Forecasting Short Term Return Distribution of S&P BSE Stock Index Using Geometric Brownian Motion: An Evidence from Bombay Stock Exchange

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Abstract

For accurate forecasting of stock price, we should have to understand the process and according to that, we can develop the model to decide the best possible values of forecast. Geometric Brownian Motion model is a mathematical model used for forecasting the future stock price and highly accurate as compared to other model and also gives high returns. It helps the investors to take further decisions on their investment. Before forecasting the stock price using Geometric Brownian Motion model, Kolmogorov-Smirnov test and Q – Q plot technique were conducted on the sample data to conclude that the data are normally distributed and feasible to forecast. The algorithm starts from calculating the stock returns, drift and volatility to predict the return distribution at specific time 't'. Simulations are performed using log volatility equation with the closest behavior to the S&P BSE closed stock price. The closest forecast simulation with actual stock closed price is selected with the more precise value of drift and volatility to proceed in Geometric Brownian Motion model. In order to determine the forecast accuracy as well as performance of the model Mean Absolute Percentage Error (MAPE) is calculated. Since MAPE < 10% i.e. 5.41 %, it implies that Geometric Brownian Motion model is highly accurate and an appropriate model for forecasting stock price.

Keywords: Stock, Brownian Motion, Drift, Volatility, Simulation and Forecast

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1.   INTRODUCTION

FINANCE is the field that deals with study of investment: it is also called as management of money. Financial markets are essential for function of any country's economy and generate two functions: firstly as decision variable for investors or as a facilitator for companies to get funding and secondly as a facilitator for society to invest and one of them is stock market.

Stock market is a place where investors either Indians or foreigners can invest or take their funds for capital appreciation. It is also able to give high rate of profit return which interests the investors. However stock trading has high rate of risk. The fluctuations of stock price affect the investor's decision on invest their capital. Any mistake in decision making will result loss for investor. Thus to minimize the high risk, investor need information as a reference for decision making of which stock they should buy, sell and maintain for future.

The stock price reflects all information about the stocks and also the expectations of the future performance of corporate sector. For this cause Indian stock market plays a significant role in the growth of Indian economy and every movement on it puts an impact on the performance of the economy.

The Indian stock market had seen various up-down since 1991, after the government implemented the LPG, i.e., Liberalization, Privatization and Globalization. According to the Economic Survey 2018-19, India's growth rate is forecasted to be between 7.0 -7.5 percent for the FY19 as compared to 6.80 percent for FY18 and 7.20 in FY17. The government has planned to achieve a fiscal target of 3.40 percent of GDP and 44.50 percent debt to GDP ratio with the 3.40 percent inflation for the FY18. Over the years, India has attracted many investments by different economies and a large consumption base. With this growth rate, India also became the fastest growing economy in FY18.

The changes in stock prices make the stock market more volatile and difficult to predict because of economic factors of the country. When investor buys shares from stock market, it does not guarantee any returns, but it makes risky in investment and can give high returns. Therefore, Geometric Brownian Motion will help the investors to give a better view and predict the stock price in a short period of time.

Bombay Stock Exchange (BSE) - A Bird's Eye View

Established in 1875, the BSE (formerly known as Bombay Stock Exchange Ltd.) is Asia's oldest stock exchange was founded by an influential businessman Premchand Roychand, located at Dalal Street, Mumbai. The BSE is the world's 10th largest stock exchange with an overall market capitalization of more than $2.2 trillion. Today, BSE is the world's number one exchange in the world in terms of the number of listed companies (over 4900). It is the world's 5th most active in terms of number of transaction handle through its electronic trading system. On 31st August1957, the BSE becomes the first stock exchange to be recognised by Indian Government under the Securities Contracts Regulation Act. SENSEX is India's first and most popular stock

market benchmark index of Bombay Stock Exchange. Exchange traded funds (ETF) on SENSEX are listed in BSE and Hong Kong. SENSEX, first compiled in 1986, was calculated on a "Market Capitalization-Weighted" methodology of 30 component stocks representing large, well-established and financially sound companies across key sectors. It is being calculated on a free-float market capitalization methodology. The free-float market capitalization-weighted methodology is a widely followed index construction methodology on which majority of global equity indices are based; all major index providers like MSCI, FTSE, STOXX, S&P and Dow Jones uses the free float methodology. In the same year, it developed the S&P BSE SENSEX index giving the BSE a means to measure the overall performance of the exchange. In 2000, the BSE used this index to open its derivative market, trading S&P BSE SENSEX future contracts. The development of S&P BSE SENSEX options along with equity derivatives expanding the BSE's trading platform.

