Consider the transition system given in Fig. 3.6.

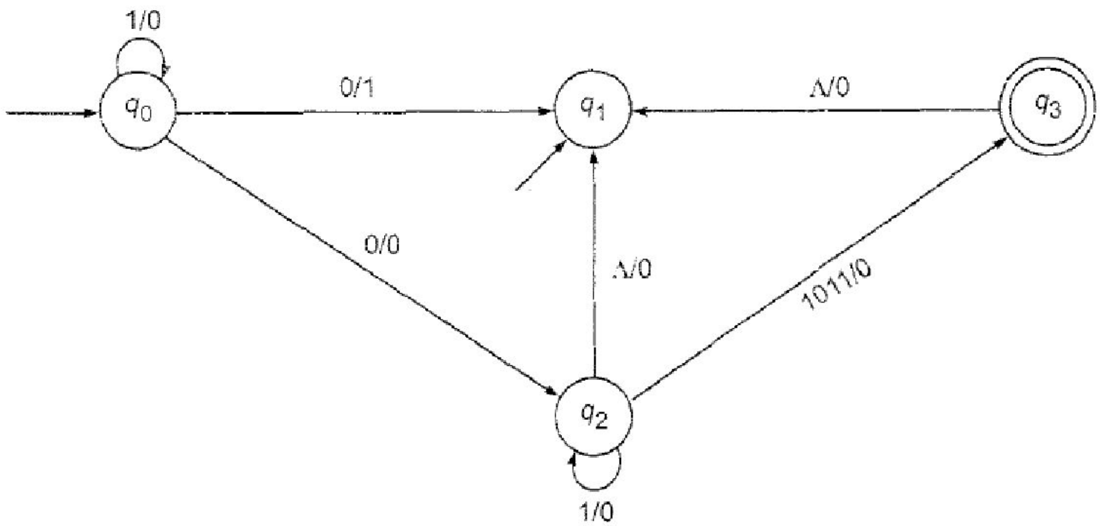


Fig. 3.6 Transition system for Example 3.2.

Determine the initial states, the final states, and the acceptability of 101011, 111010.

Consider the finite state machine whose transition function δ is given by Table 3.1 in the form of a transition table. Here, $Q = \{q_0, q_1, q_2, q_3\}$, $\Sigma = \{0, 1\}$, $F = \{q_0\}$. Give the entire sequence of states for the input string 110001.

TABLE 3.1 Transition Function Table for Example 3.5

State	*Input* 0	1
$\rightarrow q_0$	q_2	q_1
q_1	q_3	q_0
q_2	q_0	q_3
q_3	q_1	q_2

Construct a deterministic automaton equivalent to

$$M = (\{q_0, \quad q_1\}, \{0, 1\}, \delta, q_0, \{q_0\})$$

where δ is defined by its state table (see Table 3.2).

TABLE 3.2 State Table for Example 3.6

State/Σ	0	1
$\rightarrow q_0$	q_0	q_1
q_1	q_1	q_0, q_1

Find a deterministic acceptor equivalent to

$$M = (\{q_0, q_1, q_2\}, \{a, b\}, \delta, q_0, \{q_2\})$$

where δ is as given by Table 3.4.

TABLE 3.4 State Table for Example 3.7

State/Σ	*a*	*b*
$\rightarrow q_0$	q_0, q_1	q_2
q_1	q_0	q_1
q_2		q_0, q_1

Construct a deterministic finite automaton equivalent to

$$M = (\{q_0, q_1, q_2, q_3\}, \{0, 1\}), \delta, q_0, \{q_3\})$$

where δ is given by Table 3.6.

TABLE 3.6 State Table for Example 3.8

State/Σ	a	b
$\rightarrow q_0$	q_0, q_1	q_0
q_1	q_2	q_1
q_2	q_3	q_3
q_3		q_2

Consider the Mealy machine described by the transition table given by Table 3.10. Construct a Moore machine which is equivalent to the Mealy machine.

