G.H. Raisoni College of Engineering and Management, Wagholi
(An Autonomous Institute Affiliated to Savitribai Phule Pune University)

TAE 1

Diagonalization of a matrix

Matrix diagonalization is the process of taking a square matrix and converting it into a special type of matrix--a so-called diagonal matrix--that shares the same fundamental properties of the underlying matrix.
Matrix diagonalization is equivalent to transforming the underlying system of equations into a special set of coordinate axes in which the matrix takes this canonical form.
Diagonalizing a matrix is also equivalent to finding the matrix's eigenvalues, which turn out to be precisely the entries of the diagonalized matrix. Similarly, the eigenvectors make up the new set of axes corresponding to the diagonal matrix.

Definition of diagonalizable matrix.

Definition Let **A** be a **K x K** matrix. We say that **A** is diagonalizable if and only if it is similar to a diagonal matrix.

In other words, when **A** is diagonalizable, then there exists an invertible matrix such that

$$D = P^{-1}AP$$

where **D** is a diagonal matrix, that is, a matrix whose non-diagonal entries are zero.

How to diagonalize a matrix

Suppose we are given a matrix **A** and we are told to diagonalize it. How do we do it?

1. Compute the eigenvalues of **A**.
2. Check that no eigenvalue is defective. If any eigenvalue is defective, then the matrix cannot be diagonalized. Otherwise, you can go to the next step.
3. For each eigenvalue, find as many linearly independent eigenvectors as you can (their number is equal to the geometric multiplicity of the eigenvalue).
4. Adjoin all the eigenvectors so as to form a full-rank matrix **P**.
5. Build a diagonal matrix whose diagonal elements are the eigenvalues of **A**.
6. The diagonalization is done: $D = P^{-1}AP$

Importantly, we need to follow the same order when we build **P** and **D**: if a certain eigenvalue has been put at the intersection of the k-th column and the k-th row of **D**, then its corresponding eigenvector must be placed in the k-th column of **P**.

- **Example: Find a matrix P, if possible, that diagonalizes**

$$A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$$

The eigenvalues and eigenvectors are given by
$\lambda = 1$ with corresponding eigenvector

$$p1 = \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$$

and $\lambda = 2$ with corresponding eigenvectors

$$p2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \text{ and } p3 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

Since the matrix is 3x3 and has 3 eigenvectors, then A is diagonalizable and

$$P = \begin{bmatrix} -2 & -1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

and

$$P^{-1}AP = \begin{bmatrix} -1 & 0 & -1 \\ 1 & 0 & 2 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix} \begin{bmatrix} -2 & -1 & 0 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

Reference -
https://mathworld.wolfram.com/MatrixDiagonalization.html
https://math.libretexts.org/Bookshelves/Linear_Algebra/

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TAE Assesment Sheet for FY Btech					
SR.NO	ROLL NO	DIVISION	NAME OF THE STUDENT	TAE NO	POSTER/PRESENTATION
1	A72	A	Pratik Rajesh Jade	1	Poster
Rubrics For Assesement					
CATEGORY	Contents	Presentation	Spelling and pronunciation	Oral Presentation	TOTAL
MARKS	2	1	1	1	5
			Teacher sign (With name and date)		