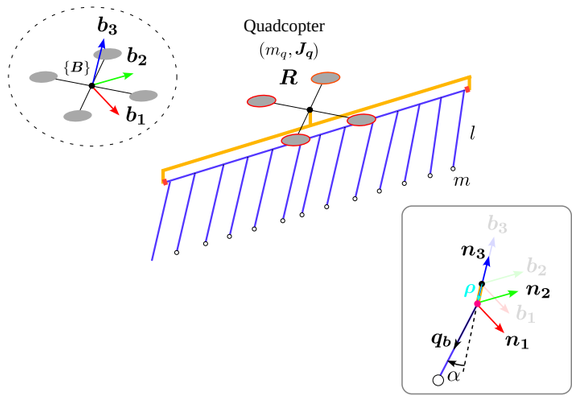
MRS NET WITH Control Link Only



i31 -> 0 | 1 | 2 | 3 | m+1 | m + 2

j32-> 1 | j32-> 1 | j32-> 1

i41 -> 0 | 1 | 2 | 3 | m+1 | m + 2

j43-> 1 | j43-> 1 | j43-> 1

	M_{00}	M_{011} (j32, i31)	M_{021}	M_{031}	$-R\widehat{A_1}$	$c_1 R q_t$
j40 1 / j42 1	$M_{011}\widehat{q_{11}^2}$ (j42, j40)	$-M_{111}$ (j40, j42, j43)	0_3	0_3	$-K_{11}$	$M_{011} l \widehat{q_{11}^2} R q_t$
j40 2 / j42 1	$M_{021}\widehat{q_{21}^2}$	0_3	$-M_{211}$	0_3	$-K_{21}$	$M_{021} l \widehat{q_{21}^2} R q_t$
j40 3 / j42 1	$M_{031}\widehat{q_{31}^2}$	0_3	0_3	$-M_{311}$	$-K_{m1}$	$M_{031} l \widehat{q_{31}^2} R q_t$

i51 -> 0 | 1 | 2 | 3 | m+1 | m + 2

j53-> 1 | j53-> 1 | j53-> 1

A_2	$K_{R_{11}} R^T$ (j53, i51)	$K_{R_{21}} R^T$	$K_{R_{31}} R^T$	$\bar{J}$	$A_3 q_t$

i61 -> 0 | 1 | 2 | 3 | m+1 | m + 2

j73-> 1 | j73-> 1 | j73-> 1

$c_1 q_t^T R^T$	$M_{011} l q_t^T R^T$ (j73, i61)	$M_{021} l q_t^T R^T$	$M_{031} l q_t^T R^T$	$-l q_t^T \widehat{A_4}$	$c_1 l$

$\ddot{x}_q$
$\ddot{q}_{11}$
$\ddot{q}_{21}$
$\ddot{q}_{31}$
$\dot{\Omega}$
$\ddot{\alpha}$

$$M_{00} = \left(m_q + m_r + \sum_{i=1}^{m} m_{i1} + \sum_{i=1}^{m} m_i\right), \qquad M_{ijk} = m_{i1} l_{ij} l_{ik}, \qquad M_{0ij} = m_{i1} l_{ij}$$

$$\bar{J} = \left(J - \frac{1}{2} m_r \widehat{\rho}^2 - \frac{1}{6} m_r l_r^2 \widehat{e_2}^2 - \frac{1}{2}\sum_{i=1}^{m} m_{i1}([\rho + \widehat{\rho_i} + l q_b])^2 - \frac{1}{2}\sum_{i=1}^{m} m_i([\rho + \widehat{\rho_i} + l q_b])^2\right)$$

$$A_1 = m_r \rho + \sum_{i=1}^{m} m_{i1}(\rho + \rho_i + l q_b) + \sum_{i=1}^{m} m_i(\rho + \rho_i + l q_b)$$

