Assignment Questions

BL2 – Understanding Level Questions

1. Explain the Markov property with a suitable example.
2. Describe the difference between a Markov Model and a Hidden Markov Model.
3. What are the key components of a Markov Model?
4. Describe the role of transition, emission, and initial probabilities in an HMM.
5. Differentiate between fully connected (ergodic) and left-to-right HMMs.
6. Explain the stationary assumption in a Markov process.
7. What is the purpose of the Forward Algorithm in HMMs?
8. Interpret the meaning of emission probability in the context of HMMs.
9. Explain how HMMs are useful for modeling sequential data.
10. Summarize the applications of HMMs in speech and handwriting recognition.

BL3 – Applying Level Questions

Q1. Given States: S_1 = Sunny, S_2 = Cloudy, S_3 = Rainy
Transition matrix A:

From\To	**S_1**	**S_2**	**S_3**
S_1	0.6	0.3	0.1
S_2	0.2	0.5	0.3
S_3	0.3	0.3	0.4

Find the probability of moving from Sunny → Cloudy → Rainy.

Q2. If A =$\begin{bmatrix} 0.7 & 0.2 & 0.1 \\ 0.4 & 0.4 & 0.2 \\ 0.3 & 0.3 & 0.4 \end{bmatrix}$
What is the probability of transitioning from state 3 → 1 → 2?

Q3. Hidden states: Rainy (S_1), Sunny (S_2)
Observations: Walk (O_1), Shop (O_2), Clean (O_3)

State	**Walk**	**Shop**	**Clean**
Rainy	0.1	0.4	0.5
Sunny	0.6	0.3	0.1

What is the probability of observing "Clean" if the state is "Rainy"?

Q4. Hidden states: Rainy (R), Sunny (S)
Observations: Walk (W), Shop (Sh), Clean (C)
Given:
π = [0.6, 0.4]
A = \[0.7,0.3,[0.4,0.6]]
B =

State	**W**	**Sh**	**C**
R	0.1	0.4	0.5
S	0.6	0.3	0.1

Find probability of "Walk → Shop".

Q 5. Given transition and emission matrices, calculate the probability of an observation sequence using the Forward Algorithm. S_1 = Rainy (R), S_2 = Sunny (S) and **three observation symbols** {Walk (W), Shop (Sh), Clean (C)}.

- Initial distribution: $\pi = [0.6,\ 0.4]$
- Transition matrix $A = \begin{bmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{bmatrix}$
- Emission matrix B(rows = states R,S; columns = W,Sh,C):

$$B = \begin{bmatrix} 0.1 & 0.4 & 0.5 \\ 0.6 & 0.3 & 0.1 \end{bmatrix}$$

Observation sequence: $O = [\,W,\ Sh,\ C\,]$.

Compute the probability $P(O \mid \text{model})$using the **Forward algorithm**. Show all intermediate α values.

Q 6. Consider an HMM with two hidden states
S_1 = **Rainy (R)** and S_2 = **Sunny (S)**, and three observation symbols {Walk (W), Shop (Sh), Clean (C)}.Initial distribution: $\pi = [0.6,\ 0.4]$

- Transition matrix A:

$$A = \begin{bmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{bmatrix}$$

- Emission matrix B(rows = R,S; columns = W,Sh,C):

$$B = \begin{bmatrix} 0.1 & 0.4 & 0.5 \\ 0.6 & 0.3 & 0.1 \end{bmatrix}$$

Observation sequence: $O = [\,W,\ Sh,\ C\,]$.

Q 7. Write state transition matrix and transition diagram for Expression: A → B → C→ A

Q 8. Write state transition matrix for expression: S1 → S2, S1 → S3, S2 → S4, S3 → S4,

Q9. Hidden states: Bull (B), Bear (R)
Observations: Up (U), Down (D)
Initial distribution: $\boldsymbol{\pi} = [\mathbf{0.5},\ \mathbf{0.5}]$

- **Transition matrix $\boldsymbol{A}$(rows = from, cols = to):**

$$\boldsymbol{A} = \begin{bmatrix} \mathbf{0.8} & \mathbf{0.2} \\ \mathbf{0.3} & \mathbf{0.7} \end{bmatrix}$$

- **Emission matrix $\boldsymbol{B}$(rows = Bull, Bear; cols = Up, Down):**

$$\boldsymbol{B} = \begin{bmatrix} \mathbf{0.7} & \mathbf{0.3} \\ \mathbf{0.4} & \mathbf{0.6} \end{bmatrix}$$

Observation sequence: $\boldsymbol{O} = [\,\boldsymbol{U},\ \boldsymbol{U}\,]$(two consecutive Up days).

(a) Compute $P(O \mid \text{model})$using the Forward algorithm.
(b) Find the most probable hidden-state sequence for Ousing the Viterbi algorithm

Q9. Hidden states: $S_1 = A,\ S_2 = B$
Observations: symbols $\{x, y, z\}$indexed $[0, 1, 2]$

$$\pi = [0.5,\ 0.5]$$

$$A = \begin{bmatrix} 0.6 & 0.4 \\ 0.3 & 0.7 \end{bmatrix}, B = \begin{bmatrix} 0.5 & 0.4 & 0.1 \\ 0.1 & 0.3 & 0.6 \end{bmatrix}$$

Observation sequence: $O = [x, y, z]$→ indices $[0, 1, 2]$.

Using Forward algorithm — compute $P(O \mid \text{model})$

Q10. Hidden states: Bull (B), Bear (R)
Observations: Up (U)=0, Down (D)=1

$$\pi = [0.7,\ 0.3]$$

$$A = \begin{bmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{bmatrix}, B = \begin{bmatrix} 0.8 & 0.2 \\ 0.3 & 0.7 \end{bmatrix}$$

Observation sequence: $O = [U,\ D,\ U]$→ indices $[0, 1, 0]$.

Identify appropriate hidden state sequence.

🔍 BL4 – Analyzing Level Questions

(Compare, Examine, Categorize, Justify, Break down relationships)

1. Analyze how the Markov property simplifies modeling of sequential systems.
2. Compare the working of the Forward and Viterbi algorithms.
3. Examine the differences between the three problems of HMM — Evaluation, Decoding, and Learning.
4. Analyze how HMMs overcome the limitations of traditional classifiers for sequential data.
5. Differentiate between HMMs and Conditional Random Fields (CRFs) in terms of dependency modeling.

6. Investigate why left-to-right HMMs are particularly suited for speech recognition.
7. Evaluate how emission probabilities influence the accuracy of the Viterbi algorithm.
8. Compare sequence classification using HMM and CRF in natural language processing tasks.
9. Examine how temporal dependencies are captured in HMM-based sequence classification for bioinformatics.

Markov Model Numerical Examples

Example 1

Weather states: Rain (R), Sun (S).
A = \[0.7,0.3, [0.4, 0.6]]
If today is sunny, find the probability that after **2 days** it will rain.

Solution:
We compute $P(S \to R) = a_{SR} + a_{SS} \times a_{SR} = 0.4 + 0.6 \times 0.4 = 0.64$? Wait let's compute:
From S→R in 2 steps:
= a(S→S)*a(S→R) + a(S→R)*a(R→R)
= (0.6×0.4) + (0.4×0.7) = **0.52**

✅ Probability = **0.52**

Example 2

Given initial π = [1,0] and transition matrix
A = \[0.8,0.2,[0.3,0.7]]
Find probability sequence “State1 → State2 → State2”.
Solution:
P = 1 × 0.2 × 0.7 = **0.14**