APL720 - Computational Fluid Dynamics

Lab 1: 1D Steady-State Heat Diffusion

Problem Statement

Consider a 1D rod of length $L = 1\,\text{m}$, with the following conditions:

- **Governing Equation:**

$$\frac{d^2T}{dx^2} + \frac{q}{k} = 0 \tag{1}$$

where q is the uniform heat generation rate, and k is the thermal conductivity of the material.

- **Boundary Conditions:**

$$T(0) = T_0, \quad T(L) = T_L.$$

Expected Outcomes

1. Write a program to solve the resulting system of algebraic equations for N grid points (excluding the boundary points) using any second-order finite-difference method. Your code should take the values of N, q, k, T_0 and T_L as user inputs and should plot the temperature distribution along the rod (T vs x).

2. Now, obtain the numerical solution for $N = 11$, $N = 21$, $N = 41$ and $N = 81$ for fixed but non-zero values of q and k. Plot the resulting logarithmic root mean square error, $\log(\epsilon_{rms})$, as a function of $\log(h)$, where h is the grid spacing given by $h = L/(N-1)$. Comment on the order of accuracy of the finite difference approximation based on the plot.

3. How would the temperature distribution change if the boundary condition at $x = L$ is changed to $\left(\frac{dT}{dx}\right)_L = 0$? Demonstrate and compare graphically with case 1 for same number of grid points.

Evaluation

- During the evaluation, students are expected to explain the algorithm and run the code with parameters provided by the evaluator. The output of the code must align with the "Expected Outcomes." Any deviations may result in a reduction of marks.

- All theoretical derivations and analyses, such as forming algebraic equations upon which the code is based, must be documented in a notebook that MUST be presented during the evaluation.

- An integral part of this lab evaluation is the student's ability to modify the code, demonstrating their understanding of the program. Evaluators may request minor changes and modifications during the evaluation, which could affect the overall score.

- Any form of suspected plagiarism in the code will result in zero marks for the lab.