$$Pr(A_1 \cup A_2 \cup \cdots \cup A_n)$$

  expressed using alternating sums of intersections.

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5. Assigning Probabilities

5.1 Classical Approach

- Coin flip: $Pr(H) = Pr(T)$
- Die roll: $Pr(\text{even})$
- Two dice: $Pr(\text{sum} = 5)$

5.2 Relative Frequency Approach

$$Pr(A) = \lim_{n\to\infty} \frac{\text{Number of times A occurs}}{n}$$

Limitations:

- Requires infinite repetitions
- Many phenomena are not repeatable

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6. Joint Probability

6.1 Motivation

Events are not always mutually exclusive. We are interested in:

$$Pr(A \cap B)$$

6.2 Notation

$$Pr(A, B) \quad \text{or} \quad Pr(A \cap B)$$

For multiple events:

$$Pr(A_1, A_2, \dots, A_n)$$

6.3 Calculation Approaches

- **Classical:** Express events using atomic outcomes and find common outcomes.
- **Relative Frequency:**

$$Pr(A, B) = \frac{n_{A,B}}{n}$$

Example 1: Card Deck Example

52 cards, 13 per suit.

Let:

- A: Red card
- B: Number card (Ace included)
- C: Heart card

Individual Probabilities

$$Pr(A) = \frac{26}{52} = \frac{1}{2}$$

$$Pr(B) = \frac{40}{52} = \frac{5}{26}$$

$$Pr(C) = \frac{13}{52} = \frac{1}{4}$$

Joint Probabilities

$$Pr(A, B) = \frac{20}{52}$$

$$Pr(A, C) = \frac{13}{52}$$

$$Pr(B, C) = \frac{10}{52}$$

Example 2: Costume Party

Tops: 3 T-shirts, 1 cape → $Pr(\text{Cape}) = \frac{1}{4}$

Bottoms: 2 pants, 4 boxers → $Pr(\text{Boxers}) = \frac{4}{6}$

Joint probability:

$$Pr(\text{Cape and Boxers}) = \frac{1}{4} \times \frac{4}{6} = \frac{1}{6}$$

Correct option: C

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7. Conditional Probability

7.1 Motivation

Occurrence of one event may depend on another.

7.2 Definition

$$Pr(A \mid B) = \frac{Pr(A, B)}{Pr(B)}, \quad Pr(B) > 0$$

7.3 Product Rule

$$Pr(A, B) = Pr(A \mid B)Pr(B) = Pr(B \mid A)Pr(A)$$

7.4 Three Events

$$Pr(A, B, C) = Pr(C \mid A, B)Pr(B \mid A)Pr(A)$$

7.5 Chain Rule

$$Pr(A_1, A_2, \dots, A_n) = Pr(A_n \mid A_1, \dots, A_{n-1}) \dots Pr(A_2 \mid A_1)Pr(A_1)$$

Example 3: Cards Without Replacement

Two cards drawn, no replacement.

- A: First card is Spade
- B: Second card is Spade

After one spade removed:

- Remaining cards = 51
- Remaining spades = 12

$$Pr(B \mid A) = \frac{12}{51}$$

Example 4: Game of Poker — Flush

Flush in Spades

$$Pr = \frac{13}{52} \times \frac{12}{51} \times \frac{11}{50} \times \frac{10}{49} \times \frac{9}{48}$$

Any Flush

$$4 \times Pr(\text{Spade Flush})$$

Example 5: The Missing Key

Given:

$$P(L) = 0.4, \quad P(R) = 0.4, \quad P(K) = 0.8$$

Find:

$$P(R \mid L^c)$$

Since key cannot be in both pockets:

$$P(R \mid L^c) = \frac{P(R)}{P(L^c)} = \frac{0.4}{0.6} = \frac{2}{3}$$

Correct option: C

End of Lecture