TABLE 3.10 Mealy Machine of Example 3.9

Present state	*Next state*			
	Input a = 0		*Input a = 1*	
	state	*output*	*state*	*output*
$\rightarrow q_1$	q_3	0	q_2	0
q_2	q_1	1	q_4	0
q_3	q_2	1	q_1	1
q_4	q_4	1	q_3	0

Construct a Mealy Machine which is equivalent to the Moore machine given by Table 3.14.

TABLE 3.14 Moore Machine of Example 3.10

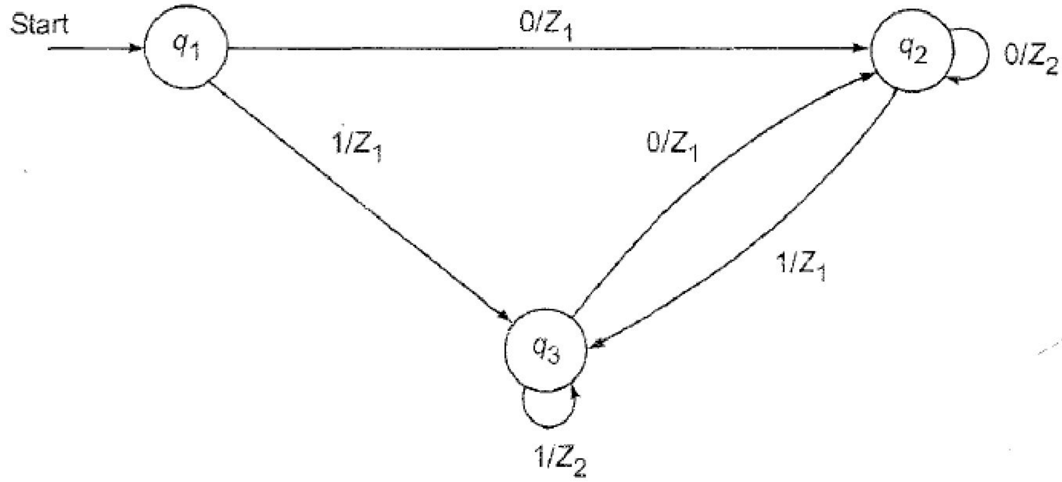
Present state	*Next state*		*Output*
	a = 0	*a* = 1	
$\rightarrow q_0$	q_3	q_1	0
q_1	q_1	q_2	1
q_2	q_2	q_3	0
q_3	q_3	q_0	0

Consider the Moore machine described by the transition table given by Table 3.16. Construct the corresponding Mealy machine.

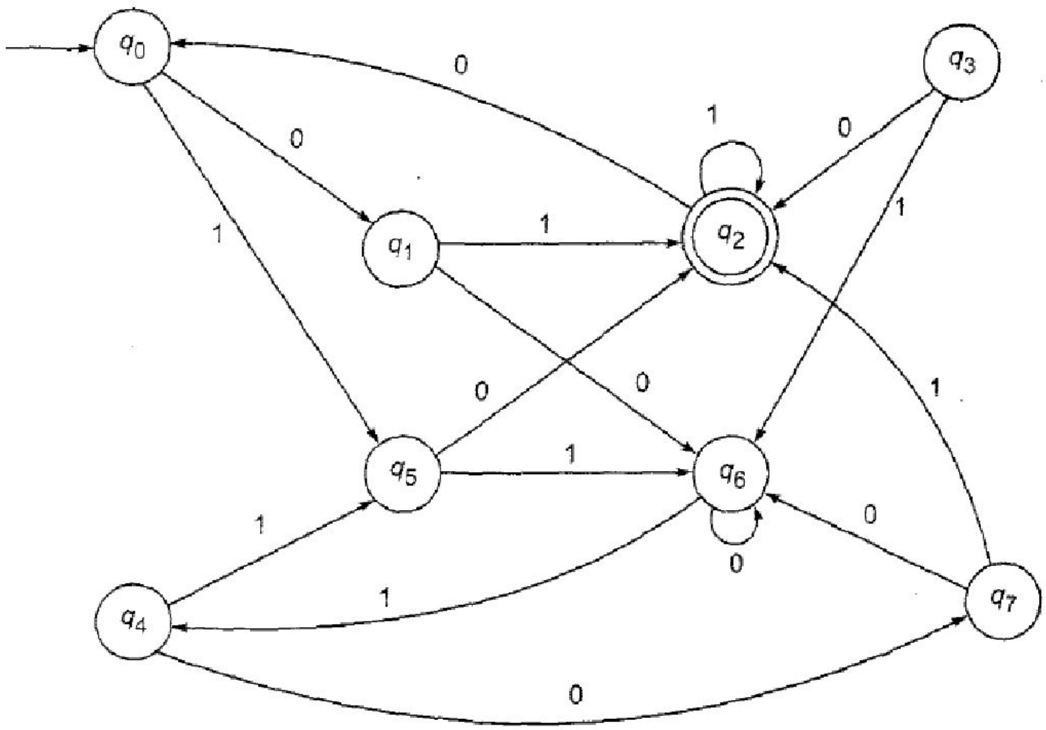
TABLE 3.16 Moore Machine of Example 3.11

