APL720 - Computational Fluid Dynamics

Lab 2: 1D Unsteady Heat Diffusion

Problem Statement

Consider a 1D rod of length $L = 1\,\text{m}$, with the following conditions:

- **Governing Equation:**

$$\frac{\partial T}{\partial t} = \alpha \, \frac{\partial^2 T}{\partial x^2} \tag{1}$$

  where $\alpha = 1$, and $T(x, t)$ is the time dependent temperature distribution in the rod.

- **Boundary Conditions:**

$$T(0, t) = T_0, \quad T(L, t) = T_L.$$

- **Initial Conditions:**

$$T(x, 0) = T_0 \left(1 - \frac{x}{L}\right)^2. \qquad x \in (0, L)$$

Expected Outcomes

1. Write computer programs to solve the resulting system of algebraic equations for N grid points (excluding the boundary points), marching from initial time $t = 0$ to a final time $t = t_f$ using:

   a) second-order FDM in space, and explicit Euler method in time.

   b) second-order FDM in space, and implicit Euler method in time.

   c) second-order FDM in space, and Crank-Nicholson method in time.

   Your codes should take the values of N, t_f, T_0 and T_L as user inputs and should plot the temperature distribution at final time along the length of the rod. For cases (a) and (b), the time step, Δt, should be automatically chosen based on the Fourier number, $Fo = \frac{\alpha \Delta t}{h^2}$ where h is the grid spacing. For case (c), the time step can be taken same as case (a).

2. Graphically demonstrate the stability criterion for explicit Euler and Crank-Nicholson methods considering different values of time step, Δt.

3. Compare the temporal evolution of temperature at the center $T\left(\frac{L}{2}, t\right)$, using the three different schemes in a single plot. Your plot should contain three datasets (lines) corresponding to the three schemes. It should be properly labelled with a suitable legend through which visual distinction can be made.