2.   REVIEW OF LITERATURE

There is an abundance of literature surrounding the forecasting of stock return: however there is still a lot of debate as to which method is the most appropriate. Financial managers, academicians, business analysts and investors are interested in simulating the prices of stocks, options and derivatives in order to make important investment and financial decisions. Simulating the stock price means generating price paths that a stock may follow in future because future stock prices are uncertain, called stochastic but follow atleast a set of rules that can derive from historical data and knowledge of stock prices (Sengupta, 2004).

Two common approaches of forecasting stock prices are based on the theory of technical analysis and theory of fundamental analysis. Technical analysis assumes that history repeats itself, that is, past pattern of price behavior tends to reoccur in future. Fundamental analysis assumes that at any point in time an indivisual security has an intrinsic value that depends on the earning potential of the security, meaning some stocks are either over-priced or under-priced (Fama, 1995). So forecasting is the best method to predict the future of stock market. However, the rate of risk using this method is still high due to inaccurate forecast output (Omar & Jaffar, 2014).

Geometric Brownian Motion is an example of continuous time stochastic process that satisfies a stochastic differential equation and can estimate all the parameters in the modeling of stock market (Wilmott, 2000). A stochastic differential equation is a differential equation in which one or more of the terms is a random process resulting in a solution in which itself a random process (Oksendal, 2002). It is sometimes referred as a Exponential Brownian Motion in which the logarithm of the randomly varying quantity follow a Brownian Motion with drift (Ross, 2014).

Since its development, Geometric Brownian Motion has been widely recognized as a fundamental model for the valuation of various financial assets (Zhijun, 2015). The primary model used for financial modeling was invented by Merton and Scholes in 1997. This model led to boom in option trading and widely used but often with adjustment by option market participants (Zvi, Kane & Marcus, 2008). This model is

known as Black-Sholes model and the prices of stock follow stochastic process (Danjie et al. 2011, Botos & Cristina, 2012 and Prabhakaran, 2015). Regnault, (1858), Wiener, (1863), Jevons, (1870), Dancer, (1870) and Delsavx, (1877) investigated on stochastic process before Merton and Scholes and established a nexus relationship with stock market (Javanovic & Schinckus, 2017). A French mathematician Louis Bachelier mentioned in his PhD thesis that the stock price dynamics follow Brownian Motion (Bertrand, 2005), which is erratic and unpredictable. More than half a century later, Samuelson modified the Bachelier's model and states that, instead of stock price, rate of return of stocks follow Geometric Brownian Motion and avoid negative value (Zhou, 2015). He also proposed Geometric Brownian Motion associates the mathematical description of the physical phenomenon to the market efficient hypothesis generating a random process and respecting the limited liability principle (Neves, 2010). Thorsen in his paper mentioned demand for service in rapidly growing industries follow a Geometric Brownian Motion process. He also assumed that the future net prices of round wood where the real option theory is applied to describe the decision of establishing a new forest stand followed a Geometric Brownian Motion process (Marathe & Ryan, 2005). In 1993, Steven Heston proposed Heston Model for stock price dynamics using Geometric Brownian Motion with stochastic volatility and assumed that volatility of assets is not constant, but follows a random process (Heston, 1993).

Hull, (2000) refers Geometric Brownian Motion process as a model for stock price and the expected returns are not independent on the value of the process. It assumes only non negative values and shows an erratic path which reflects real stock price and follow lognormal distribution (Tsaur, 2012). It has two components: Certain components and uncertain components. The certain components represents that the stock will earn over a short period of time, also referred as drift of the stock. Abidin & Jaffar, (2014) applied Geometric Brownian Motion to forecast the future stock price for the short term investment a maximum of two weeks. It is also accurate in short term forecasting (Dmouj, 2006). The uncertain component is a stochastic process including the stock volatility and the product is called Wiener process which incorporates random volatility and time interval (Brewer, Feng & Kwan, 2012). Stock price changes over non-overlapping time intervals which are independent and identically distributed with the variance of each price proportional to the length of time involved (Marathe & Ryan, 2005).

