# APL720 - Computational Fluid Dynamics
**Lab 3: Evaluation on January 30, 2025**

# Problem statement: FDM for 1D wave equation

Consider the 1D first-order wave equation under the following conditions:

## Governing equation

$$\frac{\partial u}{\partial t} + c\,\frac{\partial u}{\partial x} = 0, \tag{1}$$

where:

- $u(x,t)$ is the scalar wave function,
- $c$ is the wave speed (constant $c > 0$),
- $L = 1$ is the length of the domain.

## Boundary and initial conditions

- Boundary conditions:

$$u(0,t) = 0, \quad u(L,t) = 0, \quad t > 0.$$

- Initial condition:

$$u(x,0) = \sin\left(\frac{2\pi x}{L}\right), \quad x \in [0, L].$$

## Tasks and expected outcomes

**A.** Write computer programs to solve the resulting system of algebraic equations for $N$ grid points (excluding the boundary points), marching from initial time $t = 0$ to a final time $t = t_f$, using:

1. Forward differencing in space and forward differencing in time. Ensure the Courant-Friedrichs-Lewy (CFL) condition is satisfied:

$$\text{CFL} = \frac{c\Delta t}{\Delta x} \leq 1.$$

2. Backward differencing in space and forward differencing in time.

**B.** Compare the numerical results with the analytical solution:

$$u(x,t) = \sin\left(\frac{2\pi(x - ct)}{L}\right).$$

**C.** Comment on the numerical stability, accuracy, and any observed differences between the two schemes.