$$c_1 = \sum_{i=1}^{m} m_{i1} l + \sum_{i=1}^{m} m_i l$$

$$A_2 = m_r g \widehat{\rho} R^T + \sum_{i=1}^{m} m_{i1} g[(\rho + \widehat{\rho_i} + l q_b)] R^T + \sum_{i=1}^{m} m_i g[(\rho + \widehat{\rho_i} + l q_b)] R^T$$

$$A_3 = \sum_{i=1}^{m} m_{i1}(\rho + \widehat{\rho_i} + l q_b) l + \sum_{i=1}^{m} m_i(\rho + \widehat{\rho_i} + l q_b) l$$

$$K_{i1} = m_{i1} l_{i1} \widehat{q_{i1}^2} R(\rho + \widehat{\rho_i} + l q_b$$

$$K_{R_{i1}} = m_{i1} l_{i1}(\rho + \widehat{\rho_i} + l q_b$$

$$+R\widehat{\Omega}^2 A_1 + 2c_1\dot{\alpha} R\widehat{\Omega} q_t - c_1\dot{\alpha}^2 R q_b + M_{00} g e_3$$

$+$

j60 1 / j62 1:
$$-M_{111} q_{11} \|\dot{q}_{11}\|^2 + m_{11} l_{11} \widehat{q_{11}^2} R\widehat{\Omega}^2(\rho + \rho_1 + l q_b) + 2M_{011} l \widehat{q_{11}^2} R\widehat{\Omega} q_t \dot{\alpha} - M_{011} l\dot{\alpha}^2 \widehat{q_{11}^2} R q_b + \widehat{q_{11}^2} M_{011} g e_3$$

j60 2 / j62 1:
$$-M_{211} q_{21} \|\dot{q}_{21}\|^2 + m_{21} l_{21} \widehat{q_{21}^2} R\widehat{\Omega}^2(\rho + \rho_2 + l q_b) + 2M_{021} l \widehat{q_{21}^2} R\widehat{\Omega} q_t \dot{\alpha} - M_{021} l\dot{\alpha}^2 \widehat{q_{21}^2} R q_b + \widehat{q_{21}^2} M_{021} g e_3$$

j60 3 / j62 1:
$$-M_{311} q_{31} \|\dot{q}_{31}\|^2 + m_{31} l_{31} \widehat{q_{31}^2} R\widehat{\Omega}^2(\rho + \rho_3 + l q_b) + 2M_{031} l \widehat{q_{31}^2} R\widehat{\Omega} q_t \dot{\alpha} - M_{031} l\dot{\alpha}^2 \widehat{q_{31}^2} R q_b + \widehat{q_{31}^2} M_{031} g e_3$$

$$+\widehat{\Omega}\bar{J}\Omega + \dot{\bar{J}}\Omega - A_3\dot{\alpha}^2 q_b + \widehat{\Omega} A_3 \dot{\alpha} q_t + A_2 e_3$$

$$+\tfrac{1}{2}\Omega^T A_3 \widehat{q_t}\Omega + c_1 g q_t^T R^T e_3$$

$=$

$$\begin{bmatrix} u \\ 0 \\ 0 \\ 0 \\ \tau \\ 0 \end{bmatrix}$$

i31 ->	0	1	2	3	m+1	m + 2
		j32-> 1	j32-> 1	j32-> 1		

$$\ddot{x}_q,\ \ddot{q}_{11},\ \ddot{q}_{21},\ \ddot{q}_{31},\ \dot{\Omega},\ \ddot{\alpha}$$

$$M_{00} = \left(m_q + m_r + \sum_{i=1}^{m} m_{i1} + \sum_{i=1}^{m} m_i\right), \qquad M_{ijk} = m_{i1} l_{ij} l_{ik}, \qquad M_{0ij} = m_{i1} l_{ij}$$

$$\bar{J} = \left(J - \frac{1}{2} m_r \widehat{\rho}^2 - \frac{1}{6} m_l l_r^2 \widehat{e_2}^2 - \frac{1}{2}\sum_{i=1}^{m} m_{i1}([\rho + \widehat{\rho_i} + l q_b])^2 - \frac{1}{2}\sum_{i=1}^{m} m_i([\rho + \widehat{\rho_i} + l q_b])^2\right)$$

