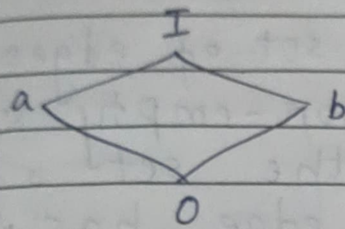
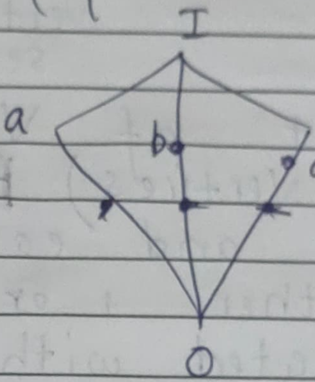
## * Complemented Lattice :-

Lattice $L$ is called complemented if it is bounded and every element has complement. (Atleast one)

eg:-




{If any elements contains more than one complement, then it is not distributive la}

## * Distributive Lattice

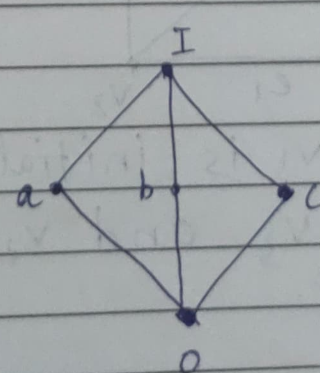
Lattice $L$ is called Distributive Lattice, if for every element $a, b, c \in L$, it satisfy distributive property.

① $a \vee (b \wedge c) = (a \vee b) \wedge (a \vee c)$

② $a \wedge (b \vee c) = (a \wedge b) \vee (a \wedge c)$

Atmost one complement

eg:-




$a \vee (b \wedge c)$
$= a \vee 0 = a$

$(a \vee b) \wedge (a \vee c)$
$= I \wedge I$
$= I$

$a \vee (b \wedge c) \neq (a \vee b) \wedge (a \vee c)$

# Graph

* Graph is $G = (V, E)$

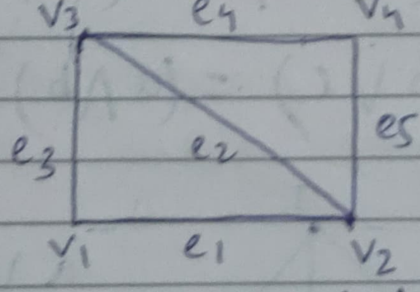
$V \to$ vertex set, $E \to$ set of edges.

consist of V (non-empty set of vertices) E is the set of edges and each edge has either 1 or 2 vertices associated with it.

vertices $\to$ (end pt of the edge)

eg: ① $V = \{v_1, v_2, v_3, v_4\}$

$E = \{e_1, e_2, e_3, e_4, e_5\}$

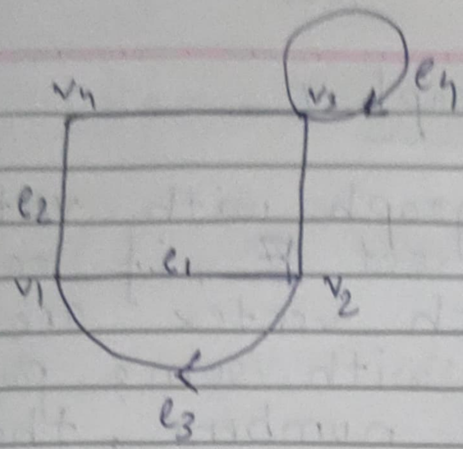



For $e_1$, $v_1$ is initial & $v_2$ is terminal

for $v_1$, $v_3$ and $v_2$ are adjacent. ~~edges~~

for $v_3$, $v_2, v_4, v_1$ are adjacent

(2)

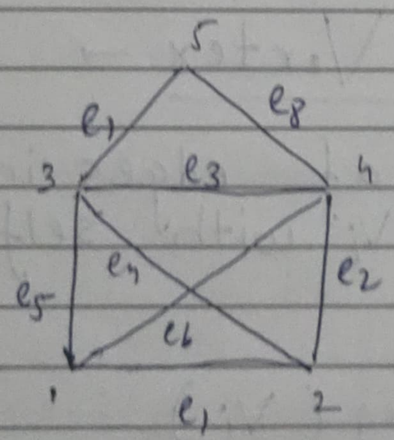



## ★ Types of Graph :-

### (1) Simple graph :->

A graph that has neither self-loop nor ||el edges.

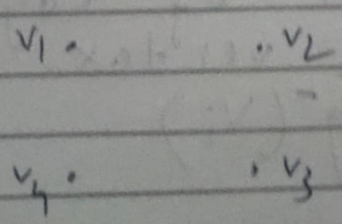
eg-(1)




~~1, 2 & 3, 4~~

$e_3$ and $e_1$ are not parallel edges cause it is having different vertices 1 and 2 & 3 ~~&4~~ and 4.

ex eg-(2) ★ NULL graph

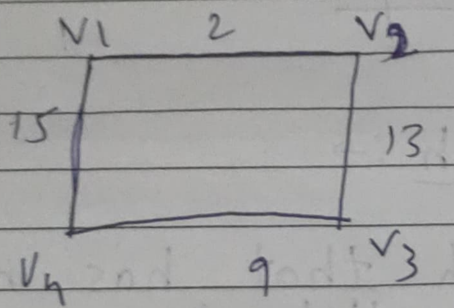



Not edges but it is graph.

(2) **Weighted Graph :-**

→ Let G be graph with vertex set V and edge set E, if each edge or each vertex are associated with some positive ~~integers~~ real numbers, then the graph is called weighted graph.

eg:-




## * Degree of a Vertex →

→ The number of edges incident on the vertex $V_i$ with self-loop counted twice.

Denoted as $deg(V_i)$

## * Indegree of Vertex :-

→ Is the number of edges as their terminal vertex.