Reddy & Clinton, (2016) used Capital Asset Pricing Model (CAPM) to estimate expected annual return and standard deviation of the daily return of stock price for the empiric volatility in simulation. Hsu et al. (2009) authored Markov Fourier Grey Model, simply called MFG Model is a combination of Grey model, Fourier series and Markov process and concludes that it can predict share price accurately. Sometimes, Geometric Brownian Motion unable to capture some features including long range correlations and heavy tailed distribution (Brigo et al. 2007). But it remains the single most crucial model in financial modeling as there is no alternative has been proposed (Gadja & Wylomanska, 2012).

3.   OBJECTIVE OF THE STUDY

The main objective of the study is to develop a model for forecasting the short term return distribution of S&P BSE closed stock price of Bombay Stock Exchange. The present study is focused on following objectives.

- Check whether the observed S&P BSE closed stock price is normal or not, with the help of Kolmogorov-Smirnov normality test and $Q - Q$ plot technique.

- Construct a Geometric Brownian Motion model to forecast the future S&P BSE closed stock price or return distribution.

- Simulated the forecast data to select the most appropriate simulation for getting the close behavior of observed data.

- Calculate Mean Absolute Percentage Error (MAPE) to determine the forecast accuracy as well as performance of the model.

4.   RESEARCH METHODOLOGY

In this paper, the past S&P BSE closed stock price data are taken from the official website of Bombay Stock Exchange. Kolmogorov-Smirnov (K-S) test and $Q - Q$ plot technique were conducted on these data to conclude that the data are normally distributed and feasible to forecast for future stock price.

The mathematical model used in this research is Geometric Brownian Motion, sometimes referred as an Exponential Brownian Motion. In order to find out it, we have to calculate the stock returns, drift and volatility to predict S&P BSE closed stock price. Simulations are done and random walk with the closest behavior to the S&P BSE closed stock price are choosen to proceed in Geometric Brownian Motion Model. From the model $S_t = S_0 \exp\left[\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma W_t\right]$, each of the value is inserted to complete the model. Sets of forecast data is then produced with the model acquired from Geometric Brownian Motion and forecast data is simulated to select the most appropriate simulation from random walk in order to getting the close behavior of S&P BSE closed stock price. Based on the drift and volatility of each of the sets of forecast simulation, it is the closest that can be acquired from simulation and compare with the actual drift and volatility. Mean Absolute Percentage Error (MAPE) is calculated in order to determine the forecast accuracy as well as performance of the model.

5.   GEOMETRIC BROWNIAN MOTION: CONCEPT & DEFINITION

In the modelling of Stock Market, Geometric Brownian Motion plays a vital role in building a statistical model. Before building the model, first introduce some concept in order to understand the indepthness of Brownian Motion in the theory of financial mathematics.

Brownian Motion is the random movement of particles suspended in a fluid or in a gas resulting from their bombardment by the fast-moving molecules in the liquid or gas. In 1827, the English botanist Robert Brown looking through a microscope that particles found in pollen grains moved through the water and the direction of the force of atomic bombardment is constantly changing at different times. The particle is hit more on one side than another, leading to the seemingly random nature of the motion. This transport phenomenon is called as "Brownian Motion" (Brown, 1828). The first mathematical rigorous construction is due to Wiener in 1923 that is why; Brownian Motion is sometimes called as Wiener Process (Ermogenous, 2006).

5.1.   Statistical layout of Geometric Brownian Motion

Let Ω be the set of all possible outcomes of any random experiment and the continuous time random process X_t, defined on the filtered probability space $(\Omega, F, \{F_t\}_{t \in T}, P)$.

where,   F is the σ-algebra of event

$\{F_t\}_{t \in T}$ denotes the information generated by the process X_t over the time interval [0, T].

P is the probability measure.

Definition: A random variable 'X' have the lognormal distribution with parameters μ and σ if log (X) is normally distributed. i.e,

$$\log(X) \sim N(\mu, \sigma^2)$$

Definition: A real valued random process $W_t = W(t, w)$ on the time interval [0, ∞] is Brownian Motion or Wiener Process if it satisfies following conditions (Karlin & Taylor, 2012 and Ross, 1996).