$$A_1 = m_r \rho + \sum_{i=1}^{m} m_{i1}(\rho + \rho_i + l q_b) + \sum_{i=1}^{m} m_i(\rho + \rho_i + l q_b)$$

$$c_1 = \sum_{i=1}^{m} m_{i1} l + \sum_{i=1}^{m} m_i l$$

i41 ->	0	1	2	3	m+1	m + 2
	j42	j43-> 1	j43-> 1	j43-> 1		
j40 1	1	$M_{011}\widehat{q}_{11}^2$	$-M_{111}$	0_3	0_3	$-K_{11}$	$M_{011} l \widehat{q}_{11}^2 R q_t$
	j42					
j40 2	1	$M_{021}\widehat{q}_{21}^2$	0_3	$-M_{211}$	0_3	$-K_{21}$	$M_{021} l \widehat{q}_{21}^2 R q_t$
	j42					
j40 3	1	$M_{031}\widehat{q}_{31}^2$	0_3	0_3	$-M_{311}$	$-K_{m1}$	$M_{031} l \widehat{q}_{31}^2 R q_t$

$$A_2 = m_l g \widehat{\rho} R^T + \sum_{i=1}^{m} m_{i1} g[(\rho + \widehat{\rho_i} + l q_b)] R^T + \sum_{i=1}^{m} m_i g[(\rho + \widehat{\rho_i} + l q_b)] R^T$$

$$A_3 = \sum_{i=1}^{m} m_{i1}(\rho + \widehat{\rho_i} + l q_b) l + \sum_{i=1}^{m} m_i(\rho + \widehat{\rho_i} + l q_b) l$$

$$K_{i1} = m_{i1} l_{i1} \widehat{q}_{i1}^2 R(\rho + \widehat{\rho_i} + l q_b$$

$$K_{R_{i1}} = m_{i1} l_{i1}(\rho + \widehat{\rho_i} + l q_b$$

i51 ->	0	1	2	3	m+1	m + 2
		j53-> 1	j53-> 1	j53-> 1		
	A_2	$K_{R_{11}} R^T$	$K_{R_{21}} R^T$	$K_{R_{31}} R^T$	$\bar{J}$	$A_3 q_t$

i61 ->	0	1	2	3	m+1	m + 2
		j73-> 1	j73-> 1	j73-> 1		
	$c_1 q_t^T R^T$	$M_{011} l q_t^T R^T$	$M_{021} l q_t^T R^T$	$M_{031} l q_t^T R^T$	$-l q_t^T \widehat{A_4}$	$c_1 l$

$$+R\widehat{\Omega}^2 A_1 + 2c_1 l \dot{\alpha} R \widehat{\Omega} q_t - c_1 l \dot{\alpha}^2 R q_b + M_{00} g e_3$$

$$+\widehat{\Omega}\bar{J}\Omega + \dot{\bar{J}}\Omega - A_3 \dot{\alpha}^2 q_b + \widehat{\Omega} A_3 \dot{\alpha} q_t + A_2 e_3$$