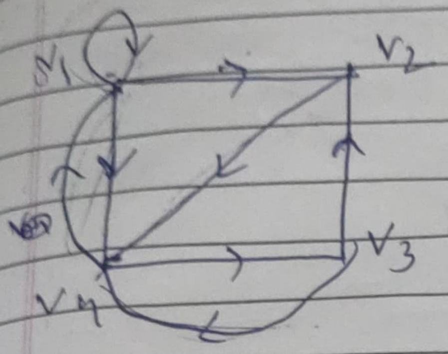
Denoted by $deg^-(V_i)$

(Kitne ~~vertex~~ edges vertex pe aare)

## Outdegree of Vertex →

→ is the no. of edges with as their initial vertex. Denoted as $deg^+(v_i)$

eg →




$deg(v_1) = 5$

$deg^-(v_1) = 2$

$deg^+(v_1) = 3$

$deg(v_4) = 5$

$deg^-(v_4) = 3$

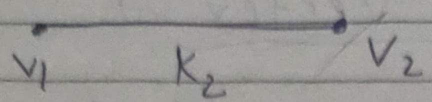
$deg^+(v_4) = 2$

---

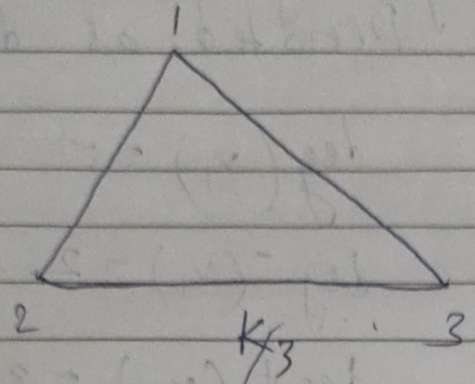
## Complete Graph →

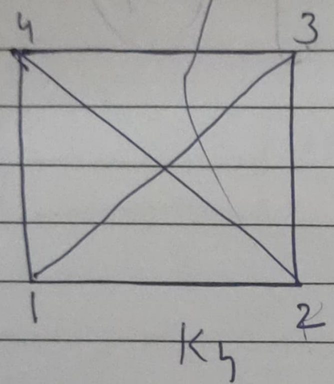
→ Let G be Simple graph with n vertices, if degree of each vertex is ~~A~~ "n-1", then the graph is called Complete graph.

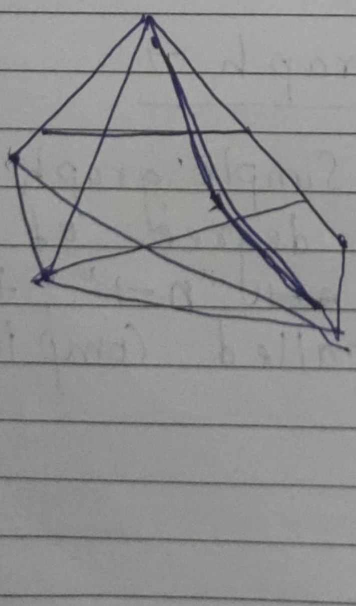
eg :-

① 


$\Rightarrow n-1 \Rightarrow 2-1 = 1$ degree

② 


③ 


④ 

## Bipartite Graph →

Let G be the graph with vertex set V and edges set E, then G is called bipartite graph if it's vertex set V can be partitioned into two disjoint subsets $V_1$ and $V_2$. such that,

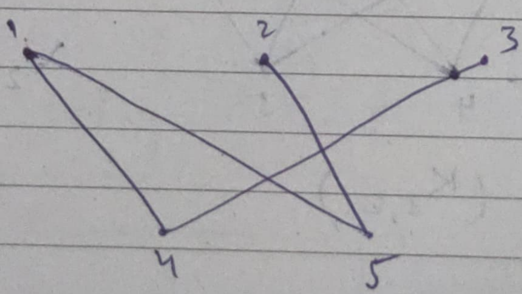
$$V_1 \cup V_2 = V \quad \text{and} \quad V_1 \cap V_2 = \phi$$

and also each edge of G joins the vertex of $V_1$ to a vertex of a $V_2$.

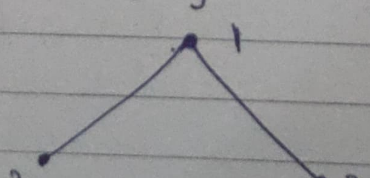
eg → $V = \{1, 2, 3, 4, 5\}$

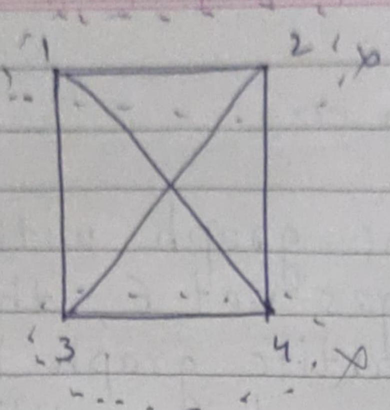
$V_1 = \{1, 2, 3\}$  $V_2 = \{4, 5\}$

(1)




(2) $V_1 = \{1\}$  $V_2 = \{2, 3\}$

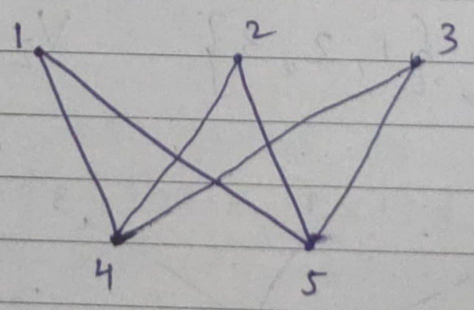


③ 


Not a bipartite graph.

## * Complete Bipartite Graph →

→ A bipartite graph is called complete bipartite graph if each vertex of $V_1$ is joint to every vertex of $V_2$ by unique edge.

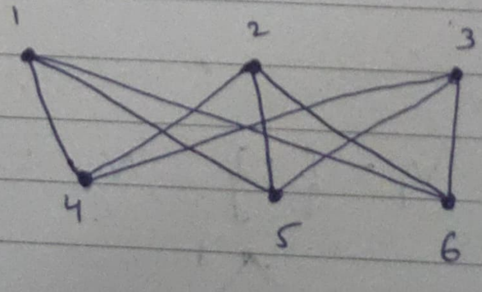
eg → ① 


$V_1 = \{1, 2, 3\}$

$V_2 = \{4, 5\}$

$(K_{3,2})$

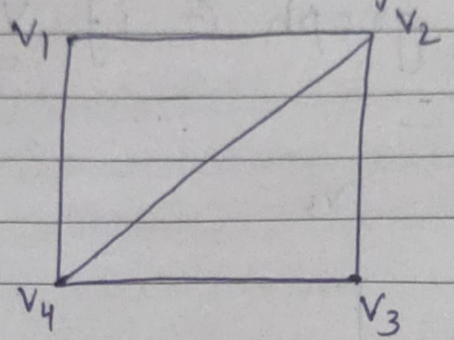
② $(K_{3,3})$



## Path →

A path in a graph is a sequence of ~~vertices~~ $V_1, V_2, ----, V_k$ of a vertices each adjacent to the next.

eg →




$V_4 V_2 V_3$ ↓ A path

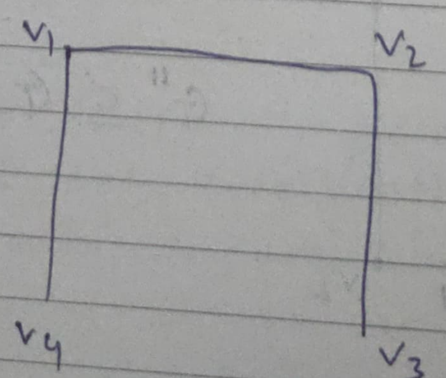
$V_1 V_3 V_4$ → Not a path.