1. Continuity: $W_0 = 0$
2. Normality: For $0 \leq s < t \leq T$, $W_t - W_s \sim N(0, t - s)$
3. Markov Property: For $0 \leq s^{'} < t^{'} < s < t \leq T$, $W_t - W_s$ *independent of* $W_{t^{'}} - W_{s^{'}}$

A stochastic process S_t is said to follow a Geometric Brownian Motion if it satisfies the following stochastic differential equation

$$dS_t = \mu S_t dt + \sigma S_t dW_t$$

where, W_t is a Wiener process (Brownian Motion) and

$\mu \, \& \, \sigma$ are constants.

Normally, μ is called the percentage drift and σ is called the percentage volatility. So consider a Brownian Motion trajectory that satisfies this differential equation. The right hand side term $\mu S_t dt$ controls the trends of this trajectory and the tern $\sigma S_t dW_t$ controls the random noise effect in the trajectory.

After applying the technique of separation of variable, the equation becomes:

$$\frac{dS_t}{S_t} = \mu dt + \sigma dW_t$$

Taking integration of both side

$$\int \frac{dS_t}{S_t} = \int (\mu dt + \sigma dW_t) dt$$

Since $\frac{dS_t}{S_t}$ relates to derivative of $\ln(S_t)$ the $It\bar{o}$ calculus becomes:

$$\ln\left(\frac{dS_t}{S_t}\right) = \left(\mu - \frac{1}{2}\sigma^2\right) t + \sigma W_t$$

Taking the exponential in both sides and plugging the initial condition S_0, the analytical solution of Geometric Brownian Motion is given by:

$$S_t = S_0 exp\left[\left(\mu - \frac{\sigma^2}{2}\right) t + \sigma W_t\right]$$

The constants μ and σ are able to produce a solution of Geometric Brownian Motion throughout time interval. For given drift and volatility the solution of Geometric Brownian Motion in the form:

$$S_t = S_0 \exp[x(t)]$$

$$\text{where,} \quad x(t) = \left(\mu - \frac{\sigma^2}{2}\right) t + \sigma W_t$$

Now the density of Geometric Brownian Motion is given by

$$f(t,x) = \frac{1}{\sigma x (2\pi t)^{1/2}} exp[-(\log x - \log x_0 - \mu t)]/2\sigma^2 t$$

mle estimation:

Suppose that a set of input: t_1, t_2, t_3 .... and a set of corresponding output: S_1, S_2, S_3..........from S_t and the set of data is in mle function $L(\Theta)$. Since Geometric Brownian Motion is a markov chain process

$$L(\Theta) = f_\Theta(x_1, x_2, x_3 \ldots \ldots \ldots) = \prod_{i=1}^{n} f_\Theta(x_i)$$

Taking logarithm on both side,

$$L(\Theta) = \sum_{i=1}^{n} \log f_\Theta(x_i)$$

Now taking derivative of the right hand side, we get

$$\bar{m} = \sum_{i=1}^{n} \frac{x_i}{n}$$

$$\bar{v} = \sum_{i=1}^{n} \frac{(x_i - m)^2}{n}$$

Where $\bar{m}$ and $\bar{v}$ are the mle of m and v respectively and $X_i = \log S(t_i) - \log S(t_i - 1)$

5.2. Mathematical layout of Geometric Brownian Motion

Suppose X is a continuous random variable follow lognormal distribution, then $v = \ln X$ is a random variable which is normally distributed with mean μ and variance σ^2. Symbolically:

$$v = \ln X \sim N(\mu, \sigma^2)$$

The probability density function from variable v becomes:

$$f(v) = \frac{1}{\sigma\sqrt{2\pi}} exp\left[\frac{-1}{2}\left(\frac{v - \mu}{\sigma}\right)^2\right] \quad for -\infty < v, \mu < \infty \ and \ \sigma > 0$$

$$for \ v = \ln X, dv = d(\ln X) = \frac{1}{x}dX$$

and

$$h(x) = \frac{f(v)dv}{dx} = \frac{1}{\sigma\sqrt{2\pi}} exp\left[\frac{-1}{2}\left(\frac{\ln x - \mu}{\sigma}\right)^2\right]\frac{1}{x} \ dx$$

Thus, probability density function becomes:

$$h(x) = \frac{1}{\sigma x\sqrt{2\pi}} exp\left[\frac{-1}{2}\left(\frac{\ln x - \mu}{\sigma}\right)^2\right] \quad ; x > 0$$

where μ and σ^2 represents mean and variance of the lognormal variable x.

