G.H. RAISONI COLLEGE OF ENGINEERING AND MANAGEMENT, WAGHOLI

(An Autonomous Institute Affiliated to Savitribai Phule Pune University)

RAISONI GROUP — a vision beyond

TAE 1

GREEN'S THEOREM

What is Green's Theorem?

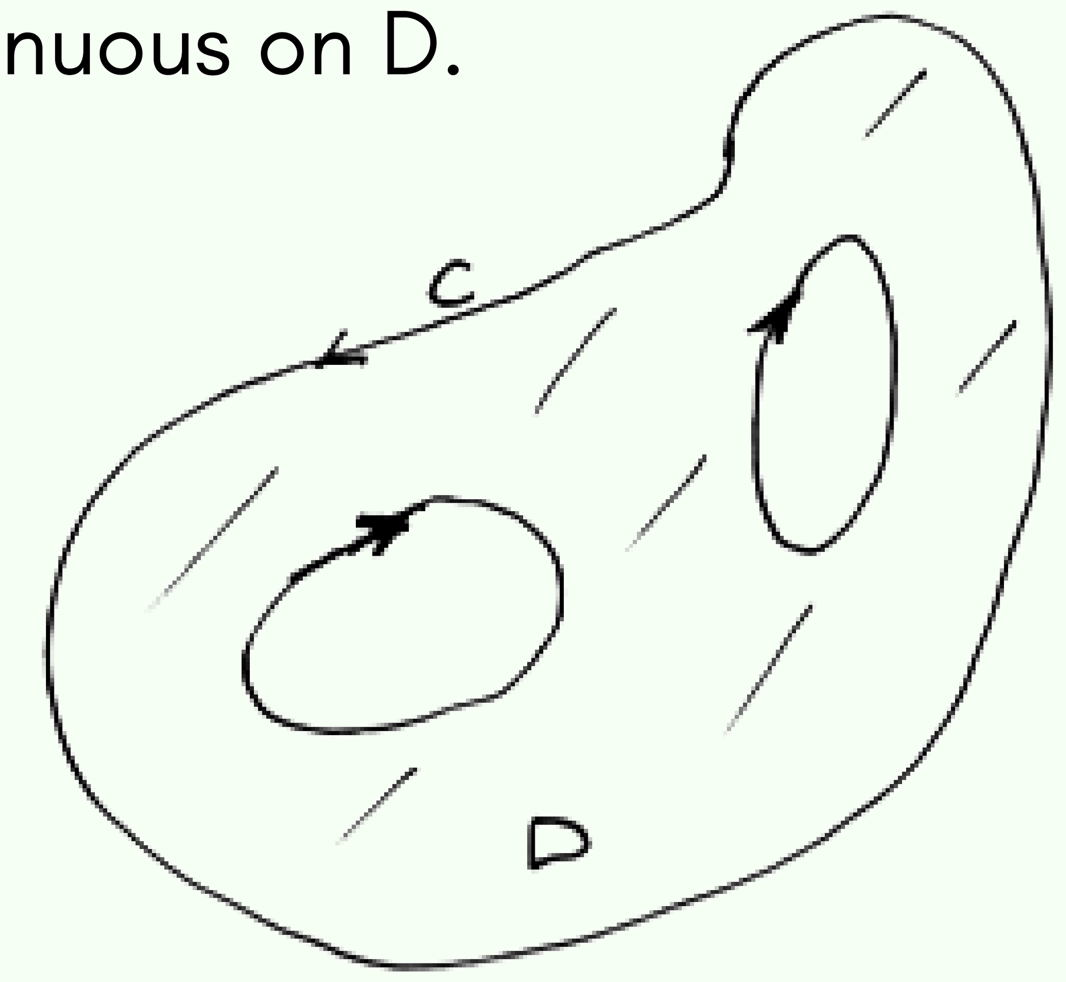
Green's Theorem gives you a relationship between the line integral of a 2D vector field over a closed path in a plane and the double integral over the region that it encloses. However, the integral of a 2D conservative field over a closed path is zero is a type of special case in Green's Theorem.

GREEN’S THEOREM:

Green’s theorem states that a line integral around the boundary of a plane region D can be computed as a double integral over D. More precisely, if D is a “nice” region in the plane and C is the boundary of D with C oriented so that D is always on the left-hand side as one goes around C (where the path integral is traversed counterclockwise along with C.), then

$$\oint_C P\,dx + Q\,dy = \iint_R \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$$

If the partial derivatives of P and Q are continuous on D.



Regions that are simultaneously of type I and II are “nice” regions, i.e., Green’s theorem is true for such regions.

EXAMPLE 1 : USING GREEN'S THEOREM

Que: Verify Green’s theorem in a plane for $\oint_C [(3x^2 - 8y^2)dx + (4y - 6xy - xy)dy]$ where C is the boundary Of the region defined by the lines x=0, y=0 and x+y=1.

Solution :

Green’s theorem is $\oint_C (P\,dx + Q\,dy) = \iint_R (Q_x - P_y)\,dx\,dy$

$$\int_C \left[\left(3x^2 - 8y^2\right)dx + \left(4y - 6xy\right)dy\right] = \iint_R 10y\,dx\,dy$$

$$L.S.\ of\ (1) = \int\limits_{\substack{AB \\ \left(\begin{smallmatrix} y=0 \\ dy=0 \end{smallmatrix}\right)}} + \int\limits_{\substack{BC \\ \left(\begin{smallmatrix} x+y=1 \\ x=1-y \\ dx=-dy \end{smallmatrix}\right)}} + \int\limits_{\substack{CA \\ \left(\begin{smallmatrix} x=0 \\ dx=0 \end{smallmatrix}\right)}} \left[\left(3x^2 - 8y^2\right)dx + \left(4y - 6xy\right)dy\right]$$

$$= \int_0^1 3x^2 dx + \int_0^1 \left(11y^2 + 4y - 3\right)dy - \int_0^1 4y\,dy$$

$$R.S.\ of\ (1) = \int_0^1 \int_0^{1-y} 10y\,dx\,dy \quad = \mathbf{5/3}$$

Applications Learning Goals:

Green's Theorem is commonly used for the integration of lines when combined with a curved plane. It is used in compute area; the tangent vector to the boundary is rotated 90° clockwise to become the outward-pointing normal vector, to derive the divergence form of Green’s theorem.

Reference :

https://www.khanacademy.org/math/multivariable-calculus

https://www.math.mcgill.ca/jakobson/courses/ma265/green.pdf

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TAE Assesment Sheet for FY Btech					
SR.NO	ROLL NO	DIVISION	NAME OF THE STUDENT	TAE NO	POSTER/PRESENTATION
1	A69	A	Soham Yugraj Tiwari	1	Poster
2	A70	A	Amaan Ayyub Nalband		
3	A71	A	Shravan Vijaypratap Singh		
4	A72	A	Pratik Rajesh Jade		
Rubrics For Assesement					
CATEGORY	Contents	Presentation	Spelling and pronunciation	Oral Presentation	TOTAL
MARKS	2	1	1	1	5
			Teacher sign (With name and date)		