Present state	*Next state*		*Output*
	a = 0	*a* = 1	
$\rightarrow q_1$	q_1	q_2	0
q_2	q_1	q_3	0
q_3	q_1	q_3	1

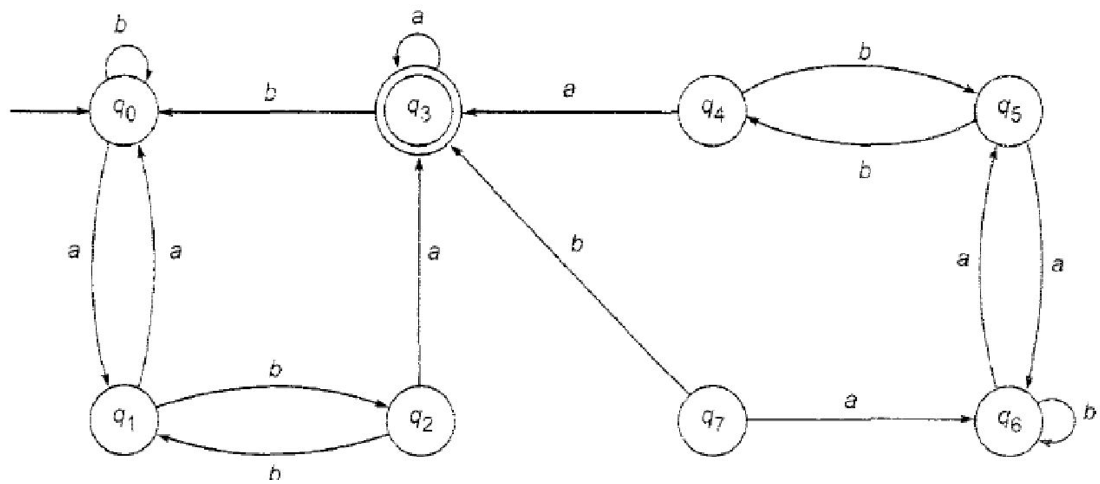
Consider a Mealy machine represented by Fig. 3.10. Construct a Moore machine equivalent to this Mealy machine.



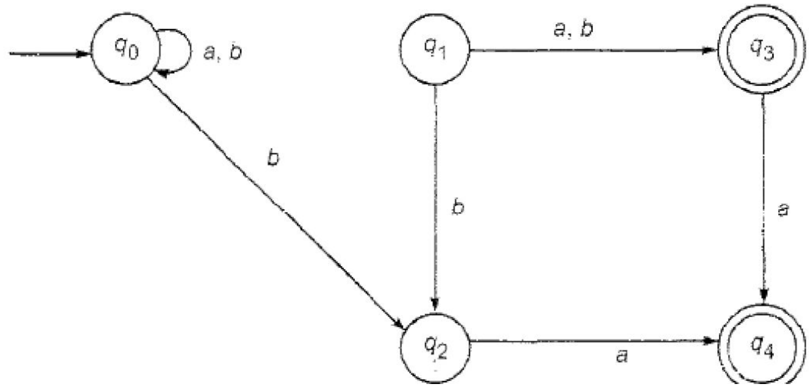
Construct a minimum state automaton equivalent to the finite automaton described by Fig. 3.12.



