APL720 - Computational Fluid Dynamics

Lab 6: Evaluation on March 20, 2025

Problem 1

Consider the domain of size $L_x \times L_y$ as shown in the schematic below. A Gaussian concentration profile of a pollutant is applied at the left boundary ($x = 0$). The rest of the domain initially contains no pollutant. A constant velocity field (u, v) is imposed on this domain along with assuming that a fluid with diffusion coefficient D fills the entire domain. Simulate how the pollutant spreads until a steady state is reached.

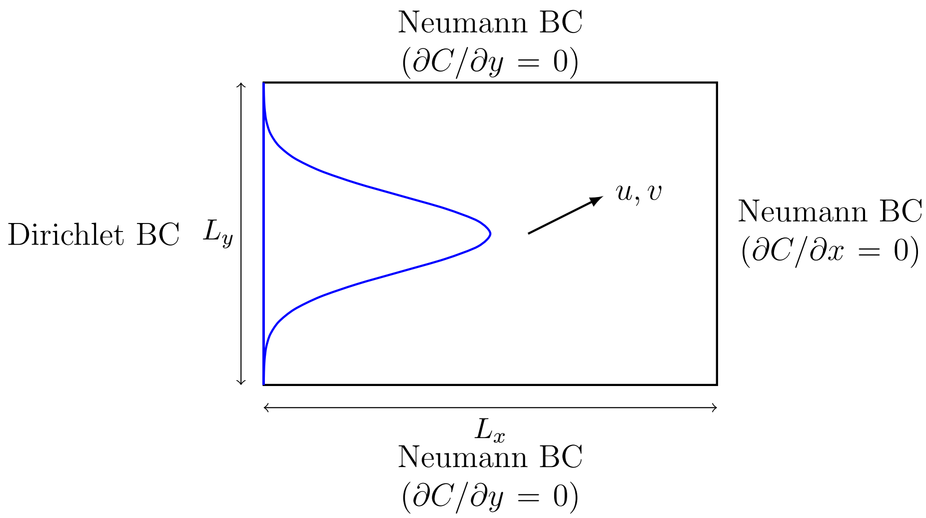

Figure 1: Schematic of the domain and boundary conditions with approximate Gaussian profile at the left boundary.

Governing equation

Solve the governing equation marked as (1).

Boundary conditions

- **Dirichlet condition at the left boundary ($x = 0$):** Use the second equation, where $C_0 = 1$ is the peak concentration, y_c is the midpoint along the y-axis, and σ controls the spread of the concentration and may be taken as $\sigma = \frac{L_y}{5}$.
- **Neumann conditions at other boundaries:** See the third equation.

Expected outcomes

1. Solve the governing equation using FVM with QUICK scheme for discretization. Plot the steady-state concentration field as a contour plot. Inputs for L_x, L_y, N_x, N_y, u, v and D will be provided at the time of evaluation.
2. Compare the concentration profile obtained at the right boundary with that at the left boundary.

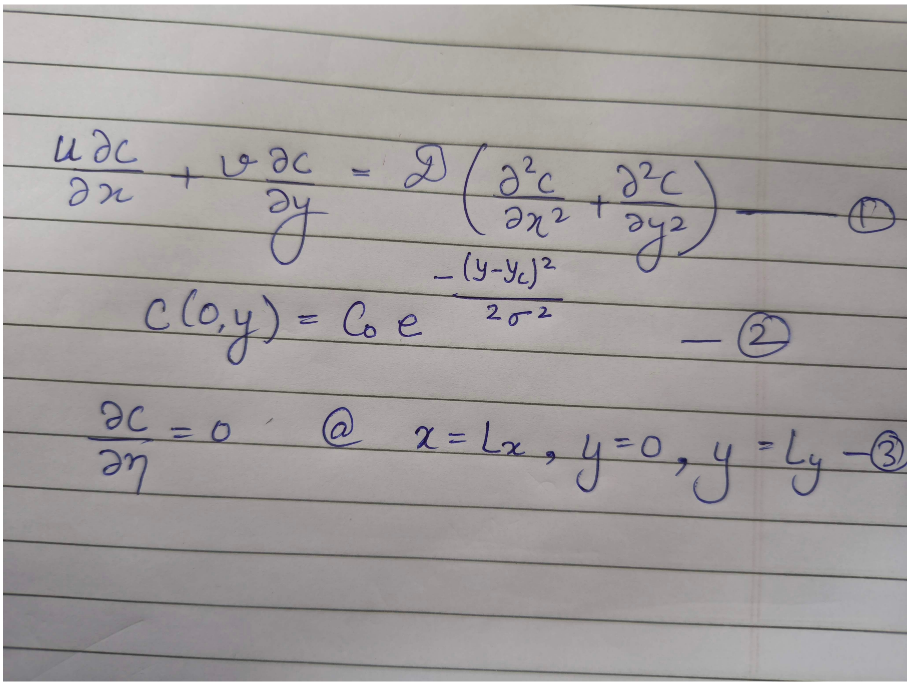

Figure 2: Equations used for problem 1.

Problem 2

The Laplace equation below is to be solved using the Gauss-Seidel (GS) method for $\phi(x, y)$ in a square domain: $0 < x < 1, 0 < y < 1$. The boundary condition is $\phi = 0$ on all boundaries.

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0. \tag{1}$$

(a) Program the Gauss-Seidel method using equal number of grid cells in x and y directions i.e. $\Delta x = \Delta y$. The initial guess for the solution is $\phi = sin(\pi x) + sin(\pi y)$. Plot the relative error defined by $E(k) = \frac{||\epsilon^{(k)}||}{||\epsilon^{(0)}||}$ as a function of number of iterations, n_{it}. Here, $||\epsilon^{(k)}|| = \sqrt{\sum_{i,j}(\phi_{i,j}^{k+1} - \phi_{i,j}^{k})^2}$ is the root-square error and $\sum_{i,j}$ implies summation over all grid points in the domain. How many number of iterations would it take for reducing the error by two orders of magnitude i.e. $E(k) = 10^{-2}$ using 32 x 32 grid ?

(b) Vary the grid size and see how the convergence rate changes for various grids.

(c) Repeat part (a) for an initial guess, $\phi = sin(8\pi x) + sin(8\pi y)$.