

1. Mathematical Logic

A. Activities

1. Carry out the following activity.

p	q	r	$q \rightarrow r$	$r \rightarrow p$	$(q \rightarrow r) \vee (r \rightarrow p)$
T	T	T	T	T	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	T	F	T
F	T	T	T	T	T
F	T	F	F	F	F
F	F	T	T	T	T
F	F	F	T	T	T

The given statement pattern is ...**Tautology**

2. Using algebra of statements, complete the following activity to show

$$[p \vee (\sim p \vee \sim q)] \wedge [p \vee (q \wedge r)] \equiv p \vee (q \wedge r)$$

$$\text{LHS} \equiv [p \vee (\sim p \vee \sim q)] \wedge [p \vee (q \wedge r)]$$

$$\equiv [(p \vee \sim p) \vee \sim q] \wedge [p \vee (q \wedge r)] \text{ (Associative law)}$$

$$\equiv [(T \vee \sim q) \wedge [p \vee (q \wedge r)]] \text{ (Identity law)}$$

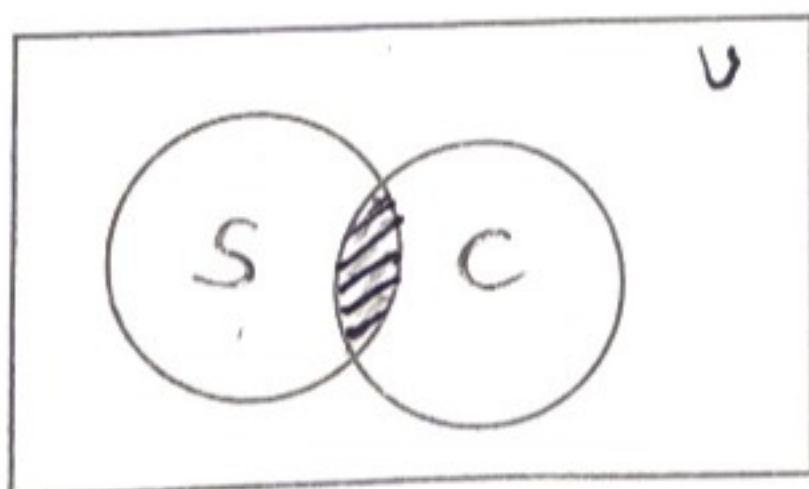
$$\equiv t \wedge [p \vee (q \wedge r)] \text{ (Identity law)}$$

$$\equiv p \vee [q \wedge r] \text{ (Identity law)}$$

$$\equiv \text{RHS}$$

3. Complete the following venn diagrams.

- i) Some share brokers are chartered accountants.



Where U : Set of all human beings.

S : Set of all share brokers.

C : Set of all chartered accountants

(denote the corresponding sets and represent region)

- ii) 'Some integers are natural numbers'



Where U : Set of all real numbers.

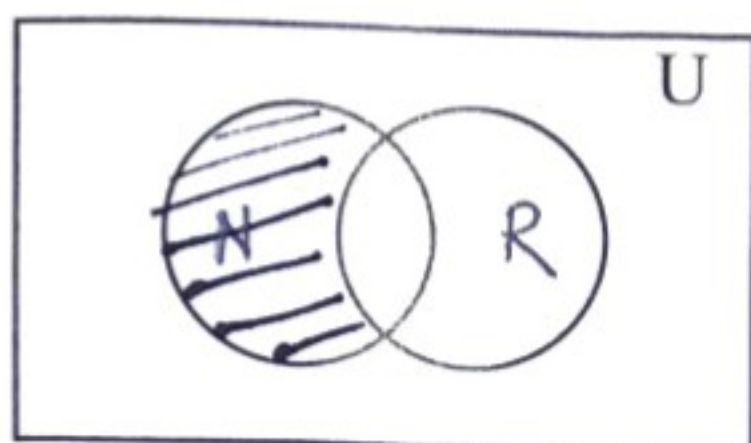
I : Set of all integers.