$$+\tfrac{1}{2}\Omega^T A_3 \widehat{q}_t \Omega + c_1 g q_t^T R^T e_3$$

$$= \begin{bmatrix} u \\ 0 \\ 0 \\ 0 \\ \tau \\ 0 \end{bmatrix}$$

j60	j62					
1	1	$-M_{111} q_{11} \|\dot{q}_{11}\|^2 + m_{11} l_{11} \widehat{q}_{11}^2 R \widehat{\Omega}^2(\rho + \rho_1 + l q_b) + 2M_{011} l \widehat{q}_{11}^2 R \widehat{\Omega} q_t \dot{\alpha} - M_{011} l \dot{\alpha}^2 \widehat{q}_{11}^2 R q_b + \widehat{q}_{11}^2 M_{011} g e_3$				
2	1	$-M_{211} q_{21} \|\dot{q}_{21}\|^2 + m_{21} l_{21} \widehat{q}_{21}^2 R \widehat{\Omega}^2(\rho + \rho_2 + l q_b) + 2M_{021} l \widehat{q}_{21}^2 R \widehat{\Omega} q_t \dot{\alpha} - M_{021} l \dot{\alpha}^2 \widehat{q}_{21}^2 R q_b + \widehat{q}_{21}^2 M_{021} g e_3$				
3	1	$-M_{311} q_{31} \|\dot{q}_{31}\|^2 + m_{31} l_{31} \widehat{q}_{31}^2 R \widehat{\Omega}^2(\rho + \rho_3 + l q_b) + 2M_{031} l \widehat{q}_{31}^2 R \widehat{\Omega} q_t \dot{\alpha} - M_{031} l \dot{\alpha}^2 \widehat{q}_{31}^2 R q_b + \widehat{q}_{31}^2 M_{031} g e_3$				

(7)

(8)

) (9)

) (10)

References

$$
\begin{bmatrix}
M_{00} & & & M_{021} & M_{031} & -R\widehat{A}_1 & c_1\boldsymbol{R}\boldsymbol{q_t} \\
M_{011}\widehat{q}_{11}^2 & & & 0_3 & 0_3 & -K_{11} & M_{011}l\widehat{q}_{11}^2\boldsymbol{R}\boldsymbol{q_t} \\
M_{021}\widehat{q}_{21}^2 & & & -M_{211} & 0_3 & -K_{21} & M_{021}l\widehat{q}_{21}^2\boldsymbol{R}\boldsymbol{q_t} \\
M_{031}\widehat{q}_{31}^2 & & 0_3 & 0_3 & -M_{311} & -K_{m1} & M_{031}l\widehat{q}_{31}^2\boldsymbol{R}\boldsymbol{q_t} \\
\boldsymbol{A_2} & & & \boldsymbol{K}_{R_{21}}\boldsymbol{R}^T & \boldsymbol{K}_{R_{31}}\boldsymbol{R}^T & \bar{\boldsymbol{J}} & \boldsymbol{A_3}\boldsymbol{q_t} \\
c_1\boldsymbol{q}_t^T\boldsymbol{R}^T & M & T & M_{021}l\boldsymbol{q}_t^T\boldsymbol{R}^T & M_{031}l\boldsymbol{q}_t^T\boldsymbol{R}^T & -l\boldsymbol{q}_t^T\widehat{\boldsymbol{A_4}} & c_1 l
\end{bmatrix}
\begin{bmatrix} \ddot{\boldsymbol{x}}_q \\ \ddot{\boldsymbol{q}}_{11} \\ \ddot{\boldsymbol{q}}_{21} \\ \ddot{\boldsymbol{q}}_{31} \\ \dot{\boldsymbol{\Omega}} \\ \ddot{\alpha} \end{bmatrix}
$$