Now,

$$E(x) = \int_{-\infty}^{\infty} xh(x)dx = \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} exp\left[\frac{-1}{2}\left(\frac{\ln x - \mu}{\sigma}\right)^2\right] dx \dots\dots\dots\dots\dots\dots (1)$$

If $y = \ln x - \mu$, then $dy = \frac{1}{x}dx$ and equ. (1) becomes

$$E(e^{y+\mu}) = \int_{-\infty}^{\infty} \frac{1}{\sigma x\sqrt{2\pi}} exp(y + \mu) \ exp\left[\frac{-1}{2}\left(\frac{y}{\sigma}\right)^2\right] dy$$

$$= e^{\mu}e^{\frac{\sigma^2}{2}} \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} exp\left[\frac{-1}{2}\left(\frac{y - \sigma^2}{\sigma}\right)^2\right] dy \dots\dots\dots\dots\dots\dots\dots\dots (2)$$

If $z = \frac{y-\sigma^2}{\sigma}$, *then* $dz = \frac{1}{\sigma} dy$ *and equ.* (2) *becomes*:

$$E\left[e^{z\sigma+\sigma^2+\pi}\right] = exp\left(\mu + \frac{\sigma^2}{2}\right) \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} exp\left[\frac{-z^2}{2}\right] dz \ldots\ldots\ldots\ldots (3)$$

The integral part of the above equation is a probability density function of standard normal distribution subject to the conditions integral $\ln x = -\infty$ becomes $y = -\infty$ and $\ln x = +\infty$ becomes $y = +\infty$ in *equ.* (2) and integral $y = -\infty$ becomes $z = -\infty$ and $y = +\infty$ becomes $z = +\infty$ in *equ.* (3).

Now the expected stock price:

$$S_t = S_0 exp\left[\left(\mu - \frac{1}{2}\sigma^2\right)t + \sigma B_t\right] \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (4)$$

is called the Geometric Brownian Motion with drift. Where,

S_0 = Actual beginning stock price.

μ = Mean of lognormal distribution.

σ^2 = Variance of lognormal distribution.

B_t = Brownian Motion at time 't' with $\mu = 0$ and defined as,

$$B_t = \mu t + \sigma W_t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

where, W_t = Wiener process at time 't'.

$$Now \quad E(B_t) = \mu t + E(\sigma W_t) = \mu t \ as \ E(W_t) = 0 \ and$$

$$Var(B_t) = E(B_t)^2 - [E(B_t)^2] = \sigma^2 t$$

Hence Brownian Motion with drift is normally distributed with mean μt and variance $\sigma^2 t$. Symbolically:

$B_t \sim N(\mu t, \sigma^2 t)$

Thus

$\ln S_t \sim N(lnS_{t-1} + \left(\mu - \frac{1}{2}\sigma^2\right)t, \sigma^2 t)$

Hence expected stock price at time 't' for future stock is

$$E(S_t) = S_0 \ exp\left[\left(\mu - \frac{1}{2}\sigma^2\right)t\right] \ and$$

$$Var(S_t) = S_0^2 \ exp(2\mu + \sigma^2 t)[exp(\sigma^2 t) - 1]$$

With 95% confidence interval, S_t becomes:

$$exp\left[\ln S_0 + \left(\mu - \frac{1}{2}\sigma^2\right)t - 1.96\sigma\sqrt{t}\right] \leq S_t$$
$$\leq exp\left[\ln S_0 + \left(\mu - \frac{1}{2}\sigma^2\right)t + 1.96\sigma\sqrt{t}\right]$$

6. RESEARCH OUTPUT & DISCUSSION

In this paper, the past S&P BSE closed stock price data from 1st January, 2019 to 31st December, 2019 is used to find out that the data are normally distributed and feasible to forecast for future stock price and the data from 1st January, 2020 to 31st January, 2020 will be used as validation for compare with the forecast data.