$V_1 V_2 V_3 V_4 V_1$ → Circuit / Cycle.

## Connected Graph →

A graph is said to be connected if their exist a path b/w every pair of vertices.

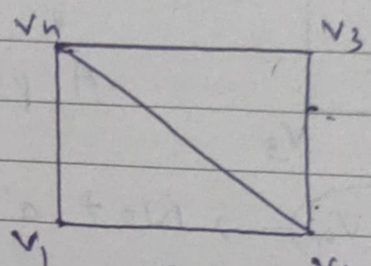
eg →



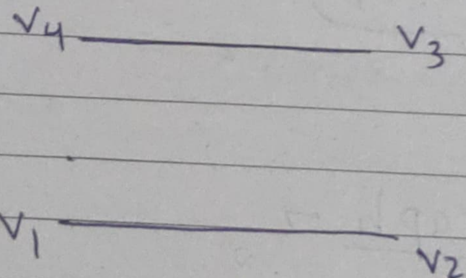
## *Subgraph of Graph →

Let $G = (V, E)$, then the graph $G' = (V', E')$ is called subgraph of graph $G$ if $V' \subseteq V$ and $E' \subseteq E$.

eg → ①



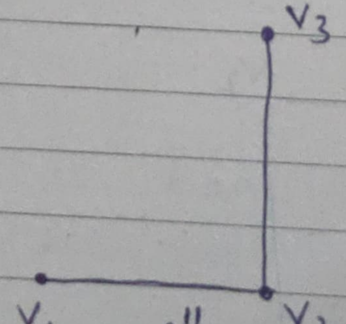

G




G'

So, $G' \subseteq G$

Also,




$G'' \subseteq G$

G''

Goodluck | Page No. | Date

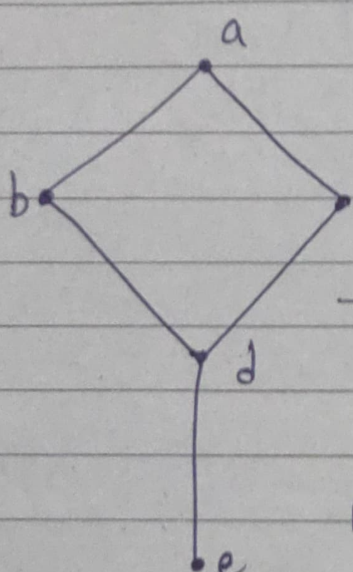
*__Euler Path__ →

A path in a graph G is Euler path if it includes every edge exactly once cover's.

*__Hamiltonian Path__ →

It includes every vertex exactly once cover's

__eg__ →




→ Euler path

⇓

[edbacd]

Not hamiltonian cauze d is coming twice.

Now if,

bacde

↓

Now this is hamiltonian path. but not euler path cauze bd path is missing!

25th Nov

## Shortest Path Algorithm →

(valid for both directed and undirected)

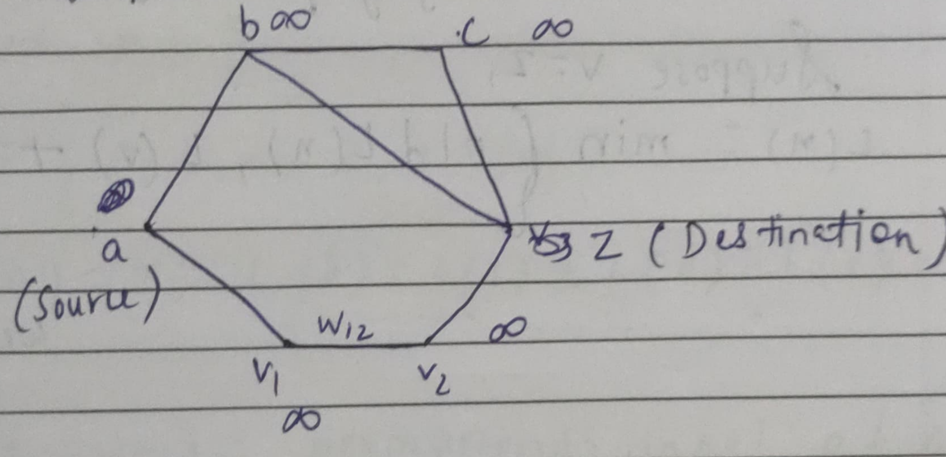
$G = (V, E)$

Let a and z be any two vertex of graph

$L(x)$
↓
Label of vertex $(x)$ of graph G.

"$w_{ij}$" denotes weight of edge $V_{i \to j}$.

eg 1 →




Step 1 →

$P = \phi$ (P denotes set of vertices having permanent label)

T
↓
set of vertices of graph G.

Source vertex will be zero.

$L(a) = 0$, $L(x) = \infty$, $x \in T$, $x \neq a$

Step 2 → Select vertex in T having smallest label. This label is called permanent label.

$v \to$ smallest label.
$V \to$ permanent label.