Construct the minimum state automaton equivalent to the transition diagram given by Fig. 3.14.



Construct a DFA equivalent to the NDFA M whose transition diagram is given by Fig. 3.16.



Construct a DFA equivalent to an NDFA whose transition table is defined by Table 3.27.

TABLE 3.27 Transition Table of NDFA for Example 3.16

State	a	b
q_0	q_1, q_3	q_2, q_3
q_1	q_1	q_3
q_2	q_3	q_2
q_3	—	—

Construct a DFA accepting all strings w over $\{0, 1\}$ such that the number of 1's in w is 3 mod 4.

Solution

Let M be the required NDFA. As the condition on strings of $T(M)$ does not at all involve 0, we can assume that M does not change state on input 0. If 1 appears in w $(4k + 3)$ times, M can come back to the initial state, after reading 4 1's and to a final state after reading 3 1's.

The required DFA is given by Fig. 3.17.

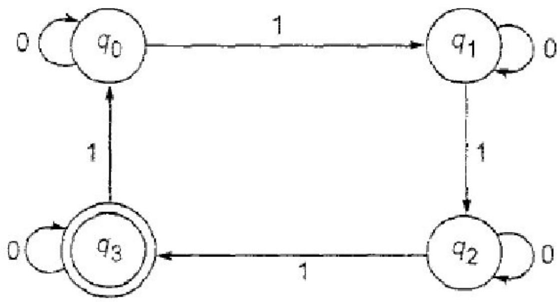


Fig. 3.17 DFA of Example 3.17.

Construct a DFA accepting all strings over $\{a, b\}$ ending in ab.

Solution

We require two transitions for accepting the string ab. If the symbol b is processed after aa or ba, then also we end in ab. So we can have states for

remembering aa, ab, ba, bb. The state corresponding to ab can be the final state in our DFA. Keeping these in mind we construct the required DFA. Its transition diagram is described by Fig. 3.18.

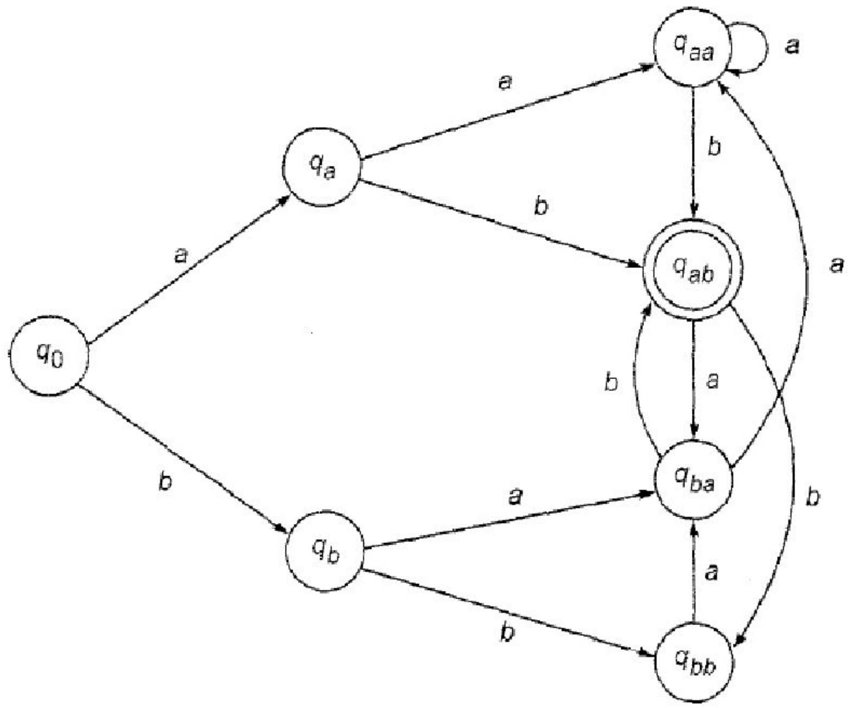


Fig. 3.18 DFA of Example 3.18.

Construct a minimum state automaton equivalent to an automaton whose transition table is defined by Table 3.29.

TABLE 3.29 DFA of Example 3.20

State	*a*	*b*
$\to q_0$	q_1	q_2
q_1	q_1	q_3
q_2	q_3	q_4
q_3	q_1	q_5
q_4	q_4	q_2
q_5	q_6	q_6

Construct a minimum state automaton equivalent to a DFA whose transition table is defined by Table 3.30.

TABLE 3.30 DFA of Example 3.21

State	*a*	*b*
$\to q_0$	q_1	q_2
q_1	q_4	q_3
q_2	q_4	q_3
q_3	q_5	q_6
q_4	q_7	q_6
q_5	q_3	q_6
q_6	q_6	q_6
q_7	q_4	q_6

Solution

$$Q_1^0 = \{q_3, q_4\}, \quad Q_2^0 = \{q_0, q_1, q_2, q_5, q_6, q_7\}$$

$$\pi_0 = \{\{q_3, q_4\}, \{q_0, q_1, q_2, q_5, q_6, q_7\}\}$$

q_3 is 1-equivalent to q_4. So, $\{q_3, q_4\} \in \pi_1$.

q_0 is not 1-equivalent to q_1, q_2, q_5 but q_0 is 1-equivalent to q_6.

Hence $\{q_0, q_6\} \in \pi_1$. q_1 is 1-equivalent to q_2 but not 1-equivalent to q_5, q_6 or q_7. So, $\{q_1, q_2\} \in \pi_1$.

q_5 is not 1-equivalent to q_6 but to q_7. So, $\{q_5, q_7\} \in \pi_1$

Hence,

$$\pi_1 = \{\{q_3, q_4\}, \{q_0, q_6\}, \{q_1, q_2\}, \{q_5, q_7\}\}$$

q_3 is 2-equivalent to q_4. So, $\{q_3, q_4\} \in \pi_2$.

q_0 is not 2-equivalent to q_6. So, $\{q_0\}$, $\{q_6\} \in \pi_2$.

q_1 is 2-equivalent to q_2. So, $\{q_1, q_2\} \in \pi_2$.

q_5 is 2-equivalent to q_7. So, $\{q_5, q_7\} \in \pi_2$.

Hence,

$$\pi_2 = \{\{q_3, q_4\}, \{q_0\}, \{q_6\}, \{q_1, q_2\}, \{q_5, q_7\}\}$$

q_3 is 3-equivalent to q_4; q_1 is 3-equivalent to q_2 and q_5 is 3-equivalent to q_7. Hence,

$$\pi_3 = \{\{q_0\}, \{q_1, q_2\}, \{q_3, q_4\}, \{q_5, q_7\}, \{q_6\}\}$$

As $\pi_3 = \pi_2$, the minimum state automaton is

$$M' = (Q', \{a, b\}, \delta', [q_0], \{[q_3, q_4]\})$$

where δ' is defined by Table 3.31.

TABLE 3.31 Transition Table of DFA for Example 3.21

State	*a*	*b*
$[q_0]$	$[q_1, q_2]$	$[q_1, q_2]$
$[q_1, q_2]$	$[q_3, q_4]$	$[q_3, q_4]$
$[q_3, q_4]$	$[q_5, q_7]$	$[q_6]$
$[q_5, q_7]$	$[q_3, q_4]$	$[q_6]$
$[q_6]$	$[q_6]$	$[q_6]$