6.1. Kolmogorov-Smirnov Test and Q – Q Plot Technique

The Kolmogorov-Smirnov (K-S) statistic is used to measure how well the distribution of a random sample $(X_1, X_2, \ldots \ldots \ldots X_n)$ agrees with a theoretical distribution (Kolmogorov, 1933; Smirnov, 1939; Feller, 1948; Doob, 1949; Durbin, 1968; Epanechnikov, 1968; Steck, 1971; No'e, 1972; Niederhausen, 1981; Calabrese & Zenga, 2010; Simard & L'Ecuyer, 2011 and Carvalho, 2015). It is defined as

$$D_n = \overset{sup}{\underset{x}{}} |F_n(x) - F(x)|$$

Where n is the sample size, $F_n(x)$ be the empirical distribution function of $(X_1, X_2, \ldots \ldots \ldots X_n)$ and $F(x)$ be the cumulative distribution function of a pre-specified theoretical distribution, under the null hypothesis that the sample $(X_1, X_2, \ldots \ldots \ldots X_n)$ comes from $F(x)$ or the data are normally distributed (Massey, 1951).

The Kolmogorov-Smirnov (K-S) statistic of S&P BSE closed stock price for 245 days from 1st January, 2019 to 31st December, 2019 is illustrated in Table – 1. Based on the above test, $K - S\ Statistic \geq 0.05$. So it can be conclude that daily observed S&P BSE closed stock price is normally distributed and feasible to do stock price forecast.

Table 1: Kolmogorov-Smirnov Normality Test

Accuracy Measurement	K-S Statistic
Value	0.0667

Since the distribution of D_n is difficult, Smirnov (1948), suggested, either $D_n^+ = \overset{sup}{\underset{x}{}} |F_n(x) - F(x)|$, the maximum of first half, or $D_n^- = \overset{sup}{\underset{x}{}} |F(x) - F_n(x)|$, the maximum of the second half have a common easier distribution. For statistical computing, Knuth (1998) recommended to use $K_n^+ = \sqrt{n}D_n^+$ and $K_n^- = \sqrt{n}D_n^-$ on the ground that they seems most convenient for computer use.

The graphical representation of Q-Q plot technique indicates that the data set plausibly came from normal distribution and the points forming a line that's roughly straight (Figure 1).
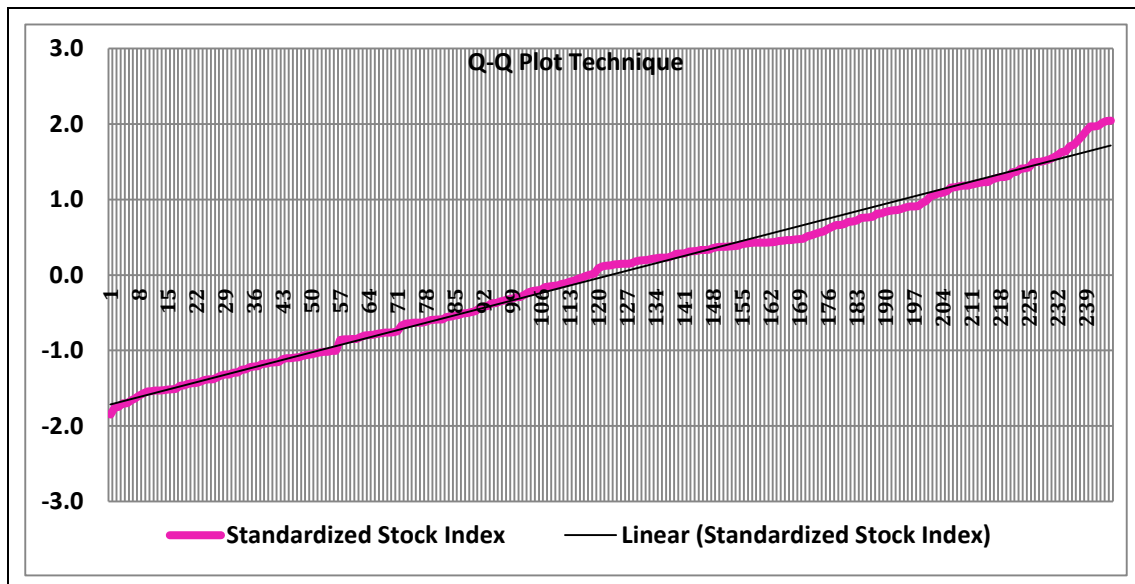
Figure 1: Q – Q Plot Technique