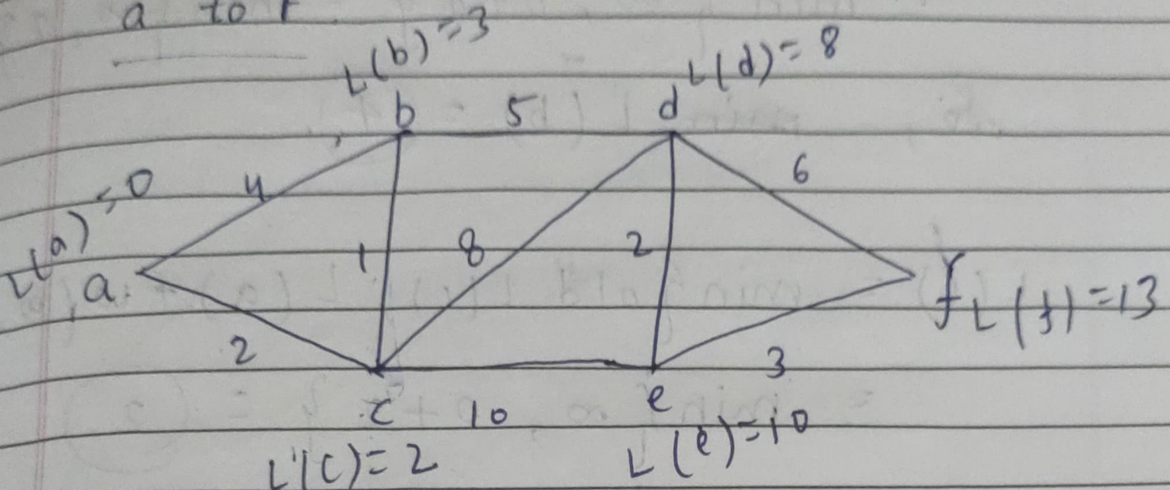
$T = \{$set of all vertices of graph G$\} - V$

Suppose $v = z$,

$$L(x) = \min\{\text{old } L(x),\ L(v) + \underbrace{w(v,x)}_{\text{weight}}\}$$

# Problem

Using Dijkstra's Algorithm to find shortest ~~vertex~~ path from path a to f




Step ① Let $p = \phi$,

$T = \{a, b, c, d, e, f\}$

$L(a) = 0, \quad L(b) = L(c) = L(d) = L(e) = \infty$

Step ② Let $v = a$, permanent label of a is 0.

$p = \{a\} \quad T = \{b, c, d, e, f\}$.

The label of vertex $x$ in T is given by $\rightarrow$

$L(x) = \min\{\text{old } L(x),\ L(v) + w(v, x)\}$
$\cancel{v(b)} = v = b$, permanent label of $b = 4$
$L(b) = \min\{\text{old } L(b),\ L(a) + w(\infty, b)\}$

$= \min\{\infty, 0 + 4\} = \min\{\infty, 4\}$

So, ~~min~~ $L(b) = 4$.

$L(c) = \min\{\text{old } L(c),\ L(a) + w(v, x)\}$

$= \min\{\infty, 0 + 2\} = 2$

$L(d) = \min\{\infty, \cancel{0+5}\ \infty\} = \infty$

As d, e, f are not adjacent to a, so,

$L(d) = L(e) = L(f) = \infty$

* Check for a's permanent label then add ( ) + 4 then it will be $L(b)$.

Step 3 → $v = c$, the permanent label of $c = 2$. $L(c) = 2$

$P = \{a, c\}$ $T = \{b, d, e, f\}$

$L(f) = \infty$

$L(b) = \min\{4, 2+1\} = 3$

$L(d) = ~~\min\{~~ 10$, $L(e) = 12$.

Now, $v = b$,

$P = \{a, c, b\}$, $T = \{d, e, f\}$

$L(d) = 8$, $L(e) = 12$, $L(f) = \infty$

Now,

$P = \{a, c, b, d\}$, $T = \{e, f\}$.