$$
+\begin{bmatrix}
+\boldsymbol{R}\widehat{\boldsymbol{\Omega}}^2\boldsymbol{A_1} + 2c_1\dot{\alpha}\boldsymbol{R}\widehat{\boldsymbol{\Omega}}\boldsymbol{q_t} - c_1\dot{\alpha}^2\boldsymbol{R}\boldsymbol{q_b} + M_{00}g\boldsymbol{e_3} \\
-M_{111}\boldsymbol{q_{11}}\|\dot{\boldsymbol{q}}_{11}\|^2 + m_{11}l_{11}\widehat{\boldsymbol{q}}_{11}^2\boldsymbol{R}\widehat{\boldsymbol{\Omega}}^2(\boldsymbol{\rho}+\boldsymbol{\rho}_1+l\boldsymbol{q_b}) + 2M_{011}l\widehat{\boldsymbol{q}}_{11}^2\boldsymbol{R}\widehat{\boldsymbol{\Omega}}\boldsymbol{q_t}\dot{\alpha} - M_{011}l\dot{\alpha}^2\widehat{\boldsymbol{q}}_{11}^2\boldsymbol{R}\boldsymbol{q_b} + \widehat{\boldsymbol{q}}_{11}^2M_{011}g\boldsymbol{e_3} \\
-M_{211}\boldsymbol{q_{21}}\|\dot{\boldsymbol{q}}_{21}\|^2 + m_{21}l_{21}\widehat{\boldsymbol{q}}_{21}^2\boldsymbol{R}\widehat{\boldsymbol{\Omega}}^2(\boldsymbol{\rho}+\boldsymbol{\rho}_2+l\boldsymbol{q_b}) + 2M_{021}l\widehat{\boldsymbol{q}}_{21}^2\boldsymbol{R}\widehat{\boldsymbol{\Omega}}\boldsymbol{q_t}\dot{\alpha} - M_{021}l\dot{\alpha}^2\widehat{\boldsymbol{q}}_{21}^2\boldsymbol{R}\boldsymbol{q_b} + \widehat{\boldsymbol{q}}_{21}^2M_{021}g\boldsymbol{e_3} \\
-M_{311}\boldsymbol{q_{31}}\|\dot{\boldsymbol{q}}_{31}\|^2 + m_{31}l_{31}\widehat{\boldsymbol{q}}_{31}^2\boldsymbol{R}\widehat{\boldsymbol{\Omega}}^2(\boldsymbol{\rho}+\boldsymbol{\rho}_3+l\boldsymbol{q_b}) + 2M_{031}l\widehat{\boldsymbol{q}}_{31}^2\boldsymbol{R}\widehat{\boldsymbol{\Omega}}\boldsymbol{q_t}\dot{\alpha} - M_{031}l\dot{\alpha}^2\widehat{\boldsymbol{q}}_{31}^2\boldsymbol{R}\boldsymbol{q_b} + \widehat{\boldsymbol{q}}_{31}^2M_{031}g\boldsymbol{e_3} \\
+\widehat{\boldsymbol{\Omega}}\bar{\boldsymbol{J}}\boldsymbol{\Omega} + \dot{\bar{\boldsymbol{J}}}\boldsymbol{\Omega} - \boldsymbol{A_3}\dot{\alpha}^2\boldsymbol{q_b} + \widehat{\boldsymbol{\Omega}}\boldsymbol{A_3}\dot{\alpha}\boldsymbol{q_t} + \boldsymbol{A_2}\boldsymbol{e_3} \\
+\tfrac{1}{2}\boldsymbol{\Omega}^T\boldsymbol{A_3}\widehat{\boldsymbol{q_t}}\boldsymbol{\Omega} + c_1g\boldsymbol{q}_t^T\boldsymbol{R}^T\boldsymbol{e_3}
\end{bmatrix}
=
\begin{bmatrix} \boldsymbol{u} \\ \boldsymbol{0} \\ \boldsymbol{0} \\ \boldsymbol{0} \\ \boldsymbol{\tau} \\ 0 \end{bmatrix}
$$

$$
M_{00} = \left(m_q + m_r + \sum_{i=1}^{m} m_{i1} + \sum_{i=1}^{m} m_i\right), \qquad M_{ijk} = m_{i1}l_{ij}l_{ik}, \qquad M_{0ij} = m_{i1}l_{ij}
$$

$$
\bar{\boldsymbol{J}} = \left(\boldsymbol{J} - \frac{1}{2}m_r\widehat{\boldsymbol{\rho}}^2 - \frac{1}{6}m_rl_r^2\widehat{\boldsymbol{e_2}}^2 - \frac{1}{2}\sum_{i=1}^{m} m_{i1}([\boldsymbol{\rho}+\widehat{\boldsymbol{\rho_i}}+l\boldsymbol{q_b}])^2 - \frac{1}{2}\sum_{i=1}^{m} m_i([\boldsymbol{\rho}+\widehat{\boldsymbol{\rho_i}}+l\boldsymbol{q_b}])^2\right)
$$

