

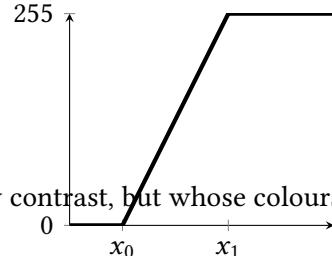
COL7(6)83 Mid-Semester Exam

14 September, 2025

1. Give a brief answer (with justification) for each of the following questions:

- (a) Consider the intensity transformation shown below, with $0 < x_0 < x_1 < 255$. What happens if it is repeatedly applied to an image?

Answer: Contrast will be increased more and more (0.5 marks)
until dark pixels tend to black and light pixels tend to white
(0.5 marks)



- (b) Explain how to enhance an RGB image which has good intensity contrast, but whose colours appear dull and faded and should be made more vibrant.

Answer: Convert to HSI and increase saturation (1 mark) OR something similar in RGB (1 mark), e.g. move each color $c = (r, g, b)$ away from equivalent grey $c' = (\frac{r+g+b}{3}, \frac{r+g+b}{3}, \frac{r+g+b}{3})$ by changing to $c + k(c - c')$.

- (c) Is bilateral filtering effective at removing salt-and-pepper noise? Why or why not?

Answer: No (0.5 marks) because for salt or pepper pixels no nearby pixels will be given significant weights (0.5 marks).

- (d) What is the Radon transform $g_\theta(\rho)$ of a disk function, $f(x, y) = \begin{cases} A & \text{if } x^2 + y^2 \leq r^2, \\ 0 & \text{otherwise?} \end{cases}$

Answer: For any θ , $g_\theta(\rho)$ is integral of f over line at distance ρ (0.5 marks) which is $\begin{cases} 2A\sqrt{r^2 - \rho^2} & \text{if } \rho < r \\ 0 & \text{otherwise} \end{cases}$ (0.5 marks).

If solution not found, 0.5 marks for just giving $g_\theta(\rho) = \iint f(x, y) \delta(x \cos \theta + y \sin \theta - \rho) dx dy$.

2. Suppose we wish to design a spatial kernel w such that convolution with it sharpens an image; specifically, fine details are enhanced while the intensity in constant regions is unaffected.

- (a) It is claimed that for such a kernel, (i) its entries should sum to 1, and (ii) all its entries should be nonnegative. Are these claims true? Justify your answer for both claims separately.

Answer: (i) Yes, entries should sum to 1 (0.5 marks) because if $f(x, y) = c$ then output is $g(x, y) = \sum_{i,j} c w_{ij} = c \sum_{i,j} w_{ij}$ (0.5 marks).

(ii) No, not all entries should be nonnegative (0.5 marks), if they were then outcome would be weighted averaging and cause smoothing/blurring (0.5 marks).

- (b) Give the kernel that corresponds to unsharp masking with a 5×5 mean filter.

Answer: Unsharp masking: $g = f + k(f - h * f)$ where h is smoothing filter (0.5 marks).

For 5×5 mean filter, $h = \begin{bmatrix} \frac{1}{25} & \frac{1}{25} & \dots \\ \frac{1}{25} & \frac{1}{25} & \dots \\ \vdots & \vdots & \ddots \end{bmatrix}$ (0.5 marks). Corresponding filter: $g = w * f$ where

$$w = \delta + k(\delta - h) = \begin{bmatrix} \frac{-k}{25} & \frac{-k}{25} & \dots \\ \frac{-k}{25} & \frac{-k}{25} & \dots \\ \vdots & \vdots & \ddots \end{bmatrix} \text{ with middle entry } (1+k) - \frac{k}{25} \text{ (1 mark for either correct formula or correct result).}$$

3. (a) Suppose a function f is separable, i.e. $f(t, z) = f_1(t)f_2(z)$, and the corresponding 1D continuous Fourier transforms are $f_1 \Leftrightarrow F_1, f_2 \Leftrightarrow F_2$. Using the definition of the 2D Fourier transform, find F such that $f(t, z) \Leftrightarrow F(\mu, v)$.