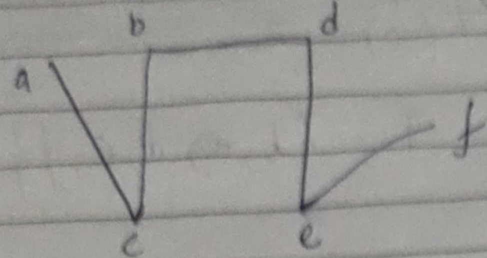
$L(e) = 10$, $L(f) = 14$

Now, also,

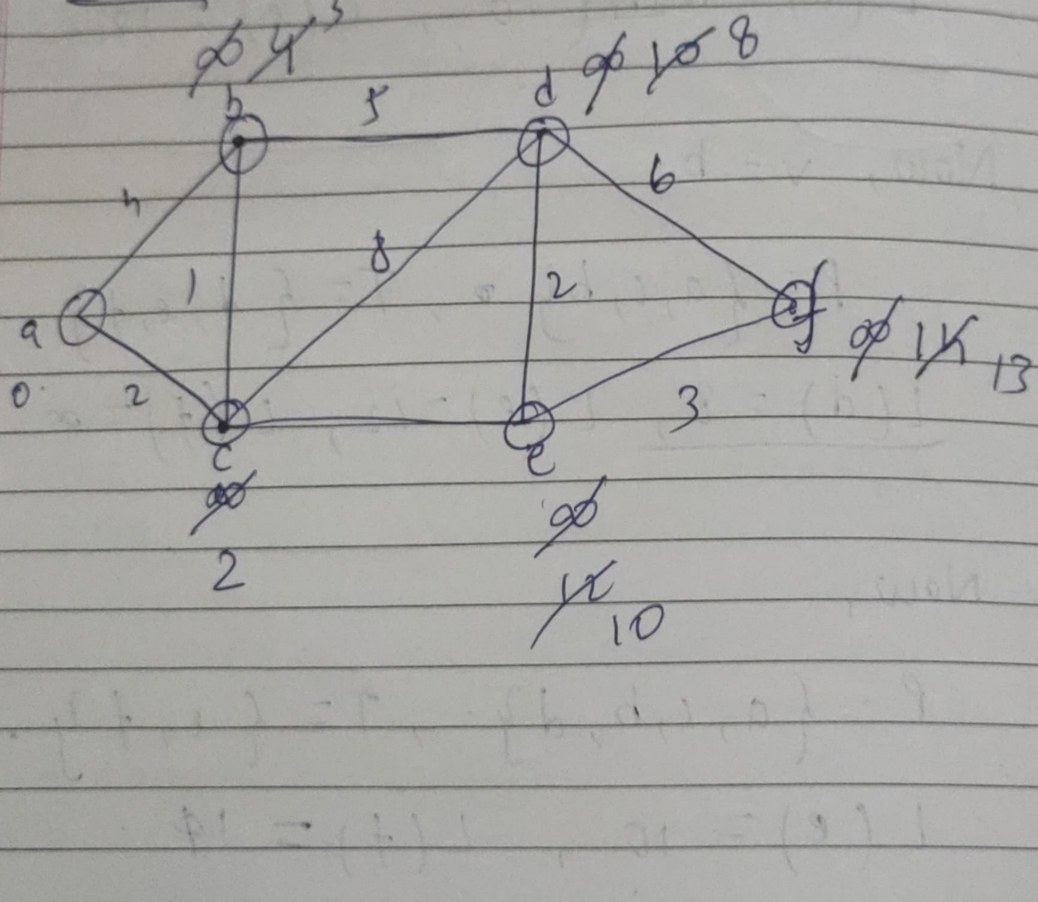
$P = \{a, c, b, d, e\}$, $T = \{f\}$

$L(f) = 13$

So, shortest path is→



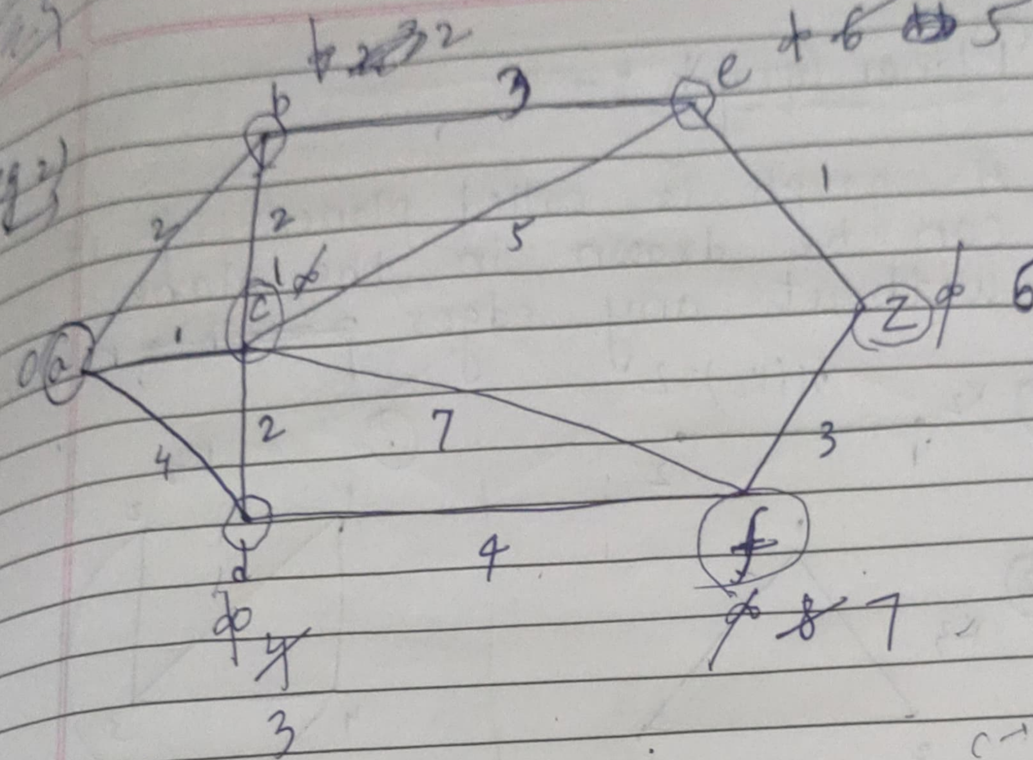

Trick:



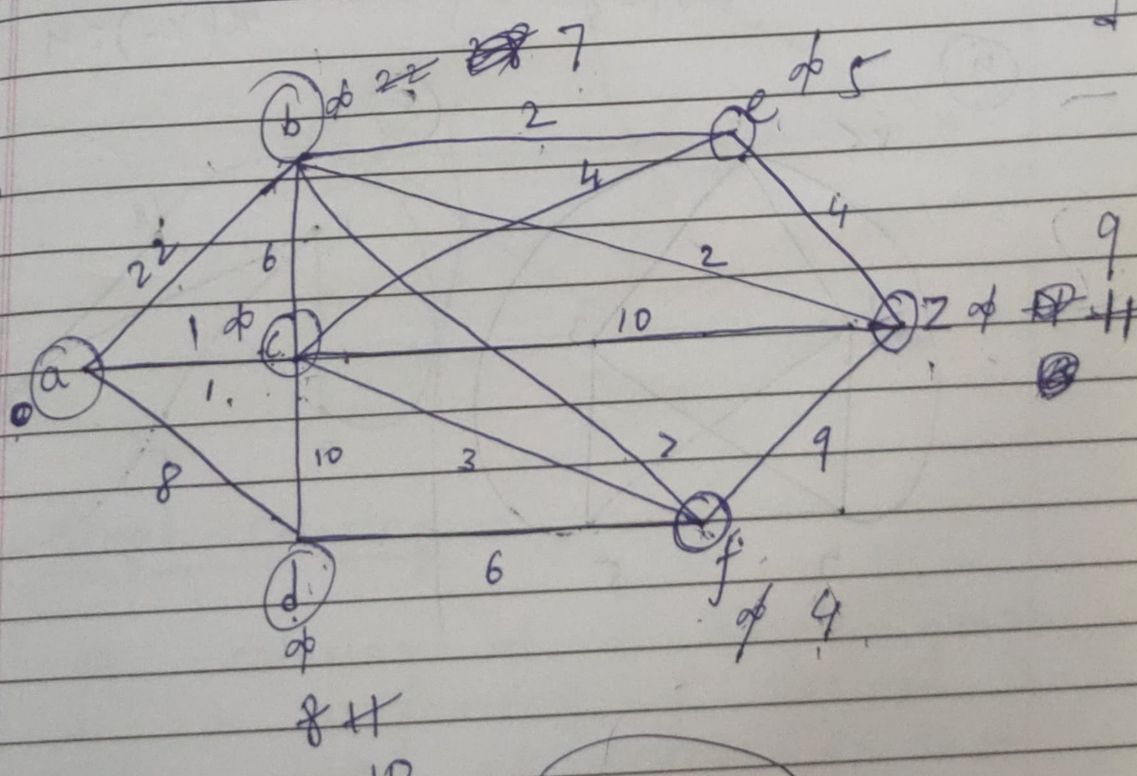

b, c, d

① assign infinity except source
② if min than than $V_i$ same change not

eg 2)




eg 3)