6.2. Forecasting the Return Distribution

From the methodology stated above, simulation of S&P BSE closed stock price using Geometric Brownian Motion for a period of 23 days, from 1st January, 2020 to 31st January, 2020 is performed and illustrated in Figure 2.
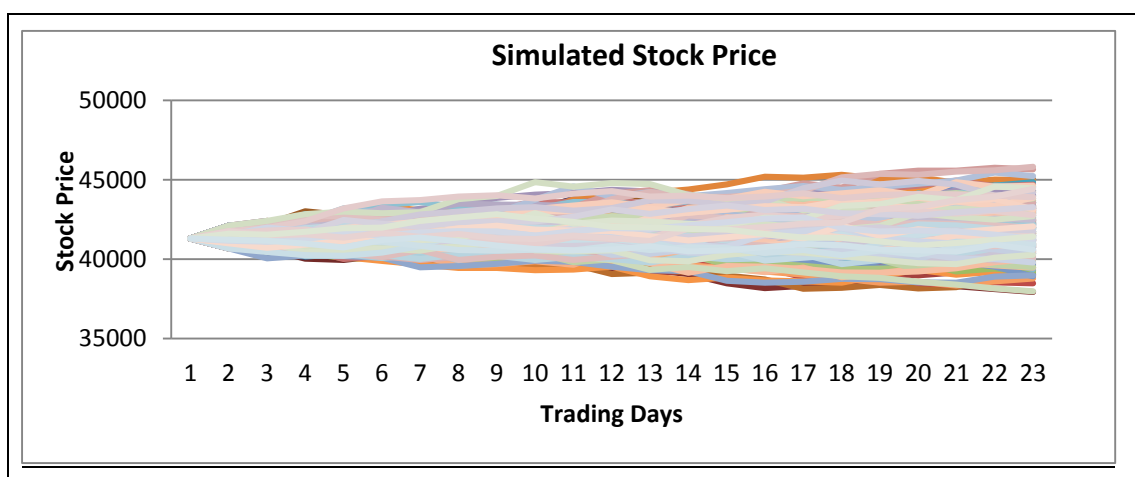
**Figure 2:** Stock Price Simulation Model

Simulation is made for stock price forecasting with 255 realization of trajectory that

may be from Geometric Brownian Motion with twenty three iterations. The actual and forecast drift and volatility based on actual and forecast value is presented in Table 2.