N : Set of all natural numbers

$N \subset I$

(Show the sets and shade the region at the appropriate place)

iii) Some non resident Indians are not rich



Where U : Set of all Indians.

N : Set of all non resident Indians.

R : Set of all rich Indians.

(Show/represent the appropriate region)

4. Determine the truth value of the following statement by completing the activity. Satara is in Maharashtra or $3 \times 7 = 21$

Statements	Truth Values
p : Satara is in Maharashtra	<input type="text" value="T"/>
q : $3 \times 7 = 21$	<input type="text" value="T"/>

The symbolic form of given statement is . Its truth value is

5. a) Find the negation of the following by completing the activity.

$$\sim [(p \leftrightarrow q) \vee (\sim q \rightarrow \sim r)]$$

$$\equiv \sim [p \leftrightarrow q] \wedge \sim [\sim q \rightarrow \sim r] \text{ (Negation of disjunction)}$$

$$\equiv [(p \wedge \boxed{\sim q}) \vee (q \wedge \boxed{\sim p})] \wedge [\sim q \wedge \sim (\boxed{\sim r})]$$

(Negation of implication and Negation of biconditional)

$$\equiv [(p \wedge \sim q) \vee (\boxed{q} \wedge \sim p)] \wedge [\sim q \wedge \boxed{r}]$$

(Negation of negation)

b) Write the dual of the following by completing the following activity.

i) Dual of $(\sim p \wedge q) \vee (p \wedge \sim q) \vee (\sim p \wedge \sim q)$ is $(\sim p \boxed{\vee} q) \boxed{\wedge}$

$(p \vee \sim q) \boxed{\wedge} (\sim p \vee \boxed{\sim q})$

ii) Dual of $(p \rightarrow q) \vee (q \rightarrow p)$ is dual of $(\sim p \vee q) \boxed{\vee} (\sim q \boxed{\vee} p)$

$(\sim p \wedge q) \wedge (\sim q \wedge p)$

B. Solve the Following

Q.1. Using truth table, examine whether $(p \rightarrow q) \leftrightarrow (\sim p \vee q)$ is a tautology, contradiction or neither.

p	q	$p \rightarrow q$	$\sim p$	$(\sim p \vee q)$	$(p \rightarrow q) \leftrightarrow (\sim p \vee q)$
T	T	T	F	T	T
T	F	F	F	F	T
F	T	T	T	T	T
F	F	T	T	T	T

From the last column, the truth table contains all true values i.e 'T'

∴ The given statement pattern is Tautology.

Q.2. Prove using truth table that : $(p \leftrightarrow q) \equiv [(\sim p \vee q) \wedge (\sim q \vee p)]$

Solⁿ:

①	②	③	④	⑤	⑥	⑦	⑧
P	q	$p \leftrightarrow q$	$\sim p$	$(\sim p \vee q)$	$\sim q$	$(\sim q \vee p)$	$A \leftrightarrow B$
T	T	T	F	T	F	T	T
T	F	F	F	F	T	T	F
F	T	F	T	T	F	F	F
F	F	T	T	T	T	T	T

From Table, column no ③ and ⑧ have identical Truth values

∴ $(p \leftrightarrow q) \equiv [(\sim p \vee q) \wedge (\sim q \vee p)]$ hence verified.

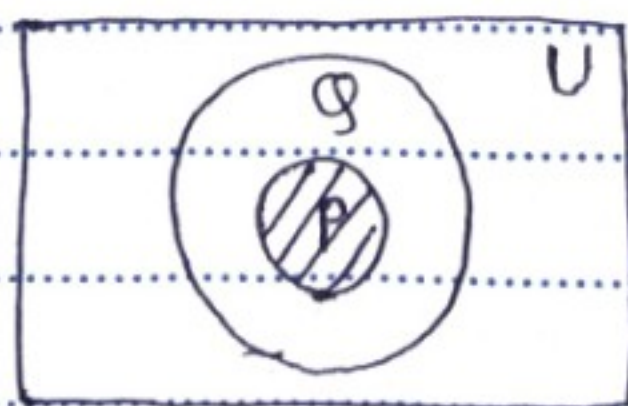
Q.3. Represent the following statements by Venn diagrams.