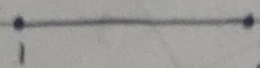

acbz

## Planer Graph :-

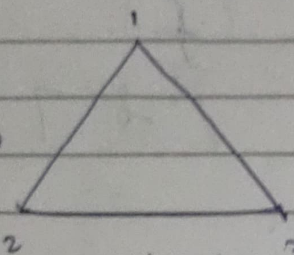
→ A graph is called planer if it can be drawn in the plane without any edges ~~possible~~ crossing

eg :- ① $K_2$ 

$n(K_2) = 2$

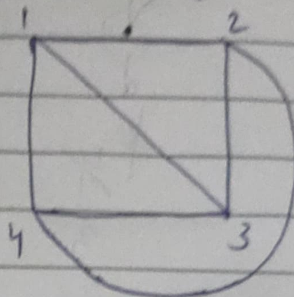



② $K_3$



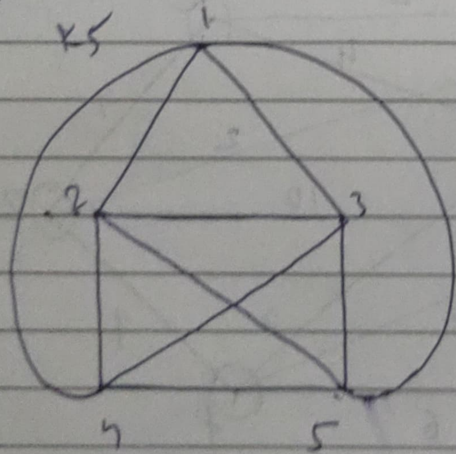

$n(K_3) = 3$

③ $K_4$

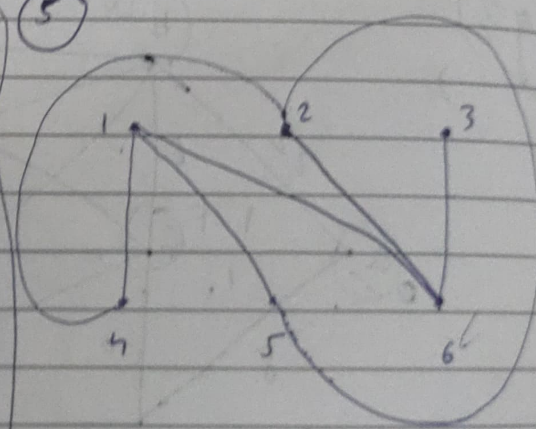



$n(K_4) = 4$

④ $K_5$

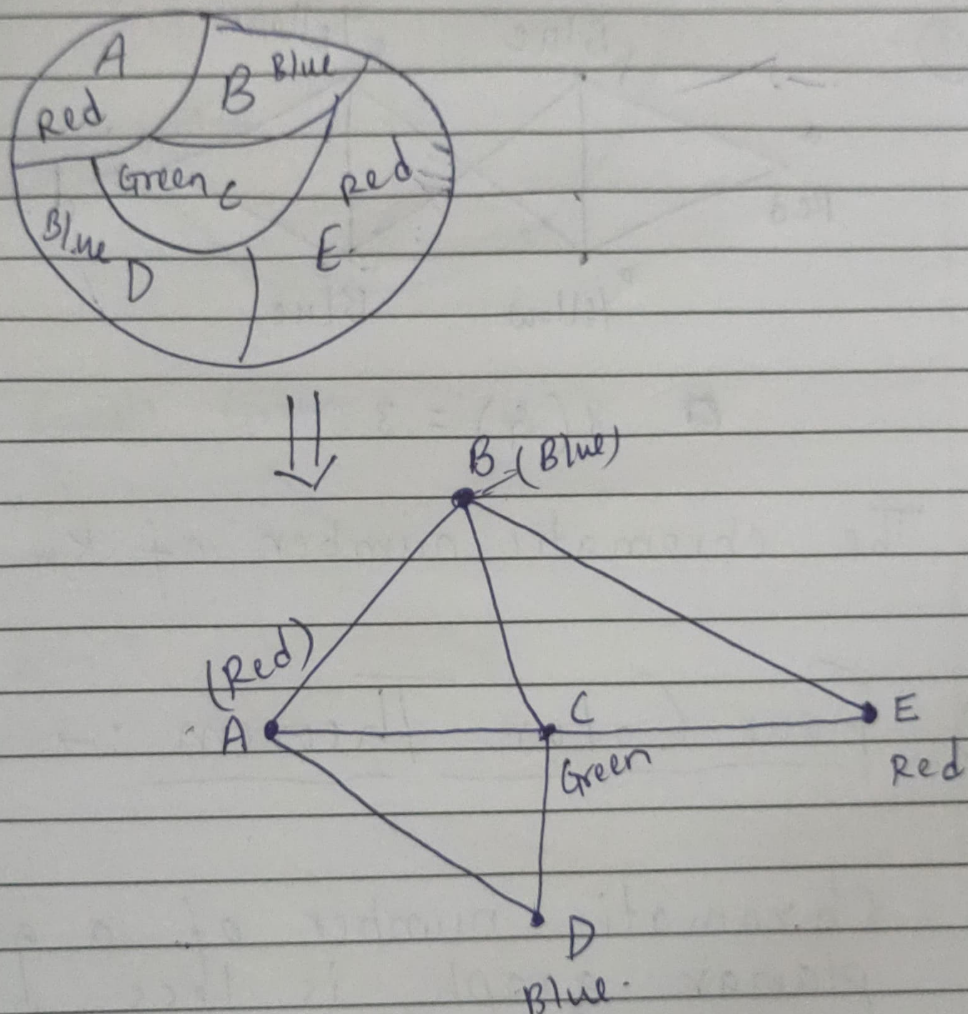



⑤



## Graph coloring :-

### Map Coloring :-



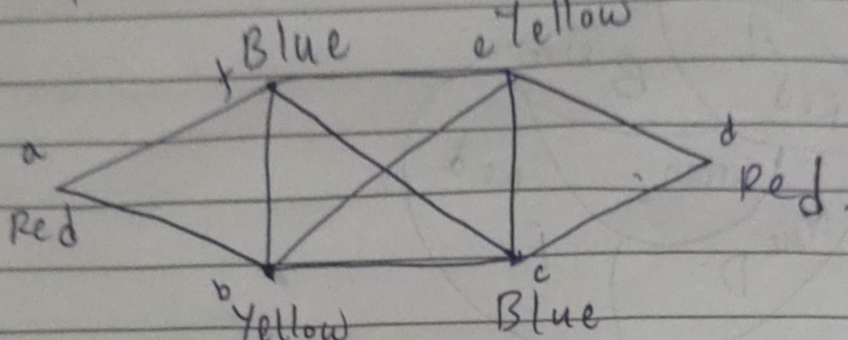

### * Graph Coloring :-

~~A~~ Coloring of a simple graph is a assignment of color to each vertex of graph so that no two adjacent vertices are assigned the same color.