Answer: $F(\mu, v) = \iint f(t, z) \exp(-2\pi i(\mu t + v z)) dt dz$ (0.5 marks for definition)

$$= \iint f_1(t)f_2(z) \exp(-2\pi i\mu t) \exp(-2\pi i v z) dt dz = \left(\int f_1(t) \exp(-2\pi i\mu t) dt \right) \left(\int f_2(z) \exp(-2\pi i v z) dz \right) = F_1(\mu)F_2(v) \text{ (2 marks)}$$

- (b) Let g_1 and g_2 be Gaussian functions of standard deviation σ_1 and σ_2 respectively. A continuous signal f is smoothed by first convolving with g_1 and then with g_2 . Using the Fourier transform, show that the result is equivalent to convolving f once with a Gaussian, and find its standard deviation.

You may use the fact that $k \exp(-at^2) \Leftrightarrow K \exp(-\pi^2 \mu^2/a)$; the exact value of K shouldn't matter.

Answer: Result is $h = g_2 * (g_1 * f) = (g_2 * g_1) * f$, in frequency domain $H = G_2G_1F$ (0.5 marks, different order is also OK). Gaussians are $g_i(t) = k_i \exp(-t^2/(2\sigma_i^2))$ (0.5 marks), so in frequency domain $G_i(\mu) = K_i \exp(-2\pi^2 \sigma_i^2 \mu^2)$.

Equivalent filter is $G_3(\mu) = G_1(\mu)G_2(\mu) = K_1K_2 \exp(-2\pi^2(\sigma_1^2 + \sigma_2^2)\mu^2)$ (0.5 marks for correctly multiplying, even if G_1 and G_2 themselves were incorrect).

Corresponds to spatial filter $g_3(t) = k_3 \exp(\mu^2/(2(\sigma_1^2 + \sigma_2^2)))$, which is Gaussian with s.d. $\sigma = \sqrt{\sigma_1^2 + \sigma_2^2}$ (0.5 marks)

4. Suppose you are given a 800×600 image which is corrupted by sinusoidal horizontal stripes with a period of 10 pixels.

- (a) At what frequency(ies) (u, v) will there be spikes in the Fourier transform? Give a concrete formula for a notch filter $H(u, v)$ that will remove them.

Answer: Assuming x -axis is vertical and image height is 800 (please permit any other coordinate convention if student has specified), stripes are of form $\eta(x, y) = A \sin(2\pi x/10)$. DFT expresses image as sum of sinusoids of form $\exp(2\pi i(ux/M + vy/N))$; comparing coefficients we have $u/M = 1/10 \implies u = M/10 = 80$ and $v = 0$ (1 mark, if they got $u = 1/10$ using continuous FT then 0.5 marks). By symmetry we have spikes at both $(80, 0)$ and $(-80, 0)$ (0.5 marks). Answer of $(M/2 \pm 80, N/2)$ is also valid if DFT is centered.

For notch filter, any answer is permitted if equivalent to $H(u, v) = HP(u - 80, v)HP(u + 80, v)$ (0.5 marks) with HP being some high-pass filter e.g. $1 - \exp(-(u^2 + v^2)/2D_0^2)$ (0.5 marks).

- (b) Now say the image has both periodic noise which you intend to remove using the notch filter above, and additive Gaussian noise which you intend to remove using a 5×5 mean filter. Will

you get better results if you apply the notch filter first, or the mean filter first, or will you get identical results either way? Justify your answer.

Answer: Notch filter = multiplication with H in frequency domain, mean filter = convolution with w = multiplication with W in frequency domain (0.5 marks). So final result is same, $G = HWF = WHF$ in both cases (1 mark).

5. Suppose you have a camera that always takes blurry and noisy photographs. The blur can be modeled as a fixed (but unknown) linear position-invariant degradation, while the noise is additive with a fixed (unknown) distribution. Design a procedure by which you can improve the quality of the images as much as possible. It may involve taking photographs of known scenes to calibrate the procedure, but after calibration, the procedure should be completely automated. Give full technical details of both the initial calibration steps, and the process for restoring a new photograph taken by the camera.

Answer: Calibration of noise: take image of constant intensity scene, find intensity variance = $\text{var}(\eta)$ (1 mark).

Calibration of blur: take image of small bright point light, resulting image is noisy version of PSF h and its Fourier transform is H (1 mark). Student should give some suggestion for reducing noise (0.5 marks), e.g. taking multiple images of point light and averaging.

Automated restoration: Apply Wiener filter $W = H^*/(|H|^2 + S_\eta^2/S_f^2)$. (1 mark). Student should give some way of setting S_η^2/S_f^2 (0.5 marks), e.g. saying $S_\eta^2(u, v)$ is proportional to $\text{var}(\eta)$, or replacing with constant $S_\eta^2/S_f^2 = K$ chosen for best results on some representative scenes.