Table 2: Comparison between actual and forecast drift and volatility

Rate	Drift (μ)	Volatility (σ)
Actual	-0.0562	0.7476
Forecast	-0.5697	0.6983

Short-term return distribution using simulation, which is more optimistic in nature, is the closest forecast simulation with actual value of S&P BSE closed stock price is selected with more precise value of drift and volatility and presented in Figure – 3. The selecting process between all the simulations is to ensure the forecast data in return distribution are closest in the pattern and trend according to upward and downward movement in the S&P BSE closed stock price. Such simulated return distribution can be used as a tool in managing investment risk and allowing investors to take further decisions.
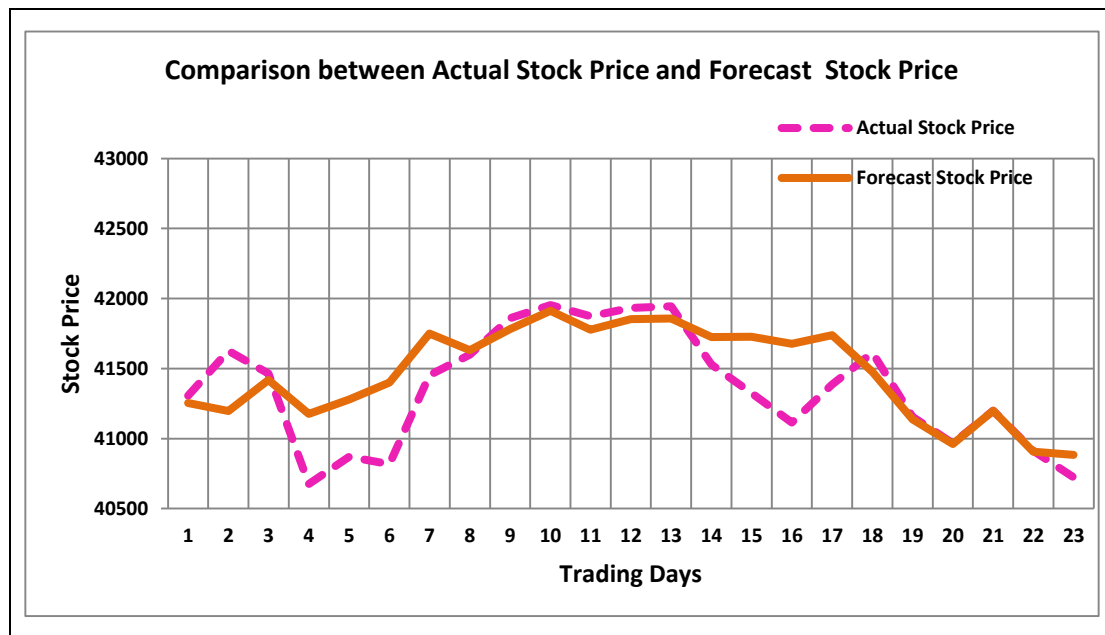
Figure 3: Comparison between Actual Stock Price and Forecast Stock Price

The correlation coefficient between simulated stock price of return distribution and actual stock price has a positive correlation($\rho = 0.524$), which represents a strong relationship between actual stock price and forecast stock price due to stabilise its mean value in long-run.

6.3.   Forecasting Accuracy

The Mean Absolute Percentage Error (MAPE) also referred as Mean Absolute Percentage Deviation (MAPD) is one of the most popular measures of the forecast accuracy due to its advantages of scale-independency and interpretability (Hanke & Reitsch, 1995 and Bowerman, O'Connell & Koehler, 2004). It produces infinite or undefined values, when the actual values are zero or closer to zero. If the actual value is very small (less than one), the MAPE yields extremely large percentage errors or outliers (Makridakis, Wheelwright & Hyndman, 1998). Makridakis (1993), attempts to resolve this problem by excluding outliers that have actual value less than one or Absolute Percentage Error (APE) values greater than the MAPE plus three S.D.

Let A_t and F_t denotes the actual and forecast value at specified time 't' respectively. Then MAPE is defined as (Tofallis, 2015 and Kim et al. 2016)

$$MAPE = \frac{1}{N} \sum_{t=1}^{N} \left| \frac{A_t - F_t}{A_t} \right|$$

Where, N is the number of data points. When MAPE is multiplied by 100 %, it is called as APE. This measure is generally only used when quantity of interest is strictly positive. The statistical test value of MAPE of S&P BSE closed stock price for 23 days, from 1st January, 2020 to 31st January, 2020 is illustrated in Table 3.

Table – 3: Mean Absolute Percentage Error (MAPE)

Accuracy Measurement	MAPE
Value	5.41

The scale of judgement using MAPE equation given by Lawrence et al. (2009) is shown in Table 4.

Table 4: Scale of Judgement of Forecast Accuracy

Sl No	MAPE	Judgement of Forecast Accuracy
1	< 10 %	Highly Accurate Forecast
2	11 % - 20 %	Good Forecast
3	21 % - 50 %	Reasonable Forecast
4	> 51 %	Inaccurate Forecast

Since the MAPE < 10%, it implies that the Geometric Brownian Motion model is highly accurate for forecasting the S&P BSE stock price. Due to random behaviour of stock price Geometric Brownian Motion model is highly suitable for short term forecasting.

7.   CONCLUSIONS

The advantage of Geometric Brownian Motion model indicates that the simulated return distribution can be applied not only as an investment decision tool but also in managing investment risk. The technique allows the investors to predict the size of losses which may occur for taking further decision on investment risk. As a form of recommendation, the predictive power of Geometric Brownian Motion model should be used to forecast daily stock prices over short period as it gives a highly accurate result.

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