$V - e + r = 2$

## Chromatic Number of graph ( $\chi(G)$ ) :-

→ It is minimum number of colors required to color the graph.

eg-① 


$\chi(G) = 3$.

The chromatic number of $K_n$ is $n$.

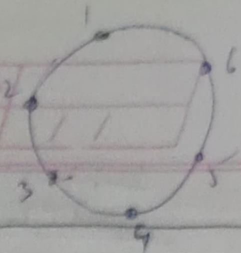
## Four Color Theorem :-

→ Chromatic number of a ~~gra~~ planar graph is less than or equal to 4.

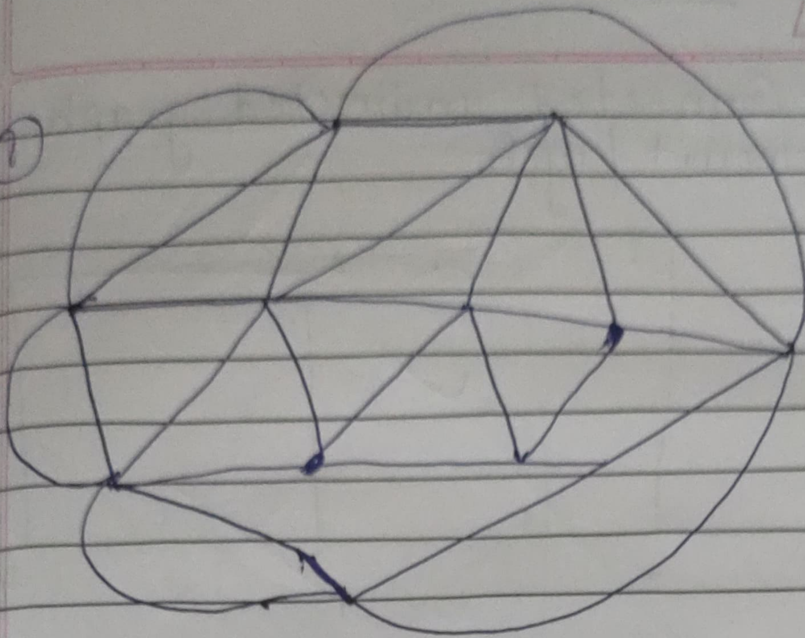
$\chi(G) = 4$.

$v - e + r = 2$

$\boxed{\text{2 colors}}$

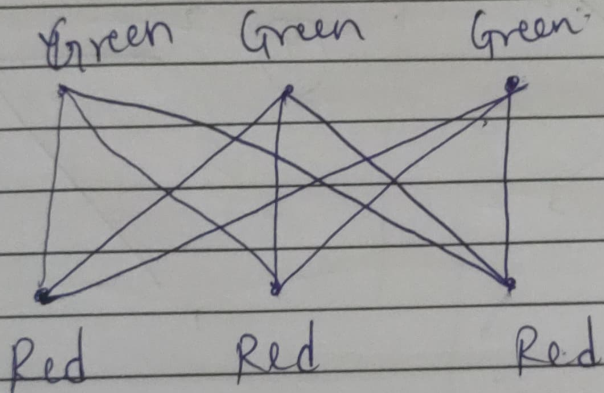



eg 1 ①



$\Rightarrow \chi(K_{11}) = 4$

eg ②

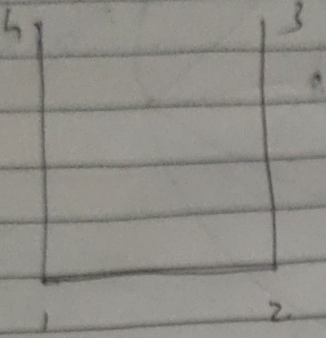


# Trees

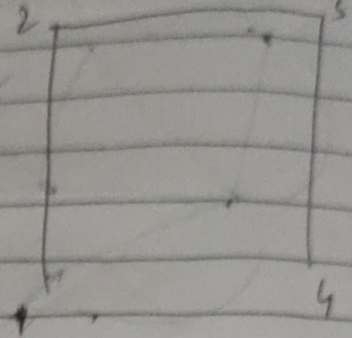
**Tree** → Connected, undirected graph with no circuit/cycle.

eg →

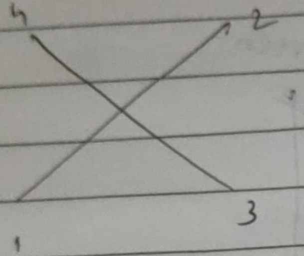
1



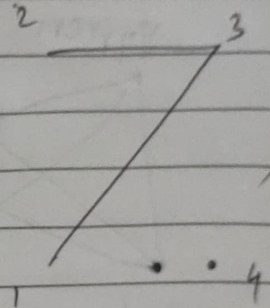

2




3



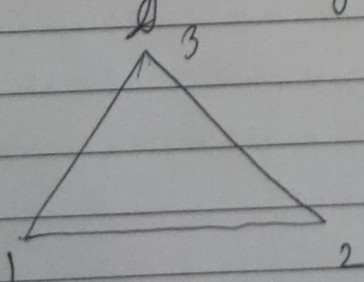

4



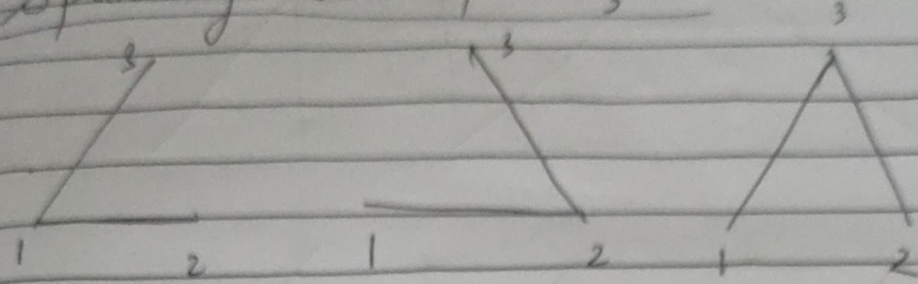

## * Spanning Tree →

→ Let G be the simple graph, spanning tree of graph G is subgraph of G that is tree containing every vertices of G.