$$
\boldsymbol{A_1} = m_r\boldsymbol{\rho} + \sum_{i=1}^{m} m_{i1}(\boldsymbol{\rho}+\boldsymbol{\rho}_i+l\boldsymbol{q_b}) + \sum_{i=1}^{m} m_i(\boldsymbol{\rho}+\boldsymbol{\rho}_i+l\boldsymbol{q_b})
$$

$$
c_1 = \sum_{i=1}^{m} m_{i1}l + \sum_{i=1}^{m} m_il
$$

$$
\boldsymbol{A_2} = m_rg\widehat{\boldsymbol{\rho}}\boldsymbol{R}^T + \sum_{i=1}^{m} m_{i1}g[(\boldsymbol{\rho}+\widehat{\boldsymbol{\rho_i}}+l\boldsymbol{q_b})]\boldsymbol{R}^T + \sum_{i=1}^{m} m_ig[(\boldsymbol{\rho}+\widehat{\boldsymbol{\rho_i}}+l\boldsymbol{q_b})]\boldsymbol{R}^T
$$

$$
\boldsymbol{A_3} = \sum_{i=1}^{m} m_{i1}(\boldsymbol{\rho}+\widehat{\boldsymbol{\rho_i}}+l\boldsymbol{q_b})l + \sum_{i=1}^{m} m_i(\boldsymbol{\rho}+\widehat{\boldsymbol{\rho_i}}+l\boldsymbol{q_b})l
$$

$$
\boldsymbol{K_{i1}} = m_{i1}l_{i1}\widehat{\boldsymbol{q}}_{\boldsymbol{i1}}^2\boldsymbol{R}(\boldsymbol{\rho}+\widehat{\boldsymbol{\rho_i}}+l\boldsymbol{q_b}
$$

$$
\boldsymbol{K}_{\boldsymbol{R_{i1}}} = m_{i1}l_{i1}(\boldsymbol{\rho}+\widehat{\boldsymbol{\rho_i}}+l\boldsymbol{q_b}
$$

$$
\boldsymbol{A}_{(4)} = \sum_{i=1}^{m} m_i(\boldsymbol{\rho}+\widehat{\boldsymbol{\rho_i}}+l\boldsymbol{q_b})
$$

$$\boldsymbol{A_2} = m_l g\widehat{\boldsymbol{\rho}}\boldsymbol{R}^T + \sum_{i=1}^{m} m_{i1} g[(\boldsymbol{\rho} + \widehat{\boldsymbol{\rho_i}} + l\boldsymbol{q_b})]\boldsymbol{R}^T + \sum_{i=1}^{m} m_i g[(\boldsymbol{\rho} + \widehat{\boldsymbol{\rho_i}} + l\boldsymbol{q_b})]\boldsymbol{R}^T \tag{7}$$

$$\boldsymbol{A_3} = \sum_{i=1}^{m} m_{i1}(\boldsymbol{\rho} + \widehat{\boldsymbol{\rho_i}} + l\boldsymbol{q_b})l + \sum_{i=1}^{m} m_i(\boldsymbol{\rho} + \widehat{\boldsymbol{\rho_i}} + l\boldsymbol{q_b})l \tag{8}$$

$$\boldsymbol{K_{i1}} = m_{i1} l_{i1} \widehat{\boldsymbol{q}}_{\boldsymbol{i1}}^2 \boldsymbol{R}(\boldsymbol{\rho} + \widehat{\boldsymbol{\rho_i}} + l\boldsymbol{q_b}) \tag{9}$$

$$\boldsymbol{K_{R_{i1}}} = m_{i1} l_{i1} (\boldsymbol{\rho} + \widehat{\boldsymbol{\rho_i}} + l\boldsymbol{q_b}) \tag{10}$$

References