(a) All poets are great artists.

Solⁿ: Let U : Set of all human beings

P : Set of all poets

Q : Set of great artists.



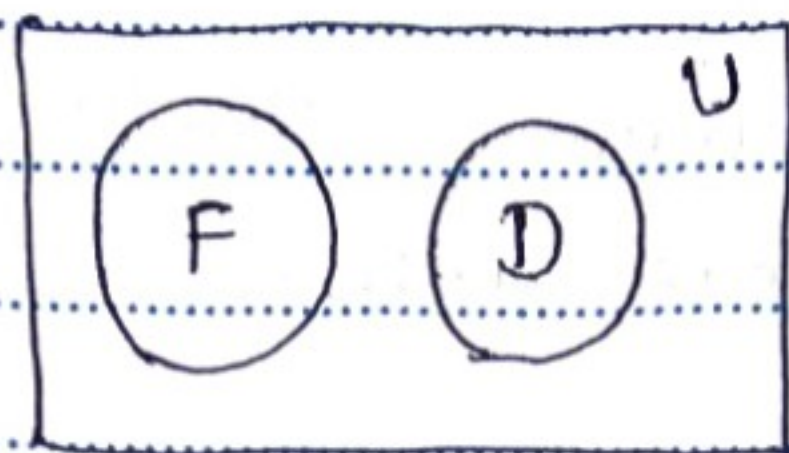
$P \subset Q$.

(b) No Filmstar is a director.

Solⁿ: Let U : Set of all human beings.

F : Set of all Filmstar

D : Set of all director.



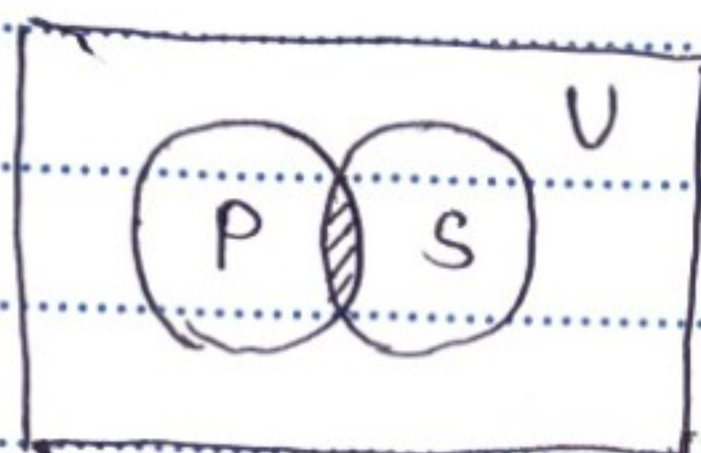
$$F \cap D = \emptyset$$

(c) Some policemen are scholars.

Solⁿ: Let U : Set of all Human beings

P : Set of all policemen

S : Set of all scholars



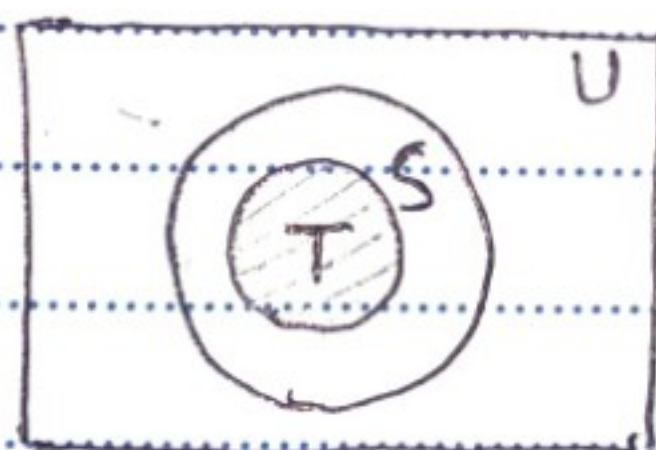
$$P \cap S \neq \emptyset$$

(d) If the person is a teacher then the person is sincere.