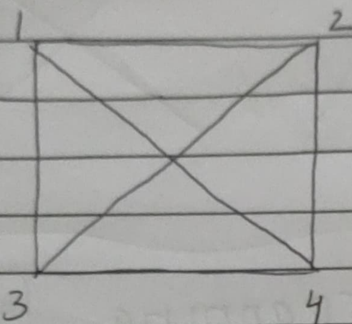
eg → 1 $K_3$



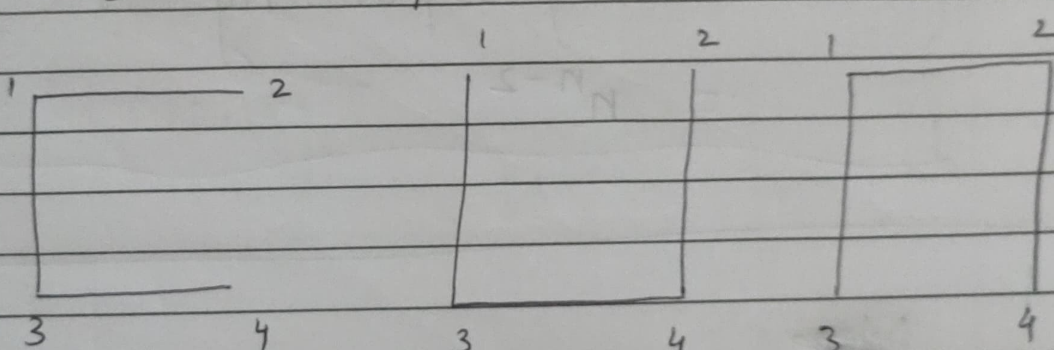
Spanning tree for $K_3$ :

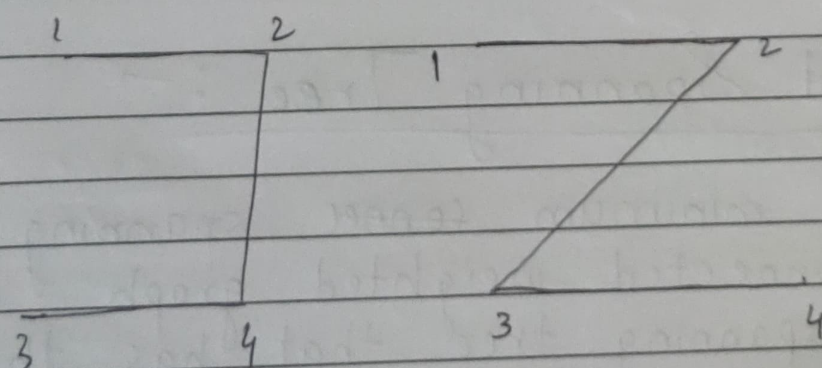


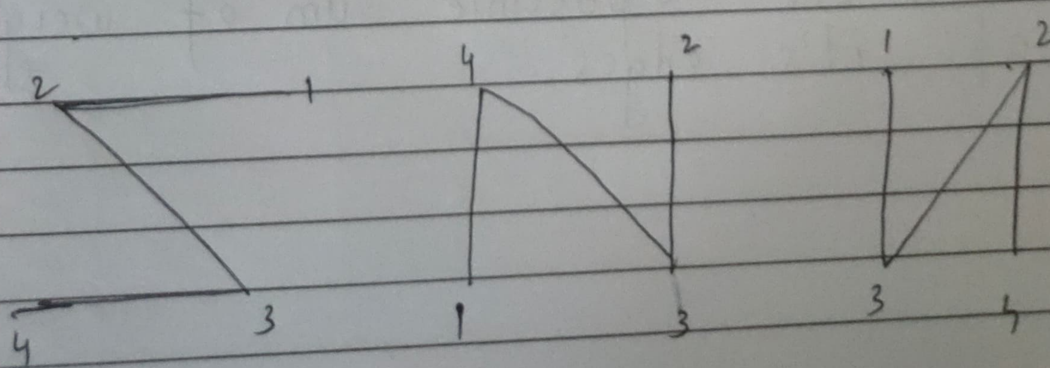

② $K_4$

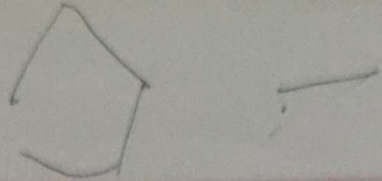


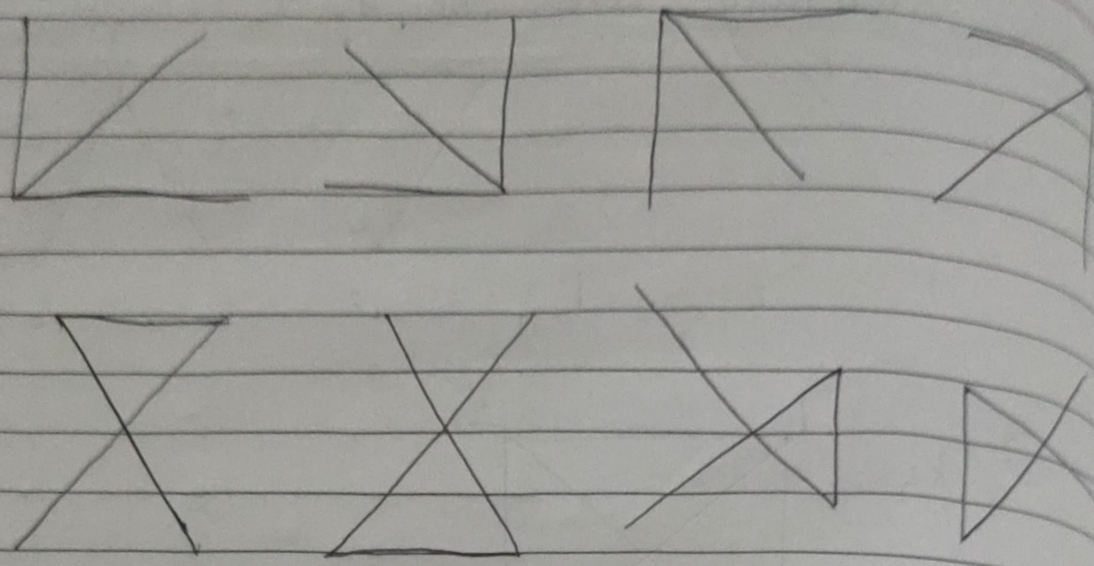

→











Number of spanning Tree $= n^{n-2}$

## * Minimal Spanning Tree :-

A minimum ~~connec~~ spanning Tree in connected weighted graph is the spanning tree that has the smallest possible sum of weight's of it's edges.