APL 720: Lab 8

Submission deadline: 17th April, 2025

The one-dimensional diffusion equation to be solved using multigrid V cycles is given by:

$$k\frac{d^2\phi}{dx^2} = -f(x), \quad \text{for } x \in [0,1]$$

with boundary conditions:

$$\phi(0) = 0, \quad \phi(1) = 1.$$

For this problem, let $k = 1$ and $f(x) = 1$.

(a) Write a MATLAB/Python function to discretize the equation using finite volume method (FVM) and generate the system of algebraic equations of the form:

$$Ux = b$$

as a function of the grid size (h) or the number of cell volumes (n). Assume a uniform grid spacing.

(b) Write another function to solve the system $Ux = b$ using the Gauss-Seidel iterative method. Ensure correctness by comparing results with the analytical solution.

(c) Implement a program for the multigrid V-cycle to improve rate of convergence. For this, you must utilize the functions from parts (a) and (b). Use a predefined number of iterations at each V-cycle level. Consider:

- The number of cell volumes in the finest grid as $n = 20$.
- The number of cell volumes in the coarse grid as $n = 10$, and the number of cell volumes in the coarsest grid as $n = 5$ for restriction and prolongation steps.