Solⁿ: Let U : Set of all Human beings

T : Set of all Teachers

S : Set of all sincere person.



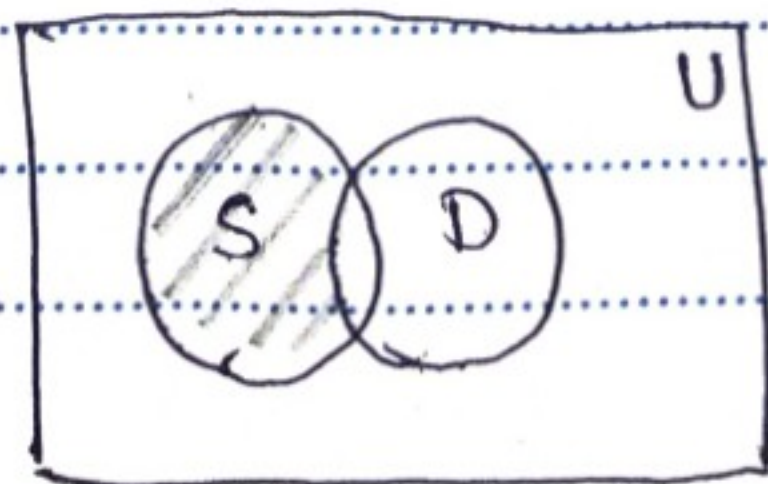
$$T \subset S$$

(e) Every sincere person may not be a doctor.

Solⁿ: Let U : Set of all human beings

S : Set of ^{all} sincere person

D : Set of ^{all} doctors



$$S - D \neq \emptyset$$

Q.4. Consider the following statements.

- If a man is rich then he buys a car.
- If a man is not rich then he does not buy a car.
- If a man buys a car then he is rich.
- If a man does not buy a car, then he is not rich.

Identify the statements having the same meaning.

Solⁿ: P : A man is rich.

Q : He buys a car

	P	Q	$\sim P$	$\sim Q$	a	b	c	d
a) $P \rightarrow Q$					$P \rightarrow Q$	$\sim P \rightarrow \sim Q$	$Q \rightarrow P$	$\sim Q \rightarrow \sim P$
b) $\sim P \rightarrow \sim Q$	T	T	F	F	T	T	T	T
c) $Q \rightarrow P$	T	F	F	T	F	T	T	F
d) $\sim Q \rightarrow \sim P$	F	T	T	F	T	F	F	T
	F	F	T	T	T	T	T	T

From Table statement (a) and (d) AND (b), (c) have same meaning.

Q.5. State the dual of each of the following statements by applying the principle of duality.

- If Shantanu passes in Accountancy, then Pratik passes in Mathematics.
- $p \vee (q \vee r) \equiv [(p \wedge q) \vee (r \vee s)]$
- $(p \rightarrow q) \vee (q \rightarrow p)$
- If Darshana drives the car, then Payal will walk. $P \rightarrow Q = \sim P \vee Q$

Solⁿ: (a) P : Shantanu passes in Accountancy

Q : Pratik passes in Mathematics.

Symbolic form is $P \rightarrow Q \equiv \sim P \vee Q$ dual is $\sim P \wedge Q$

\therefore Dual is: Shantanu does not pass in Accountancy and Pratik passes in Mathematics.

(b) $P \wedge (Q \wedge R) \equiv [(P \vee Q) \wedge (R \wedge S)]$

(c) $(P \rightarrow Q) \vee (Q \rightarrow P) \equiv (\sim P \vee Q) \vee (\sim Q \vee P)$

\therefore Dual is $(\sim P \wedge Q) \wedge (\sim Q \wedge P)$

(d) Symbolic form $P \rightarrow Q \equiv \sim P \vee Q$ dual is $\sim P \wedge Q$

\therefore Darshana ^{does not} drives the car and Payal will walk ^{does not}.

Sign of Teacher: